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Turning Dreams into Reality: Current Trends in Mathematics, Science and Computer Science Education

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The Seminar under the theme “Turning Dreams into Reality: Current Trends in Mathematics, Science and Computer Science Education” is conducted by Faculty of Mathematics and Science Education, UPI at October 19, 2013. The aim of the seminar is to provide a forum where teachers and researchers can exchange didactical, pedagogical, and epistemological ideas on mathematics, science, and computer science education which is expected to stimulate research in those areas. The seminar also provides an exceptional opportunity for all participants to contribute to the world of mathematics, science, and computer science education.

Some of outstanding scientists and educators from Germany, Australia, Hongkong, Malaysia, Singapore, Netherland, and Indonesia joined in this seminar made the seminar truly international in scope. There were 485 participants, had many fruitful discussions and exchanges that contributed to the success of the seminar. 153 papers discussed in the parallel session. The papers were distributed in 6 fields. 42 papers in mathematics or mathematics education, 19 papers in physics or physics education, 23 papers in chemistry or chemistry education, 25 papers in biology or biology education, 9 papers in computer science or computer science education, and are 18 papers in science education. Of the total number of presented papers, 153 included in this proceeding.

Generous support for the seminar was provided by SEAMEO QITEP in Science and Himpunan Sarjana dan Pemerhati Pendidikan IPA Indonesia. The support permitted us to gave an opportunity for a significant number of young scientists and persons from many universities and other institutions brought new perspectives to their fields.

All in all, the seminar was very successful. We expect that these future seminar will be as stimulating as this most recent one was, as indicated by the contribution presented in this proceeding.

Chief of Organizing Committee,

Dr. Sufyani Prabawanto, M.Ed.
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1. MATHEMATICS
Goal Programming is one that is considered appropriate mathematical model used to resolve cases that have more than one target. In this model, there is a deviation variables in the constraint functions are used to accommodate deviations completion results to be achieved against the targets aberration results in the completion of the target and also below the target. If the target is a deviation in the desired conditions, then that is minimized under the target, and vice versa if deviations below the target, then minimized above the target. It means one of the deviation variables in the objective value equal to zero. So this variable to change the meaning of the constraints being targeted to achieve the desired goals.

Tanning industry goat and sheep skins continue to experience shortages of raw materials and the leather has been no strategic solutions to overcome them. Targets to be achieved in this study is to obtain an optimal production plan with the goals targeted by the company. The use of goal programming models in this study produces an optimal number of production where the use of the amount of raw materials remain within the limits of availability of raw materials in the company. Completion goal programming optimization model using LINDO program produces output optimal solution with benefits is Rp.2,374,436,700, 00., And save production cost of capital Rp.5,456,100, 00.
Proceedings of the International Seminar on Mathematics, Science, and Computer Science Education

means one of the deviation variables in the objective value equal to zero. So this variable to change the meaning of the constraints being targeted to achieve the desired goals.

Determination of the optimal production quantities for raw materials goat skin (sheep skin) and sheep skins (goat skin), and the result of the completion of the mathematical model using goal programming optimization problem of a raw material in tannery plant will be discussed in this study.

2. METHOD
In general, the mathematical model of goal programming models can be formulated as follows [4]:

The objective function:

\[ \text{Minimum} \sum_{i=1}^{m} DA_i + DB_i \]

Constraints:

\[ a_{i1}X_1 + a_{i2}X_2 + ... + a_{in}X_n + DB_i - DA_i = h_i \]

\[ a_{m1}X_1 + a_{m2}X_2 + ... + a_{mn}X_n + DB_m - DA_m = h_m \]

and

\[ X_j, DA_i, \text{ and } DB_j \geq 0, \text{ for } i = 1, 2, ..., m \text{ and } j = 1, 2, ..., n \]

2.1. Priority Targets

Minimization sequence deviation variables will determine the order in which the target is reached. Therefore, priority setting targets to be achieved can be done by controlling the order of selection of variables that must be minimized deviation. There are three kinds of goals in the goal programming models, namely:

a. Targets with the same priority

This model assumes that all goals are equally important so that if there are targets that have to be sacrificed in order to achieve other goals. In this case, the determination of which one must be sacrificed targets or objectives which should be achieved not so important because all targets are considered to have the same price. Because each target has the same weight, then any deviation variables can be selected to be minimized in advance.

b. Targets with different priorities

Minimization sequence deviation variables can be done without having to follow a certain rule. Therefore, the selection of priority targets can be done by selecting the deviation variables associated with the target to be minimized in advance.

c. Priorities and objectives with different weights

Different objective function coefficients will determine the level of priority targets. The greater the value of the coefficient of a variable deviation, the higher the priority. However, the higher value of the coefficient is not necessarily an obstacle to make sure targets are met, and vice versa.

In settlement of a case with a priority goal programming specific target can be a notation on any deviation variables in the objective function that can sort minimum deviation variables so that the goals can be achieved in accordance with a predetermined priority. Notation used for marking priority targets are:

\[ P_i (i = 1, 2, ..., m) \]

Where \( P_i \) is not a parameter or variable but just a notation to indicate the order of priority targets to be achieved. Thus the general form of the objective function goal programming models with priority objectives are:

\[ \text{Minimum} \sum_{i=1}^{m} R_i (DA_i + DB_i) \]

2.2. LINDO program

LINDO (Linear Interactive Discrete Optimizer) is a program designed to address the cases of linear programming [4]. A case must be first converted into a linear programming mathematical model that can be processed by the LINDO program.

1. LINDO input

The program requires input a mathematical program with specific formulations.

2. LINDO output
LINDO output or processed results can basically be separated into two parts, namely:

1) Optimal completion
   a. Value of the objective function under the label of Objective Function Value.
      This information is marked with the notation "1" to indicate that the LINDO input in the
      formulation, the objective function is placed on line 1 and the constraint functions
      ranging from sequence row 2.
   b. Optimal value of the variable under Value label.
      The decision variables are labeled with the LINDO output variables. For example, the
decision variables $X_1$ and $X_2$, then the number under value and is on the line where $X_1$ is
showing optimal values of decision variables.
   c. Sensitivity $C_j$ if $X_j = 0$ under the Reduced Cost column.
      Provide information regarding the extent to which the value of $C_j$ should be lowered so
      that the value of the decision variables to be positive. This means that the cost will
      reduce the value of the variable is always zero if a positive decision and vice versa.
   d. Slack or surplus variables under the label variable Slack or Surplus.
      This information shows the slack and surplus value of each constraint as the value of the
      objective function reaches extreme values.
   e. Dual price
      This shows information about the changes that will occur in the value of the objective
      function when changing the value of the right side of the unit constraints.

2) Sensitivity analysis
   a. Sensitivity analysis of objective function coefficients ($C_j$).
      $C_j$ sensitivity analysis to explain changes in the value of $C_j$ that will not change the
      optimal value of the decision variables.
   b. Value Right sensitivity analysis section
      Price reflects the dual objective function value changes attributable to changes in unit
      value of each right-hand side constraint active. In this case, the sensitivity analysis of the
      value of the right side (Right Hand Side) explains the right-hand side value change
      interval which ensure the validity of the dual price.

LINDO processed results also provide information on the number of iterations required to
find the optimal solution.

3. RESULT AND ANALYSIS
RESULT AND ANALYSIS

The data obtained in this study of secondary data from PT Adira Universe Industry in April
2011 for 4 (four) with an average period of approximately 1 week for each period, which includes:

a. Inventories of raw materials and leather goat leather sheep in units of the sheet can be seen in
Table 1 [7].

<table>
<thead>
<tr>
<th>No.</th>
<th>Skin type</th>
<th>Total inventory per period (sheet)</th>
<th>Total (sheet)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>1</td>
<td>Sheep skin</td>
<td>29.440</td>
<td>9.202</td>
</tr>
<tr>
<td>2</td>
<td>Goat skin</td>
<td>7.253</td>
<td>6.800</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td><strong>36.693</strong></td>
<td><strong>16.002</strong></td>
</tr>
</tbody>
</table>

Goatskin purchase price = Rp. 50,000.00 per sheet.
Sheepskin purchase price = Rp. 60,000.00 per sheet.
Purchase of raw materials each skin type in units of sheets, while sales in units sf (square feet). Where
according to SNI (Indonesian National Standard); 1SF = 0.092903 m2 = 30.48 cm x 30.48 cm.

Raw materials to be used in the production process been qualified / not defect to produce a
quality product as well with the expected quantity. On average each raw material goatskin produced 5
sf per sheet, while the raw material sheepskin to produce 6 sf per sheet with a depreciation of 5% for
each raw material leather.

With existing inventories, the estimated number of goat skin and sheep skin can be seen in
Table 2 and Table 3 [7].
Table 2: Raw Material Inventory Goat Skin

<table>
<thead>
<tr>
<th>Period</th>
<th>Total inventories</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before production (sheet)</td>
<td>After production (sf)</td>
<td>Depreciation (5%)</td>
<td>After Depreciation (sf)</td>
</tr>
<tr>
<td>I</td>
<td>29.440</td>
<td>147.200</td>
<td>7.360</td>
<td>139.840</td>
</tr>
<tr>
<td>II</td>
<td>9.202</td>
<td>46.010</td>
<td>2.301</td>
<td>43.710</td>
</tr>
<tr>
<td>III</td>
<td>8.086</td>
<td>40.430</td>
<td>2.022</td>
<td>38.409</td>
</tr>
<tr>
<td>IV</td>
<td>12.190</td>
<td>60.950</td>
<td>3.048</td>
<td>57.903</td>
</tr>
<tr>
<td>Σ</td>
<td>58.918</td>
<td>294.590</td>
<td>14.731</td>
<td>279.862</td>
</tr>
</tbody>
</table>

Then starting goatskin purchase price:
= \( 50,000 \times 58.918 = \text{Rp. 2,945,900,000.00} \)
= \( \frac{\text{Rp.2,945,900,000.00}}{279\,862} = \text{Rp. 10,526.26 per sf} \)

Table 3: Raw Material Inventory Sheep Skin

<table>
<thead>
<tr>
<th>Period</th>
<th>Total inventories</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before production (sheet)</td>
<td>After production (sf)</td>
<td>Depreciation (5%)</td>
<td>After depreciation (sf)</td>
</tr>
<tr>
<td>I</td>
<td>7.253</td>
<td>43.518</td>
<td>2.176</td>
<td>41.342</td>
</tr>
<tr>
<td>II</td>
<td>6.800</td>
<td>40.000</td>
<td>2.040</td>
<td>38.760</td>
</tr>
<tr>
<td>III</td>
<td>11.345</td>
<td>68.070</td>
<td>3.404</td>
<td>64.667</td>
</tr>
<tr>
<td>IV</td>
<td>8.733</td>
<td>52.398</td>
<td>2.620</td>
<td>49.778</td>
</tr>
<tr>
<td>Σ</td>
<td>34.131</td>
<td>204.786</td>
<td>10.240</td>
<td>194.547</td>
</tr>
</tbody>
</table>

Then the purchase price commencing sheepskin:
= \( 60,000 \times 34.131 = \text{Rp. 2,047,860,000.00} \)
= \( \frac{\text{Rp.2,047,860,000.00}}{194\,547} = \text{Rp. 10,526.3 per sf} \)

b. Total demand for each type of skin during a four month period in April 2011 is shown in Table 4 [7].

Table 4: Number of Requests Skin

<table>
<thead>
<tr>
<th>No</th>
<th>Skin Type</th>
<th>Number of requests per period (sf)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>1</td>
<td>Sheep skin</td>
<td>136.609</td>
<td>45.000</td>
</tr>
<tr>
<td>2</td>
<td>Goat skin</td>
<td>44.000</td>
<td>38.679</td>
</tr>
<tr>
<td>Jumlah</td>
<td>180.609</td>
<td>85.679</td>
<td>103.300</td>
</tr>
</tbody>
</table>
c. Production cost of each type of skin with raw skin dose of 500 sheets shown in Table 5 [7].

<table>
<thead>
<tr>
<th>No</th>
<th>Skin type</th>
<th>Sheep skin</th>
<th>Goat skin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material</td>
<td>Consumption (kg)</td>
<td>Price (Rp.)/kg</td>
</tr>
<tr>
<td>1</td>
<td>Salt</td>
<td>390.6</td>
<td>1.100</td>
</tr>
<tr>
<td>2</td>
<td>Gelon PK</td>
<td>6.3</td>
<td>38.700</td>
</tr>
<tr>
<td>3</td>
<td>Sodium Format</td>
<td>8.4</td>
<td>6.500</td>
</tr>
<tr>
<td>4</td>
<td>Sodium Bicarbonate</td>
<td>17.18</td>
<td>3.300</td>
</tr>
<tr>
<td>5</td>
<td>Oropon BRS</td>
<td>16.8</td>
<td>43.200</td>
</tr>
<tr>
<td>6</td>
<td>Formic Acid</td>
<td>4.2</td>
<td>14.000</td>
</tr>
<tr>
<td>7</td>
<td>H$_2$SO$_4$</td>
<td>4.26</td>
<td>2.250</td>
</tr>
<tr>
<td>8</td>
<td>Sodium Chlorite</td>
<td>16.8</td>
<td>14.000</td>
</tr>
<tr>
<td>9</td>
<td>Sodium Bisulph</td>
<td>2.73</td>
<td>137.50</td>
</tr>
<tr>
<td>10</td>
<td>Sodium Sulphat</td>
<td>63</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>Novaltan PF</td>
<td>8.4</td>
<td>1.250</td>
</tr>
<tr>
<td>12</td>
<td>Tannit LSW</td>
<td>6.3</td>
<td>22.080</td>
</tr>
<tr>
<td>13</td>
<td>Chromosal B</td>
<td>29.4</td>
<td>42.300</td>
</tr>
<tr>
<td>14</td>
<td>Biocid C-3</td>
<td>0.2</td>
<td>13.000</td>
</tr>
<tr>
<td>15</td>
<td>Sodium Asetat</td>
<td>8.4</td>
<td>176.53</td>
</tr>
<tr>
<td>16</td>
<td>Grassol GO</td>
<td>2.1</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>Lowatan AB</td>
<td>2.1</td>
<td>6.700</td>
</tr>
<tr>
<td>18</td>
<td>Superlutan A</td>
<td>4.2</td>
<td>38.700</td>
</tr>
</tbody>
</table>

Per 500 sheet: 3,508.35 / 2 = 3,312.118,4
Per sheet: 7,016,7 / 6,624,2
Per sf: 1,477,2 / 1,255,7
Cost production: 10,526,26 + 1,477,2 ≈ 12,000/sf

d. Sale price
Quality products for goat and sheep skin leather together, then the generalized sales price that is equal to Rp.17,000,00 per sf.

e. Advantages of each type of skin shown in Table 6 [7].

<table>
<thead>
<tr>
<th>No</th>
<th>Skin Type</th>
<th>Profit per sf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sheep skin</td>
<td>Rp. 4,800,00</td>
</tr>
<tr>
<td>2</td>
<td>Goat skin</td>
<td>Rp. 5,300,00</td>
</tr>
</tbody>
</table>

f. Budget established company PT Adira Universe Industry shown in Table 7 [7].

<table>
<thead>
<tr>
<th>Commentary</th>
<th>Corporate statutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>Rp. 2,000,000,000,00</td>
</tr>
<tr>
<td>Capital cost of production</td>
<td>Rp. 5,640,000,000,00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No</th>
<th>Skin Type</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sheep skin</td>
<td>Rp.299,022,000,00</td>
</tr>
<tr>
<td>2</td>
<td>Goat skin</td>
<td>Rp.6,000,000,00</td>
</tr>
</tbody>
</table>

g. Monthly wages of employees in the company's records as follows:
- Part of the tanning process (258, @ Rp.1,159,000,00)
- Section engineer (5 people, @ Rp.1,200,000,00)
According to the data obtained, the problem of raw material production tannery goats and sheep leather can be modeled into a goal programming models for the period of April 2011 as follows:

a. Decision variables

- \( X_{11} = \) number of goat skin production 1st period
- \( X_{12} = \) number of production goatskin 2nd period
- \( X_{13} = \) number of production goatskin 3rd period
- \( X_{14} = \) number of production goatskin 4th period
- \( X_{21} = \) number sheepskin production 1st period
- \( X_{22} = \) number of production sheepskin 2nd period
- \( X_{23} = \) number of production sheepskin 3rd period
- \( X_{24} = \) number of production sheepskin 4th period

b. Constraint functions

- Production capacity constraints for every skin type:
  \[ X_{11} + X_{12} + X_{13} + X_{14} + DB_1 - DA_1 = 279.862 \]
  \[ X_{21} + X_{22} + X_{23} + X_{24} + DB_2 - DA_2 = 194.547 \]
- Demand constraints:
  \[ X_{11} + X_{12} + X_{13} + X_{14} + DB_3 - DA_3 = 279.409 \]
  \[ X_{21} + X_{22} + X_{23} + X_{24} + DB_4 - DA_4 = 194.179 \]

c. Target function

- Maximize the number of requests:
  \[ 136.609 \]
  \[ 45.000 \]
  \[ 40.800 \]
  \[ 57.000 \]
  \[ 44.000 \]
  \[ 38.679 \]
  \[ 62.500 \]
  \[ 49.000 \]

- Profit target of Rp. 2,000,000,000.00:
  \[ 5.300 \]
  \[ 5.300 \]
  \[ 5.300 \]
  \[ 5.300 \]
  \[ 5.300 \]
  \[ 5.300 \]
  \[ 5.300 \]

- Capital minimize production cost of Rp. 5,640,000,000.00:
  \[ 11.700 \]
  \[ 11.700 \]
  \[ 11.700 \]
  \[ 11.700 \]

- Objective function

  Objective function as follows:

  Minimum
  \[ DA_1 + DA_2 + DB_4 + DB_5 + DB_6 + DB_7 + DB_8 + DB_9 + DB_10 + DB_11 + DB_12 + DB_13 + DA_14 \]

  Where:

  - \( DA_1 \) = variable deviation above the target production capacity of goat skin
  - \( DA_2 \) = variable deviation above the target production capacity of sheepskin
  - \( DB_4 \) = variable deviations below the target demand goatskin
  - \( DB_4 \) = variable deviations below the target demand sheepskin
Mathematical modeling of the problem of raw material production and goat leather tanning sheepskin with goal programming models based on constraint functions and objectives to be achieved are as follows:

1. The objective function:
   Minimum
   \[ DA_1 + DA_2 + DB_1 + DB_2 + DB_3 + DB_4 + DB_5 + DB_6 + DB_7 + DB_8 + DB_9 + DB_{10} + DB_{11} + DB_{12} + DB_{13} + DA_{14} \]

2. Constraint functions:
   \[
   \begin{align*}
   X_{11} + X_{12} + X_{13} + X_{14} + DB_{1} & - DA_{1} = 279.862 \\
   X_{11} + X_{12} + X_{13} + X_{14} + DB_{2} & - DA_{2} = 194.547 \\
   X_{11} + X_{12} + X_{13} + X_{14} + DB_{3} & - DA_{3} = 279.409 \\
   X_{21} + X_{22} + X_{23} + X_{24} + DB_{4} & - DA_{4} = 194.179 \\
   \end{align*}
   \]

3. Target function:
   \[
   \begin{align*}
   X_{11} + DB_5 - DA_5 & = 136.609 \\
   X_{12} + DB_6 - DA_6 & = 45.000 \\
   X_{13} + DB_7 - DA_7 & = 40.800 \\
   X_{14} + DB_8 - DA_8 & = 57.000 \\
   X_{31} + DB_9 - DA_9 & = 44.000 \\
   X_{32} + DB_{10} - DA_{10} & = 38.679 \\
   X_{33} + DB_{11} - DA_{11} & = 62.500 \\
   X_{34} + DB_{12} - DA_{12} & = 49.000 \\
   4.800X_{11} + 4.800X_{12} + 4.800X_{13} + 4.800X_{14} + 5.300X_{21} + 5.300X_{22} + 5.300X_{23} + 5.300X_{24} + DB_{13} - DA_{13} & = 2.000.000.000 \\
   12.000X_{11} + 12.000X_{12} + 12.000X_{13} + 12.000X_{14} + 11.700X_{21} + 11.700X_{22} + 11.700X_{23} + 11.700X_{24} + DB_{14} - DA_{14} & = 5.640.000.000 \\
   \end{align*}
   \]

Completion of optimization to the problem of raw material production and goat leather tanning sheepskin using LINDO program.

a. Objective Value: shows the objective function value equal to zero.

b. Infeasibilities: shows the value of ineligibility equal to zero. It means that the optimal solution has feasibility.

c. Total solver iterations: indicates the number of iterations required to find the optimal solution of as many as 12 iterations.

d. Variable: shows the decision variables.

e. Value: shows the optimal value of each decision variable.

1) \( DB_1 \) and \( DA_1 = 0 \). That is, the target production capacity reached goatskin.

2) \( DB_2 \) and \( DB_2 = 0 \). That is, the target production capacity reached sheepskin.

3) \( DB_3 = 0 \). That is, the target is reached goatskin request.

4) \( DB_4 = 0 \). That is, the target is reached sheepskin request.

5) \( DB_5 = 0 \). That is, the target number of requests in the 1st period goatskin is reached.

6) \( DB_6 \) and \( DA_6 = 0 \). That is, the target number of requests goatskin 2nd period is reached.

7) \( DB_7 \) and \( DB_7 = 0 \). That is, the target number of requests goatskin 3rd period is reached.

8) \( DB_8 \) and \( DA_8 = 0 \). That is, the target number of requests goatskin 4th period is reached.

9) \( DB_9 = 0 \). That is, the target number of requests in the 1st period sheepskin is reached.

10) \( DB_{10} \) and \( DA_{10} = 0 \). That is, the target number of requests sheepskin 2nd period is reached.

11) \( DB_{11} \) and \( DA_{11} = 0 \). That is, the target number of requests sheepskin 3rd period is reached.

12) \( DB_{12} \) and \( DA_{12} = 0 \). That is, the target number of requests sheepskin in 4th period is reached.

13) \( DA_{13} = 0 \). That is, the profit target is reached.

14) \( DA_{14} = 0 \). That is, the target cost of production is reached.
15) $DA_f = 453$. This means that as many as 453 sf exceeded targets for the number of requests goatskin.

16) $DA_f = 368$. This means that as many as 368 sf exceeded targets for the number of requests sheepskin.

17) $DA_f = 453$. This means that as many as 453 sf exceeded targets for the number of requests in the 1st period to the goatskin.

18) $DA_f = 368$. This means that as many as 368 sf exceeded targets for the number of requests in the 1st period to the sheepskin.

19) $DA_f = 0.3744367E+09$. Means exceeded profit targets for Rp.374,436,700, 00.

20) $DB_f = 5.4561$ million. Meaning deviations below the target cost of production by Rp.5,456,100, 00.

21) $X_{11} = 137,062$ sf met $X_{12} = 45,000$ sf met $X_{13} = 40,800$ sf met $X_{14} = 57,000$ sf met $X_{21} = 38,679$ sf met $X_{22} = 62,500$ sf met $X_{23} = 49,000$ sf met

f. Reduced Cost : provide information about the extent to which the value of Cj should be lowered so that the value of the decision variables to be positive. Reduced Cost value will always be zero if a positive decision variable values, and vice versa, Reduced Cost value will be positive if the value of the decision variable is zero.

g. Slack or Surplus : shows the slack or surplus value of each constraint as the value of the objective function reaches an extreme value. In the Row label function constraints start second order.

h. Dual Price : shows the changes that will occur in the value of the objective function when changing the value of the right side of the unit constraints.

Output of processed LINDO sensitivity analysis on the problem of raw material production and goat leather tanning sheepskins are as follows:

a. Sensitivity analysis $C_j$ (Objective Coefficient Ranges) describes changes in the value of Cj that will not change the optimal value of the decision variables. The change in the coefficient of $DA_1$, $DA_2$, $DB_3$, $DB_4$, $DB_5$, $DB_6$, $DB_7$, $DB_8$, $DB_{11}$, $DB_{12}$, $DB_{13}$ and $DA_{14}$ rise to unlimited or drops to zero will not change the optimal value of the decision variables.

b. Sensitivity analysis Segment Value Right (righthand Side Ranges).

Dual price reflects the objective function value changes attributable to changes in unit value of each right-hand side constraint active. In this case, the sensitivity analysis of the value of the right hand side describes the change interval right-hand side values that guarantee the validity of the dual price.

1) Changes to the active constraint RHS - 1 which is located on the 2nd line increased to 280,316,675 or dropped to 279,409 would not change the value of the constraints to the 1st dual price, which is 0.

2) Changes to the active constraint RHS - 2 are located on the 3rd line rose to 195,013.3 or dropped to 194,179 would not change the value of the constraint to the 2nd dual price, which is 0.

3) Changes to the active constraint RHS - 3, located on the 4th line rose to 279,682 or drop to unlimited will not change the value of the constraints to the 3rd dual price, which is 0.

4) Changes to the active constraint RHS - 4 which is located on the 5th row down to 194,547 or drop to unlimited will not change the value of the constraint to the 4th dual price, which is 0.

5) Changes to the active constraint RHS - 5 located on the 6th line rose to 137,062 or drop to unlimited will not change the value of the constraint to the 5th dual price, which is 0.

6) Changes to the active constraint RHS - 6 which is located on the 7th line rose to 45,453 or drop to zero will not change the value of the constraint to the 6th dual price, which is 0.

7) Changes to the active constraint RHS - 7 are located on the 8th line rose to 41,253 or drop to zero will not change the value of the constraint to the 7th dual price, which is 0.

8) Changes to the active constraint RHS - 8 which is located on the 9th line rose to 57,453 or drop to zero will not change the value of the constraints to the 8th dual price, which is 0.
9) Changes to the active constraint RHS - 9 which is located on line 10 rose to 44,368 or drop to zero will not change the value of the constraint to the 9th dual price, which is 0.
10) Changes to the active constraint RHS - 10 which is located on line 11, increased to 39,047 or down to be unlimited, it will not change the value of the constraint to the 10th dual price, which is 0.
11) Changes to the active constraint RHS - 11 which is located on line 12, increased to 62,868 or drop to zero will not change the value of the constraint to the 11th dual price, which is 0.
12) Changes to the active constraint RHS - 12 which is located on line 13, increased to 49,368 or drop to zero will not change the value of the constraint to the 12th dual price, which is 0.
13) Changes to the active constraint RHS - 13 which is located on line 14 rose to 237,443,670 or drop to unlimited will not change the value of the constraint to the 13th dual price, which is 0.
14) Changes to the active constraint RHS - 14 which is located on the 15th row up to the unlimited or down to 563,454,390 will not change the value of the constraint to the 14th dual price, which is 0.

4. CONCLUSIONS
After the modeling and problem solving using goal programming models, it can be concluded as follows:
1) The number of optimal production in PT Adira Universe Industry in the period April 2011 was 279,862 sf sheep skin and 194,547 sf for goat skin.
2) From the settlement of a mathematical model using goal programming models, can be obtained the following results:
   a. Profit in the period April 2011 was Rp.2,374,436,700, 0. With details: a target profit of Rp.2,000,000,000, exceeding the target of 00 and a gain of Rp.374,436,700, 00.
   b. Capital cost of production can save Rp.5,456,100, 0 of the target capital stock of Rp.5,640,000,000, 00.

The use of goal programming models in decision making to determine the optimal production of raw materials in the leather tanning factory goat and sheep leather can be applied to the next period as a solution in production planning. With the calculation in the production of raw material inventory data Month April 2011 period obtained greater benefits than the desired target.

REFERENCES
THE STOCHASTIC SIS EPIDEMIC MODEL WITH VARIABLE POPULATION SIZE

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Sebelas Maret University, UNS Surakarta.

ABSTRACT
The infectious disease is very serious problem in many countries. There are many attempts for controlling the spread of infectious disease. The spread of disease can be considered as random event so it could be constructed as stochastic model. The stochastic SIS epidemic models describe the spread of disease from which an individual could be infected more than once. This paper will discuss the stochastic SIS epidemic model when the population size N is not constant according to population growth law. To formulate the model, some assumption must be made. The population growth is assumed logistic. The model solution is obtained by using the Ito’s formula and the Kolmogorov forward differential equations. Then, we apply the model for an example by comparing constant population size and variable population size. The variable population size shows the fluctuation of N(t). It is more appropriate to represent the demographics condition.

Keywords: stochastic, SIS epidemic model, variable population size, logistic form.

1. INTRODUCTION
The healthiness is the most important need in human living. There are many researches that have been improved for increasing the quality of human’s health. The spread of infectious disease is the main part to be observed for preventing the occurrence of epidemic.

According to Hethcote[6], the mathematical models have an important role to formulate the phenomenon of disease spread to the mathematical equations. The first epidemic models had been introduced by Bernoulli, a physician who seeks to recognize the spread of chickenpox disease (smallpox). Then it had been developed by Mc. Cormack and Allen [8] who successfully predicted when the peak epidemic of diseases occurs, Brauer, and Castillo [2].

There are several models of the disease spread that are made according to the characteristics of each disease. The one models the SIS epidemic models. In the SIS epidemic model, the population is divided into two groups, the first group is susceptible (S) and the second group is infected (I). The susceptible group (S) consists of individuals who have not been infected by the disease and do not have immunity. While the infected group (I) consists of individuals who have been infected. Tassier [9].

The previous researches on SIS epidemic models are given as follows: Jin and Haque [7] discussed the SIS models that consider vaccination and isolation for dynamic population, while Wang [11] discussed the probabilistic model of SIS. Mc. Cormack and Allen [8] examined the disease spread simultaneously on multiple hosts (humans and animals).

Many researches assume that the population size is constant. That assumption does not appropriate for general condition of population. There occur birth, death and migration in population. Such that, the population size in this paper is not assumed constant but varies according to the law of population growth.
To formulate epidemic models with a population of not constant the birth rate and death rate of the population depends on the population size $N$. In this paper we will discuss the SIS stochastic epidemic models by considering the population size variable.

2. RESEARCH METHOD

This is a literature study research. Here, we study how to construct and to find the solution of the stochastic SIS epidemic model by describing the theories that have written before. The steps for reaching the research’s aims can be given below,

1. Construct the transition probability for CTMC SIS model.
2. Determine the differential equation of logistic population size.
3. Construct the SIS stochastic model.
4. Apply the transition probability and logistic population size in to SIS stochastic model.
5. Describe the differences between constant and variable population size by an example.

3. RESULT AND ANALYSIS

This section will be given the theories and concepts that support the analysis. The modeling process will be explained later. In the last part of this section, an example will be given for explaining the differences result of constant population size and variable population size.

3.1 Stochastic Processes

The spread of disease can be considered as probabilistic phenomenon. The stochastic model follows a stochastic process. As written by Finkenstadt et al. [4], a stochastic process is a random phenomenon that appears continuously that satisfies the laws of probability.

According to Goodman [5], and Taylor and Karlin [10], a stochastic process is a collection of some random variables $\{X_t(s) | t \in T, s \in S\}$ with the set of indices and sample space. The set of indices is often expressed as the set time. When $T = \{0, 1, 2, 3, \ldots\}$ is said to be discrete time stochastic process, whereas $T = [0, \infty)$ is said to be continuous time stochastic process. The stochastic processes $\{X(t), t = 0, 1, 2, 3, \ldots\}$ or $\{X(t), t \geq 0\}$ are called as Markov process if for some values $x_1, \ldots, x_n$ then the conditional probability can be written as

$$P[X(t_1) \leq x_1, X(t_2) = x_2, \ldots, X(t_n) = x_{n-1}] = P[X(t_2) \leq x_2 | X(t_1) = x_1, \ldots, X(t_n) = x_{n-1}].$$

3.2 Logistic Variable Population Size

According to Brauer et. al [3], it is assumed that the birth rate and death rate follow the logistic

$$\lambda(N) = bN$$
$$\mu(N) = b \frac{N^2}{K},$$

where is $K$ positive loading constant. Then, the population size satisfy logistic differential equation

$$\frac{dN}{dt} = \lambda(N) - \mu(N) = bN \left(1 - \frac{N}{K}\right).$$

3.3 SIS Stochastic Epidemic Model

The SIS epidemic model is formulated in a population that follows the logistic differential equation. Here are the assumptions used in the deterministic SIS epidemic models.

1. Birth and death rates depend on $N$.
2. Population homogenous and the size is not constant (dependent on $t$).
3. There were no deaths associated with the disease.
4. Every individual born in good health but prone diseases.

Allen [1] said that the SIS stochastic model describe the change of the number of individuals in group $S$ and $I$ in continuous interval time. The number of infected individuals in time $t + 1$ depends on the number of infected individuals in time $t$. This phenomenon is Markov process. The spread of disease with these characteristics can be described by models of continuous time Markov chain (CTMC) SIS. In this paper there is an additional assumption that changes in the number of susceptible and infected individuals following the Wiener process $W(t)$. The SIS stochastic model has a random variable $I(t)$. If the number of $I(t)$ is $i$, and the probability function of the number of infected individuals at time $t$ is

$$p_i(t) = \text{Prob}[I(t) = i],$$
where \( i \in [0, N] \), \( t \in [0, \infty) \). The number of infected individuals may change at anytime in the interval \( t \in [0, \infty) \).

In the time interval \( (t, t + \Delta t) \), the number of infected individuals changes from state \( i \) to state \( j \). The change from state \( i \) to state \( j \) is called a transition. The probability \( p_{ij}(\Delta t) \) of the number of infected individuals changing from state \( i \) to state \( j \) in the time interval \( \Delta t \) is called the transition probability, and it can be written as

\[
p_{ij}(\Delta t) = \text{Prob}[I(t + \Delta t) = j | I(t) = i].
\]

The transition process occurs in a very small time interval. It is assumed that there is only one individual who moves from state \( i \) to \( j \). Therefore, there are three possible transitions that occur from state \( i \) to state \( j = i + 1 \), from state \( i \) to state \( j = i - 1 \), and from state \( i \) to \( j = i \). When individuals transitioning from state \( i \) to state \( j = i + 1 \), means the number of infected individuals increased by one. On the other hand, there is an individual from group \( S \) move to \( I \). Because the population are homogenous, so that each individual in the group of \( S \) have the same probability to make contact with individuals in group \( I \). If there are \( i \) infected individuals in group \( I \), then the probability that the first group of individuals to make contact with the individual group \( S \) of \( I/N \). If the contact rate of \( \beta \) large, the transition probability from state \( i \) to state \( j = i + 1 \) in the time interval \( \Delta t \) is

\[
p_{(i),(j=i+1)\Delta t} = \frac{\beta}{N} SI \Delta t + o(\Delta t). \tag{2}
\]

When infected individuals move from state \( i \) to state \( j = i - 1 \), it means that the number of infected individuals reduced one. The reduction of one individual is caused by twoways. First, an individual from group \( I \) move to \( S \) group because it recovers with a recovery rate of \( \gamma \). Second, an individual from group \( I \) die with a death rate of \( \mu(N) \). Thus, the transition probability from state \( i \) to state \( j = i - 1 \) in the time interval \( \Delta t \) is

\[
p_{(i),(j=i-1)\Delta t} = \left( \frac{\gamma}{N} \mu(N) + \gamma I \right) \Delta t + o(\Delta t). \tag{3}
\]

Furthermore, infected individuals remain in state \( i \), means the addition or reduction of the number of infected individuals. The probability of state \( i \) to state \( j = i \), that the size of the probability of all events during the transition probabilities change in state \( i \rightarrow i + 1 \) and \( i \rightarrow i - 1 \), so it can be written as

\[
p_{(i),(j=i)\Delta t} = 1 - \left[ \frac{\beta}{N} SI + \frac{\gamma}{N} \mu(N) + \gamma I \right] \Delta t + o(\Delta t). \tag{4}
\]

At a very small time interval, there is only one possible transition individuals occur. The probability that there are more than one state transitioned is very small. Therefore, the transition probability value for the number of individuals who are transitioning more than or equal to two in the time interval \( \Delta t \) is \( o(\Delta t) \). Equation (2), (3) and (4), can be written as

\[
p_{(i),(j)\Delta t} = \begin{cases} 
\frac{\beta}{N} SI \Delta t + o(\Delta t), & j = i + 1 \\
\left( \frac{\gamma}{N} \mu(N) + \gamma I \right) \Delta t + o(\Delta t), & j = i - 1 \\
1 - \left[ \frac{\beta}{N} SI + \frac{\gamma}{N} \mu(N) + \gamma I \right] \Delta t + o(\Delta t), & j = i \\
o(\Delta t), & \text{others}
\end{cases} \tag{5}
\]

Equation system of (5) is a continuous time Markov chain models SIS (CTMC SIS) with the random variable \( I(t) \) is discrete and continuous time. According to Allen [1], SIS stochastic models have assumed random variable \( I(t) \) is continuous and continuous time, so that the CTMC SIS models in equation (5) can be considered a stochastic model of SIS.

The changes in the number of infected individuals is the difference between the number of infected individuals at time \( t + \Delta t \) and the number of infected individuals at time \( t \) that can be written as \( \Delta I = I(t + \Delta t) - I(t) \). It is assumed that \( \Delta I \) follows normal distribution with mean \( \mu(I) \Delta t \) and variance \( \sigma^2(I) \Delta t \) or \( \Delta I \sim N(\mu(I) \Delta t, \sigma^2(I) \Delta t) \). According to Allen [1], the changes in the number of infected individuals follow the Wiener process. At small interval time \( t \) to \( t + \Delta t \), the changes of the number of infected individuals can be expressed in stochastic differential equation.

Furthermore, the SIS stochastic model can be expressed as

\[
dI = \mu(I) dt + \sigma(I) dW(t). \tag{6}
\]

The equation (6) can be separated in two parts. The first part \( \mu(I) \) is called deterministic part then the second part \( \sigma(I) \) is called as stochastic part.
Based on equation (5), the transition probability \( i_j = i + 1 \) state is \( \frac{\beta}{N} SI \) and the transition probability from \( i \) state to \( j = i - 1 \) state is \( \frac{\lambda}{N} \mu(N) + \gamma I \). According to Allen [1], the expectation of \( \Delta I \) is

\[
E(\Delta I) = \sum_{j=1}^{n} \Delta t p_{ij}(\Delta t) = \frac{\beta}{N} SI - \frac{I}{N} \mu(N) - \gamma I
\]

Where the \( \Delta I \) is the changing of the number of infected individuals and \( p_{ij}(\Delta t) \) is the transition probability of the number of infected individuals. Then the variance of \( \Delta I \) is

\[
Var(\Delta I) = E(\Delta I)^2 - [E(\Delta I)]^2 = \frac{\beta}{N} SI + \frac{I}{N} \mu(N) + \gamma I
\]

Such that, the standard deviation is

\[
\sigma(\Delta I) = \sqrt{\frac{\beta}{N} SI + \frac{I}{N} \mu(N) + \gamma I}
\]

Such that, the equation (6) is stochastic SIS model with

\[
\mu(t) = \frac{\beta}{N} SI - \frac{I}{N} \mu(N) - \gamma I
\]

and

\[
\sigma(t) = \sqrt{\frac{\beta}{N} SI + \frac{I}{N} \mu(N) + \gamma I}
\]

Finally, the equation (6) could be written as

\[
dI = \frac{\beta}{N} SI - \frac{I}{N} \mu(N) - \gamma I + \sqrt{\frac{\beta}{N} SI + \frac{I}{N} \mu(N) + \gamma I} dW(t)
\]

where \( W(t) \) is the Wiener process.

3.4. The Model Solution

The solution of SIS stochastic model \( I(t) \) states the number of infected individuals at time \( t \). The unique solution is obtained by determining the initial condition \( I(t_0) = I_0 \) then the SIS stochastic model can be written as

\[
dI = \frac{\beta}{N} SI - \frac{I}{N} \mu(N) - \gamma I + \sqrt{\frac{\beta}{N} SI + \frac{I}{N} \mu(N) + \gamma I} dW(t)
\]

Furthermore, the solution model is

\[
I(t) = I(0) + \int_{0}^{t} \frac{\beta}{N} SI - \frac{I}{N} \mu(N) - \gamma I + \sqrt{\frac{\beta}{N} SI + \frac{I}{N} \mu(N) + \gamma I} dW(t)
\]

According to Allen [1], the solution of stochastic SIS epidemic model uses Ito’s formula. Suppose the stochastic processes \( I(t) \) has function \( F(I(t)) \) that satisfies Ito’s formula

\[
dF(t) = \left( \mu(I,t) \frac{\partial F(t)}{\partial t} + \frac{1}{2} \sigma^2(I,t) \frac{\partial^2 F(t)}{\partial I^2} \right) dt + \sigma(I,t) \frac{\partial F(t)}{\partial I} dW(t)
\]

Then, according to Taylor and Karlin [10], the next expected value is appropriate to Wiener process properties

\[
E \int_{0}^{t} \nu(t) \frac{\partial F(t)}{\partial I} dW(t) = 0,
\]

Then the differential equation of (8) is

\[
\frac{dE[F(t)]}{dt} = E \left( \mu(I,t) \frac{\partial F(t)}{\partial t} + \frac{1}{2} \sigma^2(I,t) \frac{\partial^2 F(t)}{\partial I^2} \right).
\]

The equation (9) could be expressed as

\[
\frac{d}{dt} \int_{-\infty}^{\infty} p(I,t) F(I) dI = \int_{-\infty}^{\infty} \left( \mu(I,t) \frac{\partial p(I,t)}{\partial t} + \frac{1}{2} \sigma^2(I,t) \frac{\partial^2 p(I,t)}{\partial I^2} \right) dI,
\]

where \( p(I,t) \) is the probability function of \( I(t) \). Furthermore equation (10) could be expressed as

\[
\int_{-\infty}^{\infty} F(t) \left( \mu(I,t) \frac{\partial p(I,t)}{\partial t} + \frac{1}{2} \sigma^2(I,t) \frac{\partial^2 p(I,t)}{\partial I^2} \right) dI = 0.
\]

The equation (10) holds for each real \( F \) function such that

\[
\frac{\partial p(I,t)}{\partial t} = \mu(I,t) \frac{\partial p(I,t)}{\partial I} + \frac{1}{2} \sigma^2(I,t) \frac{\partial^2 p(I,t)}{\partial I^2}.
\]
where \( p(0, I) = p_0(I) \). The equation (12) is called as forward Kolmogorov differential equation. If \( p(I, t) \) is the probability function of \( I(t) \) and equation (10) follows forward Kolmogorov differential equation (12), then \( I(t) \) is the solution of SIS stochastic model (7).

3.5. The Simulation Example

This numerical example refers to Brauer et al. [3]. It will be showed the disease spread pattern when the population size follows the logistic differential equation (1). For illustrative proposes, we choose \( \beta = 1, \tau = 0.25 = b \) and \( K = 100 \). Then the basic reproduction number is \( R_0 = 2 \). The epidemic stochastic SIS model with constant population size, \( N = 100 \) is compared to stochastic SIS model with variable population size \( N(t) \).

The figure (1) and (2) show the number of infective individuals along time period by considering constant population size and variable population size.

![Figure 1. The number of infective individuals with constant population size \( N = 100 \)](image1)

Figure 1 shows that the upper dash line is the constant population size. Then, the dash curve shows the number of infective individuals without the Wiener process in the model. Furthermore, the red fluctuate curve shows the number of infective individuals with the Wiener processes.

![Figure 2. The number of infective individuals with variable population size](image2)

Figure 2 shows that the population size fluctuate around 100. Then, the dash curve below shows the number of infective individuals without the Wiener process in the model and the blue fluctuate curve shows the number of infective individuals with the Wiener processes.

4. CONCLUSIONS
The stochastic SIS epidemic model consists of the birth rate and the death rate that have logistic form. Such that the total population size satisfies the logistic differential equation. The solution model with uses Ito’s formula and forward Kolmogorov differential equation. Then, we apply the model for an example by comparing constant population size and variable population size. The variable population size shows the fluctuation of $N(t)$. It is more appropriate to represent the demographics condition.

REFERENCES
THE SPACE OF REAL CONTINUOUS FUNCTIONS ON \([a,b]\) AS N-NORMED SPACE

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ABSTRACT

In this paper, we will show that \(C[a,b]\), the space of real continuous functions on \([a,b]\), can be equipped with an \(n\)-norm \(\|\cdot\|_n\), which makes \((C[a,b],n)\) a complete \(n\)-normed space. Then we will prove a Fixed Point Theorem in \((C[a,b],n)\). The proof does not use a Cauchy sequence, but uses a relationship between convergent sequences in norm and convergent sequence in the \(n\)-norm.

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1. INTRODUCTION

Let \(n \in \mathbb{N}\) and \(X\) be a real factor space of dimension \(d \geq n\). (Here \(d\) is allowed to be infinite). A real-valued function \(\|\cdot\|\) on \(X^n\) satisfying the following four properties

(1) \(\|x_1, ..., x_n\| = 0\) if and only if \(x_1, ..., x_n\) are linearly dependent;

(2) \(\|x_1, ..., x_n\|\) is invariant under permutation;

(3) \(\|x_1, ..., x_{n-1}, \alpha x_n\| = \|\alpha\|\|x_1, ..., x_{n-1}, x_n\|\) for any \(\alpha \in \mathbb{R}\);

(4) \(\|x_1, ..., x_{n-1}, y + z\| \leq \|x_1, ..., x_{n-1}, y\| + \|x_1, ..., x_{n-1}, z\|\).

is called an \(n\)-norm on \(X\) and the pair \((X,\|\cdot\|,\cdot)\) is called an \(n\)-normed space.

Note that an \(n\)-normed space \((X,\|\cdot\|,\cdot)\) satisfying the properties

(1) \(\|x_1, ..., x_n\| \geq 0;\)

(2) \(\|x_1, ..., x_n\| = \|x_1, ..., x_n + \alpha_1 x_1 + \cdots + \alpha_{n-1} x_{n-1}\|\) for all \(x_1, ..., x_n \in X\) and \(\alpha_1, ..., \alpha_{n-1} \in \mathbb{R}\).

The concept of 2-normed space was first introduced by Gähler in the 1960’s. Then 2-inner product space was introduced by Deminnie, Gähler, and White in the 1970. Afterward many researchers who study the aspects of an \(n\)-normed space, such as the topology of the \(n\)-normed space and the existence of a fixed point.

Most of the space that been studied are the space of finite dimension and standard space, for example, in [2, 3, 4], while the example for non-standard space has not been so widely studied, but one of them can be seen in [3]. This paper will discuss the space of real continuous functions on \([a, b]\), ie \(C[a, b]\) as a special case of non-standard space with infinite dimensions.

This paper will show that \(C[a,b]\), the space of real continuous functions on \([a,b]\), can be equipped with an \(n\)-norm \(\|\cdot\|_n\) which makes \((C[a,b],\|\cdot\|_n)\) a complete \(n\)-normed space. Then it will prove a Fixed Point Theorem in \((C[a,b],\|\cdot\|_n)\). The proof does not use a Cauchy sequence, but uses a relationship between convergent sequences in norm and convergent sequence in the \(n\)-norm.

2. METHOD
By using the properties of integral and determinant, define functions on \((C[a, b])^n\), that is:

\[\|f_1, \ldots, f_n\|_p := \left(\frac{1}{n!} \int_a^b \cdots \int_a^b \left| \det \left( f_i (x_j) \right) \right|^p \, dx_1 \cdots dx_n \right)^{\frac{1}{p}},\]  
untuk \(1 \leq p < \infty\)

and

\[\|f_1, \ldots, f_n\|_\infty := \sup_{a \leq x_1 \leq b} \sup_{a \leq x_n \leq b} \left| \det \left( f_i (x_j) \right) \right|,\]  
untuk \(p = \infty\)

for each \(f_1, \ldots, f_n \in C[a, b]\).

**Fact 2.1.** Inequality

\[\|f_1, \ldots, f_n\|_p \leq (nt)^{\frac{1}{p}}\|f_1\|_p \cdots \|f_n\|_p\]

and

\[\|f_1, \ldots, f_n\|_\infty \leq n! \|f_1\|_\infty \cdots \|f_n\|_\infty\]

are satisfied by all \(f_1, \ldots, f_n \in C[a, b]\).

**Proof.** Let \(S_n\) be a set of permutations from \(\{1, 2, \ldots, n\}\) and for all \(\theta \in S_n\) define

\[sg(\theta) := \begin{cases} 1, & \text{if } \theta \text{ is even permutation} \\ -1, & \text{if } \theta \text{ is odd permutation} \end{cases}\]

Then

\[\|f_1, \ldots, f_n\|_p = \left(\frac{1}{n!} \int_a^b \cdots \int_a^b \left| \sum_{\theta \in S_n} sg(\theta) f_1(x_{\theta_1}) \cdots f_n(x_{\theta_n}) \right|^p \, dx_{\theta_1} \cdots dx_{\theta_n} \right)^{\frac{1}{p}} \]

\[\leq \left(\frac{1}{n!} \int_a^b \cdots \int_a^b \left( \sum_{\theta \in S_n} |f_1(x_{\theta_1}) \cdots f_n(x_{\theta_n})|^p \right)^{\frac{1}{p}} dx_{\theta_1} \cdots dx_{\theta_n} \right)^{\frac{1}{p}} \]

\[\leq \left(\frac{1}{n!} \int_a^b \cdots \int_a^b \left( \sum_{\theta \in S_n} \left( \int_a^b |f_1(x_{\theta_1}) \cdots f_n(x_{\theta_n})|^p \, dx_{\theta_1} \right)^{\frac{1}{p}} \right)^{\frac{1}{p}} dx_{\theta_2} \cdots dx_{\theta_n} \right) \]

\[= (nt)^{\frac{1}{p}} \sum_{\theta \in S_n} \left( \int_a^b |f_1(x_{\theta_1}) \cdots f_n(x_{\theta_n})|^p \, dx_{\theta_1} \cdots dx_{\theta_n} \right)^{\frac{1}{p}} \]

\[= (nt)^{\frac{1}{p}} \left( \int_a^b |f_1(x_1) \cdots f_n(x_n)|^p \, dx_1 \cdots dx_n \right)^{\frac{1}{p}} \]

\[= (nt)^{\frac{1}{p}} \|f_1\|_p \cdots \|f_n\|_p.\]

Furthermore

\[\|f_1, \ldots, f_n\|_\infty = \sup_{a \leq x_1 \leq b} \sup_{a \leq x_n \leq b} \left| \sum_{\theta \in S_n} sg(\theta) f_1(x_{\theta_1}) \cdots f_n(x_{\theta_n}) \right| \]

\[\leq \sup_{a \leq x_1 \leq b} \sup_{a \leq x_n \leq b} \left( \sum_{\theta \in S_n} |f_1(x_{\theta_1}) \cdots f_n(x_{\theta_n})| \right) \]

\[= \sum_{\theta \in S_n} \left( \sup_{a \leq x_1 \leq b} \left| f_1(x_{\theta_1}) \cdots f_n(x_{\theta_n}) \right| \right) \]

\[= n! \|f_1\|_\infty \cdots \|f_n\|_\infty.\]

**Theorem 2.2.** The functions \(\|\cdot\|_p\) and \(\|\cdot\|_\infty\) define an \(n\)-norm on \(C[a, b]\).

**Proof.** We will verify that \(\|\cdot\|_p\) and \(\|\cdot\|_\infty\) satisfy the four properties of an \(n\)-norm.
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1. If $f_1, \ldots, f_n$ are linear dependent, then $\|f_1, \ldots, f_n\|_\infty = 0$. Conversely if $\|f_1, \ldots, f_n\|_\infty = 0$, then $\det(f_i(x_j))$ for all $x_1, \ldots, x_n \in [a, b]$. Hence $\{(f_1(x_1), \ldots, f_n(x_n)), \ldots, (f_n(x_1), \ldots, f_n(x_n))\}$ is linear dependent for all $x_1, \ldots, x_n \in [a, b]$. Thus $f_1, \ldots, f_n$ are linear dependent.

2. $\|f\|$ is invariant under permutation.

3. Observe that $\|f\|$ is invariant under permutation.

4. Observe that $\|f\|$ defines an $n$-norm on $C[a, b]$. By using Minkowski inequality, we can show that $\|f\|$ defines an $n$-norm on $C[a, b]$.

Corollary 2.3. The pair $(C[a, b], \|f\|)$ is an $n$-normed space.

Corollary 2.4. The pair $(C[a, b], \|f\|)$ is an $n$-normed space.

3. RESULT AND DISCUSSION

Let $\{a_1, \ldots, a_n\}$ be a set of linear independent in $C[a, b]$. We can define functions on $C[a, b]$, such as

$$\|f\|_p^* := \left( \sum_{i=1}^{n} \left( \|f, a_{i_1}, \ldots, a_{i_n}\|_p \right)^p \right)^{\frac{1}{p}}$$

and

$$\|f\|_\infty^* := \max_{i=1}^{n} \{\|f, a_{i_1}, \ldots, a_{i_n}\|_\infty\}, \text{untuk } p \to \infty.$$

For

$$\frac{(x-a) - (i-1)(b-a)}{b-a}, \quad a \leq x < a + \frac{(i-1)(b-a)}{n}$$

and

$$\frac{(x-a) - (i-1)(b-a)}{b-a}, \quad a + \frac{(i-1)(b-a)}{n} \leq x \leq a + \frac{i(b-a)}{n}$$

we find a fact.

Fact 3.1. The functions $\|f\|_\infty^*$ and $\|f\|_p^*$ define a norm on $C[a, b]$.

Proof. Let $f, g \in C[a, b]$. If $f = 0$ then $\|f\|_\infty^* = 0$. Conversely, if $\|f\|_\infty^* = 0$, then $\|f, a_1, a_2, \ldots, a_n\|_\infty = 0$.
\[\|f, a_2, a_3, \ldots, a_n\|_\infty = 0 \quad \text{(3.3)}\]
\[\|f, a_1, a_2, \ldots, a_{n-1}\|_\infty = 0 \quad \text{(3.4)}\]
and so on until to obtain
\[\|f, a_1, \ldots, a_{n-2}, a_n\|_\infty = 0 \quad \text{(3.5)}\]

By Equation (3.3) we find that \(f\) is linear combination from \(\{a_1, a_2, \ldots, a_{n-1}\}\), let
\[f = \sum_{i=1}^{n-1} k_i a_i. \]

By Equation (3.3) we find that \(k_1 = 0\), then by Equation (3.4) we ind that \(k_2 = 0\), and so on until we obtain that \(k_{n-1} = 0\). Thus \(f = 0\). Furthermore, for \(k \in \mathbb{R}\), we have
\[
\|kf\|_\infty^* = \max_{\{i_2, \ldots, i_n|\{1, \ldots, n\}\}} \left\{\|f, a_{i_2}, \ldots, a_{i_n}\|_\infty\right\}
= |k| \max_{\{i_2, \ldots, i_n|\{1, \ldots, n\}\}} \left\{\|f, a_{i_2}, \ldots, a_{i_n}\|_\infty\right\}
= |k|\|f\|_\infty^*.
\]

Finally,
\[
\|f + g\|_\infty^* = \max_{\{i_2, \ldots, i_n|\{1, \ldots, n\}\}} \left\{\|f + g, a_{i_2}, \ldots, a_{i_n}\|_\infty\right\}
= \max_{\{i_2, \ldots, i_n|\{1, \ldots, n\}\}} \left\{\|f, a_{i_2}, \ldots, a_{i_n}\|_\infty + \|g, a_{i_2}, \ldots, a_{i_n}\|_\infty\right\}
= \|f\|_\infty^* + \|g\|_\infty^*.
\]

Therefore \(\|\cdot\|_\infty^*\) defines a norm on \(C[a, b]\). By using Minkowski inequality, we can show that \(\|\cdot\|_p^*\) defines a norm on \(C[a, b]\).

**Fact 3.2.** Norm \(\|\cdot\|_\infty^*\) is equivalent to norm \(\|\cdot\|_\infty\) on \(C[a, b]\).

**Proof.** Note that \(\|a_1\|_{\infty} = \cdots = \|a_n\|_{\infty} = 1\). Let \(f \in C[a, b]\) and \(\{i_2, \ldots, i_n|\{1, \ldots, n\}\}\), then
\[
\|f, a_{i_2}, \ldots, a_{i_n}\|_{\infty} \leq n! \|f\|_{\infty} \|a_{i_2}\|_{\infty} \ldots \|a_{i_n}\|_{\infty} = n! \|f\|_{\infty},
\]
Thus
\[
\|f\|_{\infty}^* = \max_{\{i_2, \ldots, i_n|\{1, \ldots, n\}\}} \left\{\|f, a_{i_2}, \ldots, a_{i_n}\|_{\infty}\right\} \leq n! \|f\|_{\infty}, n! \|f\|_{\infty} = n! \|f\|_{\infty}.
\]

Furthermore
\[
\|f, a_1, \ldots, a_{n-1}\|_{\infty} = \sup_{a+b_1 \in \mathbb{R}} \sup_{a+b_2 \in \mathbb{R}} \det \begin{bmatrix} f(x_1) & f(x_2) & \cdots & f(x_n) \\ a_1(x_1) & a_1(x_2) & \cdots & a_1(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ a_{n-1}(x_1) & a_{n-1}(x_2) & \cdots & a_{n-1}(x_n) \end{bmatrix}
\geq \sup_{a+b_1 \in \mathbb{R}} \sup_{a+b_2 \in \mathbb{R}} \det \begin{bmatrix} f(x_1) & f(x_2) & \cdots & f(x_n) \\ a_1(x_1) & a_1(x_2) & \cdots & a_1(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ a_{n-1}(x_1) & a_{n-1}(x_2) & \cdots & a_{n-1}(x_n) \end{bmatrix}
= \sup_{a+b \in \mathbb{R}} \sup_{a+b \in \mathbb{R}} \det \begin{bmatrix} f(x_1) & f(x_2) & \cdots & f(x_n) \\ \vdots & \vdots & \ddots & \vdots \\ a_{n-1}(x_1) & a_{n-1}(x_2) & \cdots & a_{n-1}(x_n) \end{bmatrix}.
\]

By similar method we find that
\[
\|f, a_2, \ldots, a_n\|_{\infty} \geq \sup_{a+b \in \mathbb{R}} |f(x_2)|
\]
\[
\|f, a_1, a_3, \ldots, a_n\|_{\infty} \geq \sup_{a+b \in \mathbb{R}} |f(x_3)|
\]
and so on until we obtain that
\[
\|f, a_1, \ldots, a_{n-2}, a_n\|_{\infty} \geq \sup_{a+b \in \mathbb{R}} |f(x_n)|.
\]
Thus
Therefore we have $\|f\|_\infty \leq \|f\|^{**}_\infty \leq n! \|f\|_\infty$, so that norm $\|f\|^{**}_\infty$ is equivalent to norm $\|f\|_\infty$ on $C[a, b]$.

The main result from this paper are The Completeness of $C[a, b]$ and A Fixed Point Theorem on $C[a, b]$. First, we will prove a lemma that will help in proving The Completeness of $C[a, b]$ and A Fixed Point Theorem on $C[a, b]$.

**Lemma 3.3.** A sequence in $C[a, b]$ is convergent in the $n$-norm $\|f\|_n$ if and only if it is convergent in the norm $\|f\|_\infty$. Moreover a sequence in $C[a, b]$ is Cauchy in the $n$-norm $\|f\|_n$ if and only if it is Cauchy in the norm $\|f\|_\infty$.

**Proof.** Let $(f_n)$ is convergent sequence in $C[a, b]$ to $f \in C[a, b]$ in the norm $\|f\|_n$, then

$$\lim_{n \to \infty} \|f_n - f\|_n = 0,$$

so that for all $g_1, g_2, \ldots, g_{n-1} \in C[a, b]$ satisfy

$$\lim_{n \to \infty} \|f_n - f, g_1, g_2, \ldots, g_{n-1}\|_\infty \leq n! \|f_n - f\|_\infty \leq n! \|f\|_\infty \leq n! \|g_1\|_\infty \leq \|g_1\|_\infty = 0.$$ 

Therefore $(f_n)$ is convergent in $\|f\|_\infty$. Conversely let $(f_n)$ is convergent sequence in $C[a, b]$ to $f \in C[a, b]$ in the $n$-norm $\|f\|_n$, then

$$\lim_{n \to \infty} \|f_n - f, g_1, g_2, \ldots, g_{n-1}\|_\infty = 0 \text{ for all } g_1, g_2, \ldots, g_{n-1} \in C[a, b].$$

so that for all $\{i_2, \ldots, i_n\} \subseteq \{1, \ldots, n\}$ and $a_1, \ldots, a_n$ in the Equation (3.1) satisfy

$$\lim_{n \to \infty} \|f_n - f, a_1, a_{i_2}, \ldots, a_{i_n}\|_\infty = 0.$$

Then

$$\lim_{n \to \infty} \|f_n - f\|_\infty \leq \lim_{n \to \infty} \|f_n - f\|^{**}_\infty = 0.$$

Therefore $(f_n)$ is convergent in the norm $\|f\|_\infty$.

**Theorem 3.4.** $(C[a, b], \{\|\cdot\|_n\}_{n=0}^\infty)$ is a complete space.

**Proof.** Let $(f_n)$ is a Cauchy sequence in $C[a, b]$ in the $n$-norm $\|\cdot\|_n$, by Lemma 3.3. $(f_n)$ is a Cauchy sequence in the norm $\|\cdot\|_\infty$. Because $(C[a, b], \|\cdot\|_\infty)$ is a complete space, $(f_n)$ is convergent to a function $f \in C[a, b]$ in the norm $\|\cdot\|_\infty$, so that $(f_n)$ is convergent to $f$ in the $n$-norm $\|\cdot\|_n$.

Therefore $(C[a, b], \{\|\cdot\|_n\}_{n=0}^\infty)$ is a complete space.

Consequently, we have the following result.

**Theorem 3.5.** (Fixed Point Theorem) Let $\{a_1, \ldots, a_n\}$ is a set of linear independent in $(C[a, b], \{\|\cdot\|_\infty\}_{n=0}^\infty)$ in the Equation (3.1) and $T$ is a contraction mapping of $C[a, b]$ to itself, that is, there exists a constant $k \in (0, 1)$ such that

$$\|Tf - Tg, a_1, a_{i_2}, \ldots, a_{i_{n-1}}\|_\infty \leq k\|g, a_1, a_{i_2}, \ldots, a_{i_{n-1}}\|_\infty$$

for all $f, g \in C[a, b]$ and $\{i_2, \ldots, i_n\} \subseteq \{1, \ldots, n\}$. Then $T$ has a unique fixed point in $C[a, b]$.

We will show that $(C[a, b], \{\|\cdot\|_n\}_{n=0}^\infty)$ is Banach space before we prove Theorem 3.5.

Let $(f_n)$ is a Cauchy sequence in $C[a, b]$ in the norm $\|\cdot\|_n$. Then $(f_n)$ is a Cauchy sequence in $C[a, b]$ in the norm $\|\cdot\|_\infty$ because norm $\|\cdot\|_n$ is equivalent to norm $\|\cdot\|_\infty$ on $C[a, b]$, so that $(f_n)$ is convergent to a function $f \in C[a, b]$ in the norm $\|\cdot\|_\infty$. Then $(f_n)$ is convergent to $f$ in $C[a, b]$ in the norm $\|\cdot\|_n$ because norm $\|\cdot\|_n$ is equivalent to norm $\|\cdot\|_\infty$ on $C[a, b]$. Thus $(C[a, b], \{\|\cdot\|_n\}_{n=0}^\infty)$ is Banach space.
**Proof.** Consider that
\[
\|Tf - Tg\|_\infty^* = \max_{\{i_2, ..., i_n\} \subseteq \{1, ..., n\}} \left\{ \|Tf - Tg, a_{i_2}, ..., a_{i_n}\|_\infty \right\}
\leq \max_{\{i_2, ..., i_n\} \subseteq \{1, ..., n\}} \left\{ k \|f - g, a_{i_2}, ..., a_{i_n}\|_\infty \right\}
\leq k \max_{\{i_2, ..., i_n\} \subseteq \{1, ..., n\}} \left\{ \|f - g, a_{i_2}, ..., a_{i_n}\|_\infty \right\}
= k \|f - g\|_\infty^*.
\]

Thus $T$ is contractive mapping in the norm $\|\cdot\|_\infty^*$, so that by Fixed Point Theorem on Banach space $T$ has a unique fixed point in $C[a, b]$.

4. CONCLUSION

According to Gähler, in every normed space can be defined an $n$-norm. This thesis has shown that $C[a, b]$ can be equipped with an $n$-norm $\|\cdot, ..., \|_\infty$ so that the pair $(C[a, b], \|\cdot, ..., \|_\infty)$ is an $n$-normed space. Furthermore $(C[a, b], \|\cdot, ..., \|_\infty)$ is a complete space. It is derived from the fact that the sequence in $C[a, b]$ converges in the $n$-norm $\|\cdot, ..., \|_\infty$ if and only if the sequence is also convergent in the norm $\|\cdot\|_\infty$. Moreover the sequence in $C[a, b]$ is Cauchy in the the $n$-norm $\|\cdot, ..., \|_\infty$ if and only if the sequence is also Cauchy in the norm $\|\cdot\|_\infty$.

In addition it has been proved a Fixed Point Theorem in $(C[a, b], \|\cdot, ..., \|_\infty)$. The method of proof of Fixed Point Theorem in $(C[a, b], \|\cdot, ..., \|_\infty)$ uses a relationship between convergent sequences in norm $\|\cdot\|_\infty$ and norm $\|\cdot\|_\infty^*$ that be induced from $n$-norm $\|\cdot, ..., \|_\infty$.

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ABSTRACT
Train is public transportation which attracts many people especially in Bandung area. Because of that reason the precise scheduling is needed in order to optimize train scheduling through integer programming approach which minimize delay in path Bandung-Cicalengka. To solve the optimization model, branch and bound is systematically neglect a group of solution candidate which is not potential towards optimum solution by using upper and lower estimated bounds from optimized quantity. Based on optimization model which has been built, delay which is acquired in path Bandung-Cicalengka is 630 minutes.

Keywords:
Scheduling
Train
Integer programming
Branch and bound

1. INTRODUCTION
Train is a transportation vehicle with a power of motion, either go alone or coupled with other vehicles, which is moving on rails. Train is a mass transportation which is generally comprised of a locomotive and a series of trains or carriages. The wagon is relatively large so it can accommodate passengers or goods on a large scale. Because of its nature as an effective mass transportation, some countries try to use it as a primary means of transportation within the city, intercity and interstate.

In Indonesia, especially in Bandung area, train is one of the railway transport which attracted many people. This is because of a relatively low cost and also travel time is faster than other land transport. In addition, other advantages of train is environmentally friendly and relatively safe. Therefore, proper scheduling is needed in order to optimize the operation of train. Train scheduling system is a problem that is not easy to solve because there are a lot of rules and restrictions. One of them is a single path that used for two or more trains in different directions. Schedule contains the arrival time and departure time at each station.

In this paper, we will explain the completion of the railway scheduling problem by approach of integer linear programming. Integer linear programming or integer programming is a mathematical optimization to find solutions where each solution is an integer. In this case, we use a mathematical model to minimize the delay of trains without violating the capacity of existing lines and others noticed some limitations, including the use of time limits so two trains or more will collide. According to Suyanto (2010: 81) branch and bound is a general algorithm to search for optimal solutions of various optimization problems, especially discrete optimization. Branch and bound systematically ignore potential candidate solutions that do not lead to the optimal solution using the estimated upper limit and lower limit of optimized quantity. This method was first proposed by A. H. Land & A. G. Doig in 1960.

2. CONTENT OF STUDY
The train journey is defined as a moving train from the original station to the destination through the number of railways. In addition, every trip has a minimum and maximum dwell time at each railways. In the station, the dwell time is the time for train to stop. Whereas, for the railways which are outside...
the station, the dwell time is the amount of time for the train pass the railways. Based on the number of trains and tracks capacity, we will apply the train scheduling model that can minimizes delay.

![Diagram of railway stations A and B with rail lines 1, 2, 3, 4, and 5 indicated]

This is an example of the railway between Station A and Station B, and 1, 2, 3, 4, and 5 are the names of rail lines.

The assumptions used in this modeling are:
1. There are four types of train which they local economy trains, local patas trains, long-distance express trains and long distance economy trains. The speed of each train depends on the type of train and is considered constant. This causes the dwell time at each track is different, both inside the station and outside the station.
2. Train schedule is determined for 18 hours.
3. There is no priority trains.

3. MATHEMATICAL MODEL


The sets used are:
- \( K \): The set of trains
- \( B \): The set of rails
- \( S \): The set of stations
- \( B_k \): The set of all rail train passed by \( k \)
- \( B_s \): The set of rails that are on station
- \( B_k \setminus B_s \): The set of rails that are out of station

The parameters used are:
- \( T_{\text{trip}}_k \): Minimum time required by train \( k \) to complete the journey
- \( T_{\min}^k \): Minimum dwell time for the train \( k \) on the rails \( i \)
- \( T_{\max}^k \): Maximum dwell time for the train \( k \) on the rails \( i \)
- \( b^k_0 \): First rail used by the train \( k \)
- \( b^k_F \): Last rail used by trains \( k \)
- \( B^k_i \): Rails used by trains \( k \) before using the rails \( i \)
- \( C_{\text{kd}} \): Minimum difference of time between \( k \) and \( l \) train while using rails \( i \) sequentially

The variables used are:
- \( \text{delay}_k \): Total time delay for train \( k \) (minutes)
- \( a_{ik} \): The time when the train \( k \) enters rails \( i \) (minutes)
- \( d_{ik} \): The time when the train \( k \) left the rails \( i \) (minutes)
- \( y_{ik} \): Time delay of trains \( k \) on rails \( i \) (minutes)

The objective function that can minimize delay is formulated in the following functions:

\[
\text{Minimize} \sum_{k \in K} \text{delay}_k
\]

The constraints that must be made, are as follows:
- Each train \( k \) occupies rails \( i \) at least \( T_{\min}^k \).
- \( d_{ik} \geq a_{ik} + T_{\min}^k \forall i \in B_k \forall k \in K \)
- If train \( k \) using rails \( i \) over than \( T_{\max}^k \) the excess time is considered as delay.
Rules of the use of rails by two trains at the same time, so there must be a minimum increment of time that the two trains do not collide. 

\[ a_{ik} - d_{ij} \geq C_{ijkl} \quad \forall i \in B_k, \forall k,l \in K \]

Time for train \( k \) left the rails that had just used before entering the rails \( i \) equal to the time for train \( k \) enter the rails \( i \).

\[ d_{nik} = a_{ik} \quad \forall i \in B_k, \forall k \in K \]

Time for train \( k \) left the last rail minus time of train \( k \) enters the first line is a total delay of trains \( k \) plus the minimum time required to complete journey.

\[ d_{njk} - a_{njk} = \text{delay}_{jk} + \text{Trip}_k \quad \forall k \in K \]

Valued variables and non-negative integer.

\[ a_{ik} \geq 0 \quad \text{dan integer} \quad \forall i \in B_k, \forall k \in K \]

\[ d_{ik} \geq 0 \quad \text{dan integer} \quad \forall i \in B_k, \forall k \in K \]

\[ y_{ik} \geq 0 \quad \text{dan integer} \quad \forall i \in B_k, \forall k \in K \]

\[ \text{delay}_{jk} \geq 0 \quad \text{dan integer} \quad \forall k \in K \]

4. COMPLETION TECHNIQUE

Completion techniques that used to solve the above integer programming models are using branch and bound algorithm. The procedures of branch and bound algorithm by Hartanto:

Complete integer programming model with regular simplex method.

Observe the optimum solution if the basis variable is integer, so the optimum solution has been found. If one or more basis variables that are expected were not round, proceed to step 3.

The solution branched broken down into sub-problems. The goal is to eliminate the continuous solution that it is not integer. Branching is done through mutually exclusive constraints to find integer solution that can guarantee there are no integer solutions excluded.

For each sub-problem, the optimum solution of the objective function is consider as an upper limit. The best integer solution is consider as a lower bound. Sub-problems which have an upper limit that is less than the lower limit is not included in the subsequent analysis. An integer solution is the best or better than the upper limit for each sub-problem. If such a solution founded, a sub-problem with the best upper limit is selected for branching. Return to step 3.

5. SCHEDULE OF CASE STUDY ON RAIL TRAIN IN INDONESIA PT (Persero) DAOP 2 BANDUNG TRACK BANDUNG – CICALENGKA

This case study conducted on 56 trains that pass in the path between Bandung station to Cicalengka station Cicalengka which departed at 04.00 am until 22.00 pm. The train will pass through several stations namely Bandung station (BDG), Cikudapateuh station (CKU), Kiaracendong station (KAC), Gedebage Station (GDB), Cimekar Station (CMK), Ranchi Station (RCK), Haarpugur Station (HRP), and Cicalengka station (CCL). Each station has a different rail capacity and every train stops at certain stations. Sketch of the railway line that will be used can be seen in the following figure.

By using the software LINGO 10 obtained objective function of 630, meaning that the total delay for all trains is 630 minutes, with the details of each delay is as follows:

<table>
<thead>
<tr>
<th>Train Index</th>
<th>Name of Trains</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KRD Ekonomi</td>
<td>3 minutes</td>
</tr>
<tr>
<td>2</td>
<td>KRD Patas</td>
<td>No delay</td>
</tr>
<tr>
<td>3</td>
<td>KRD Patas</td>
<td>9 minutes</td>
</tr>
<tr>
<td>4</td>
<td>Pasundan</td>
<td>7 minutes</td>
</tr>
</tbody>
</table>
The schedule of the train can be shown in this table:
### Table of Bandung – Cicalengka train schedule

<table>
<thead>
<tr>
<th>Train Index</th>
<th>Name of Trains</th>
<th>No. Tr</th>
<th>BDG Arr</th>
<th>BDG Dep</th>
<th>CKU Arr</th>
<th>CKU Dep</th>
<th>KAC Arr</th>
<th>KAC Dep</th>
<th>GDB Arr</th>
<th>GDB Dep</th>
<th>CMK Arr</th>
<th>CMK Dep</th>
<th>RCK Arr</th>
<th>RCK Dep</th>
<th>HRP Arr</th>
<th>HRP Dep</th>
<th>CCL Arr</th>
<th>CCL Dep</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Pasundan</td>
<td>122</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5:28</td>
<td>5:30</td>
<td>5:34</td>
<td>5:36</td>
<td>5:38</td>
<td>5:39</td>
<td>5:43</td>
<td>5:45</td>
<td>5:50</td>
<td>5:55</td>
<td>5:58</td>
<td>6:00</td>
</tr>
<tr>
<td>7</td>
<td>Lodaya Pagi</td>
<td>66</td>
<td>6:58</td>
<td>7:00</td>
<td>7:02</td>
<td>7:03</td>
<td>7:05</td>
<td>7:07</td>
<td>7:10</td>
<td>7:11</td>
<td>7:23</td>
<td>7:24</td>
<td>7:27</td>
<td>7:28</td>
<td>7:31</td>
<td>7:32</td>
<td>7:35</td>
<td>7:37</td>
</tr>
<tr>
<td>8</td>
<td>KRD Patas</td>
<td>186</td>
<td>7:33</td>
<td>7:35</td>
<td>7:38</td>
<td>7:39</td>
<td>7:42</td>
<td>7:44</td>
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<td>7:51</td>
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<td>8:06</td>
<td>8:07</td>
<td>8:11</td>
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<td>9</td>
<td>Argo Wilis</td>
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<td>7:58</td>
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<td>8:03</td>
<td>8:05</td>
<td>8:07</td>
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<td>8:17</td>
<td>8:20</td>
<td>8:21</td>
<td>8:24</td>
<td>8:25</td>
<td>8:28</td>
<td>8:30</td>
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<tr>
<td>10</td>
<td>KRD Ekonomi</td>
<td>324</td>
<td>8:05</td>
<td>8:08</td>
<td>8:12</td>
<td>8:15</td>
<td>8:19</td>
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<td>8:29</td>
<td>8:30</td>
<td>8:33</td>
<td>8:35</td>
<td>8:46</td>
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<tr>
<td>17</td>
<td>Cibatu</td>
<td>338</td>
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</tbody>
</table>
# Table of Cicalengka – Bandung train schedule

<table>
<thead>
<tr>
<th>Train Index</th>
<th>Name of KA</th>
<th>No. Tr</th>
<th>CCL</th>
<th>HRP</th>
<th>RCK</th>
<th>CMK</th>
<th>GDB</th>
<th>KAC</th>
<th>CKU</th>
<th>BDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>KRD Patas</td>
<td>183</td>
<td>7:43</td>
<td>7:45</td>
<td>7:49</td>
<td>7:50</td>
<td>7:55</td>
<td>8:03</td>
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</tr>
<tr>
<td>45</td>
<td>Scrayu Pagi</td>
<td>141</td>
<td></td>
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<td></td>
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</table>
6. CONCLUSION

Train scheduling problem can be modeled in the form of integer programming models with objective function to minimize the total time delay. By applying the algorithm branch and bound, train scheduling model can be solved to produce the optimal scheduling of trains that will minimize the total time of delay. Implementation of mathematical models that are constructed for scheduling trains in case of PT Kereta Api Indonesia (Persero) Daop 2, Bandung-Cicalengka trajectory indicates that the model can be applied or well implemented.

Suggestion
The train scheduling model can be developed for example by adding constraints in order to minimize operating costs, priority trains, and so forth or using another model of integer programming to find the optimal train schedules.

REFERENCES
CONVERGENCE OF BINOMIAL AND TRINOMIAL MODEL IN PRICING EUROPEAN OPTIONS

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ABSTRACT

A European option is a financial contract which gives its holder a right (but not an obligation) to buy or sell an underlying asset from writer at the time of expiry for a pre-determined price. The continuous European options pricing model is given by the Black-Scholes formula. The discrete model can be priced using the lattice models. So, we can get the European option valuation problem either analytically, by simulation or applying suitable numerical technique. The evolution of the underlying asset can be modeled e.g. by binomial model, trinomial model, so the discrete European options can be priced using binomial model and trinomial model. We define the error simply as the difference between the binomial and trinomial approximation and the value computed by the Black-Scholes formula. An interesting property about error is how to understand the convergence of the binomial and trinomial model to the Black-Scholes model.

1. INTRODUCTION

Option is a contract between writer and holder which gives the right, not obligation for holder to buy or sell an underlying asset at or before the specified time for the specified price. The specified time called as expiration date (maturity time) and the specified price called as exercise price (strike price). Option call (put) allow holder to buy (sell) underlying asset with strike price K. Holder can exercise European-style option just only at maturity time T, whereas American-style option can be exercised anytime during before maturity time.

Option valuation models first introduced by Black and Scholes and Merton [1973]. They observe behavior lognormal from stock price and then they reduce a differential partial equation which describe option price. For European options, they have derived an closed form of the Black-Scholes formula. In this paper we study the principles of binomial and trinomial models and, since we can treat them as an approximation of the continuous time model, their convergence.

2. RESEARCH METHOD

History about option valuation started at 1900 when Louis Bachelier describe movement from stock price as Brown motion with drift $\mu = 0$. At 1973, Fischer Black and Myron Scholes publish their paper “The Pricing of Option and Corporate Liabilities”, a paper that changed rapidly theory of option pricing. In this paper, Black-Scholes makes assumptions on the market that is: stock price attend Geometric Brown Motion, with drift $\mu$ and volatility, stock trading takes place in a continuous interval, risk-free interest rates known and constant with respect to time, no-devidends were paid during the life of the option, there is
not transaction fee for the purchase or sale of assets or option, and without taxes, assets can be shared flawlessly, there may be short selling the asset (stock), and there is no possibility of arbitrage.

Black-Scholes formula for European call option at time \( t \) with strike price \( K \) and maturity time \( T \) and no dividend, that is:

\[
C_T(t, S_t) = S_t N(d_1) - K \exp \left( -r(T-t) \right) N(d_2)
\]

Black-Scholes formula for European put option at time \( t \) with strike price \( K \) and maturity time \( T \) and no dividend, that is:

\[
P_T(t, S_t) = K \exp \left( -r(T-t) \right) N(-d_2) - S_t N(-d_1)
\]

for \( S(t) > 0, T > 0 \) with

\[
d_1^t = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T-t}}
\]

\[
d_2^t = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T-t}}
\]

\[
d_2 = d_1 - \sigma \sqrt{T-t}
\]

and

\[
N(x) = \frac{1}{\sqrt{2\pi}} \exp\left( -\frac{1}{2} x^2 \right)
\]

where \( N(\cdot) \) is the cumulative standard normal distribution function.

A. Binomial Model

Binomial model which used in option valuation first time is introduced by Cox, Ross, and Rubinstein in their paper which published at 1979. In the binomial models, economicspan of trade until the option exercise date is divided into several intervals of time, called the period or step. At the end of each period the stock price can be up or down with accompanying probabilities of the stock price movement.

Binomial model is a dynamics model of the stock price has only two possible scenarios of stock price movements, \( \Omega = \{ \omega_1, \omega_2 \} \) in each period. At the end of the scenario period the stock price also has only two possible scenarios that the stock price goes up or down stock prices. The structure of the stock price movement can be described in the form of a binomial tree. A scenario of stock price movements can be seen also as a part of the structure path in the binomial tree. Each path is an illustration of the binomial random walk and the stock price follows a multiplicative binomial process with return (denoted by \( R \)) of each period \( R_i, (i = 1, 2, 3, \ldots, n) \) is expressed as follows.

\[
R_i = \begin{cases} 
  u & \text{with probability } \ p \\
  d & \text{with probability } (1 - p)
\end{cases}
\]

Multi-period binomial model is a discrete option pricing model with economic trade occurring in a limited time from \( t = 0 \) until \( t = T \). Multi-period binomial models are an extension of the one-period binomial models. As well as on the market with a binomial model of one period then the multi-period binomial models composed of two assets are risky assets, namely stocks and the risk-free asset in the form of savings deposits in banks. To facilitate further understanding, then the thing to do is to partition the period of economic trade \( [0, T] \) becomes

\[
0 = t_0 < t_1 < t_2 < \cdots < t_{n-1} < t_n = T
\]

which \( h = \Delta t = t_i - t_{i-1}, \ \forall \ i = 1, 2, 3, \ldots, n \) thus obtained

\[
h = \Delta t = \frac{T}{n}
\]
In other words, the period of economic trade $[0, T]$ partitioned into $n$ pieces of unit width interval $[t_i, t_{i+1}]$, $i = 1, 2, \cdots, (n - 1)$, the interest rates at the end of each period $t = t_{i+1}$ have been known in the beginning of each period $t = t_i$. As mentioned in the discussion section at the beginning, the structure of the stock price movement can be described in the form of a tree known as the binomial tree, so that the end of each period $[t_i, t_{i+1}]$, $0 \leq i \leq n - 1$. Therefore, the stock price movements following the binomial models.

Parameter stated that affects stock prices go up and parameter stated factors that affect stock prices fall, with risk-neutral measure $P(\tilde{p}, \tilde{q})$ that describes the opportunities of stock price increases or stock price decreases.

Model of Cox-Ross-Rubinstein (CRR) is a model of stock price movement in the context of the time which is a discrete random variable that is widely used in stock market (trading). If the movement of stock prices follows binomial model of stock price increases by a factor of $u$ and the factor share price declines by $d$ at intervals that fulfill each unit.

\begin{align*}
\hat{S}_t &= S_0 \left( u^\Delta t \right) < u
\end{align*}

so stock price at maturity time that is

\begin{align*}
S_T &= S_0 \left( u^\Delta t \right) \left( u^\Delta t \right)^{n-1} = S_0 \left( u^\Delta t \right)^n
\end{align*}

Random variable in equation (10) is a random variable that states the amount of increase in stock prices in a Bernoulli experiment with probability of rising stock prices for each time interval in the risk-neutral world is $\tilde{p}$. In other words, random variable $j$ is distributed binomial with parameters $n$ and $\tilde{q}$. Mean and variance of random variable are

\begin{align*}
E[j] &= n \tilde{p} \quad \text{and} \quad \text{Var}[j] = n \tilde{p} \tilde{q}
\end{align*}

Option pricing CRR model is part of the option pricing binomial model in a specified period which is determined by using backward induction and based on equation (10), and the equation

\begin{align*}
C_0 &= S_0 \sum_{j=0}^{n} \left( \frac{n}{j} \right) \left( p^* \right)^j \left( q^* \right)^{n-j} - K \sum_{j=0}^{n} \left( \frac{n}{j} \right) \tilde{p}^j \tilde{q}^{n-j}
\end{align*}

and formula for European put option binomial model $n$ period that is:

\begin{align*}
P_0 &= \frac{K}{e^{r \Delta t}} \sum_{j=0}^{n} \left( \frac{n}{j} \right) \tilde{p}^j \tilde{q}^{n-j} - S_0 \sum_{j=0}^{n} \left( \frac{n}{j} \right) \left( p^* \right)^j \left( q^* \right)^{n-j}
\end{align*}

with $\tilde{p} = \left[ \frac{e^{r \Delta t} - d}{u} \right]$ and $\tilde{q} = \left[ \frac{u - e^{r \Delta t}}{d} \right]$

\begin{align*}
p^* &= \frac{S_0}{e^{r \Delta t}} \quad \text{and} \quad q^* = \frac{p^*}{e^{r \Delta t}}
\end{align*}

or equation (11) can be written as:

\begin{align*}
C_0 &= S_0 \Phi(\alpha; n, p^*) - K \Phi(\alpha; n, \tilde{p})
\end{align*}

where $\Phi(\cdot)$ is the normal distribution function.

The following Cox, Ross, and Rubinstein select value for parameter $u$ and $d$

\begin{align*}
u = e^{\sigma \Delta t}
\end{align*}
\[ d = \exp\left(-\sigma \Delta t\right) \]  

with \( \sigma \) is the annual volatility of the stock price. Cox, Ross, and Rubinstein chose and values \( \Delta \) in inequality (16) and equation (17) with the intention that the time \( \Delta t \rightarrow 0 \), then the European call option price in equation (11) of the binomial CRR model can be proved to be the Black-Scholes formula for a European call option. In other words, the European option pricing binomial CRR model is an approximation of the European option pricing Black-Scholes at the time \( \Delta t \rightarrow 0 \).

### B. Trinomial Model

One proposed alternative to the binomial tree is the trinomial tree. The trinomial tree allows for a third option for a stocks movement: up, down, or middle (where the stock price stays constant over the time interval). The trinomial tree was first developed by Phelim Boyle in 1986. Since Boyle, others have attempted to formulate an improved version of the trinomial tree, most notably Kamrad and Ritchken. The trinomial tree constructed by Boyle can be compared to two binomial trees stacked on top of one another. The range of possible end values for the stock price is significantly wider than the range of values of the binomial tree. Boyle’s approach to the problem seems to be to include a wider range of values, thus better representing the possible outcome of the stock price, giving a more accurate approximation of the option’s value.

This model is an model of the stock market (trading) with one period (one time step) in other words, in this model there are only two trading times which at the time \( t = 0 \) and \( t = 1 \). Suppose \( S_0 \) denotes the stock price at the time \( t = 0 \), then at the end of period \( S_0 \) can be changed to \( S_1(\omega_1) \), \( S_2(\omega_2) \), or \( S_3(\omega_3) \). Later in the market with no periodic trinomial model is arranged without assets that are risky assets, namely stocks and risk-free assets, namely savings in the form of deposits in bank. \( B_t \) denotes the amount of savings in the form of deposits in the bank at the time \( t \). It is noted that the price of the deposit at time \( t \).

In this model, the movement of deposits be held deterministically, and can be expressed as follows:

\[ B_t = (1 + r)^t \]

where \( r \) is the risk-less (risk-free) interest rate. Moreover over know what the money market applicable interest rates on bank deposits (no periodic) rate and is assumed to be applicable the following relationship:

\[ d < 1 + r < u \]  

Equation (18) also can be expressed with:

\[ d < e^r < u \]

Strengthened by the following lemma:

**Lemma:**

A necessary and sufficient condition for no-arbitrage in the one-period trinomial model is

\[ d < 1 + r < u \]

Besides the foregoing assumptions, on the trinomial model, there are other assumptions that is:

\[ d < m < u \]  

The process of stock price movement is a stochastic process, and it can be expressed as follows

\[ S_1(\omega) = \begin{cases} S_0 u & \text{probability} \ p_u \\ S_0 m & \text{probability} \ p_m \\ S_0 d & \text{probability} \ p_d \end{cases} \]

At the end of period 1, the portfolio will become \( V_1 \) which consists of \( B_0 \) in shares of stock and something in the form of deposits or loans will be increased because \( e^r (V_0 - \theta_0 S_0) = e^r B_0 \). Portfolio at the end of period 1 can be expressed as follows:

\[ V_1(\Theta) = C_1 \]

\[ \theta_0 S_u + e^r (V_0 - \theta_0 S_0) = C_u \]

\[ \theta_0 S_m + e^r (V_0 - \theta_0 S_0) = C_m \]

\[ \theta_0 S_d + e^r (V_0 - \theta_0 S_0) = C_d \]

Equation (22) can be written in the form of:
Based on equation (23) is known that replication portfolio in first period trinomial model is a system of linear equations consisting of 3 equations and 2 variables. Therefore, a necessary and sufficient condition is that:

\[ (m - d)C_u - (u - d)C_m + (u - m)C_d = 0 \]  

This specifies a two-dimensional space of \( C = (C_u; C_m; C_d) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \). Therefore "most" contingent claims are not replicable, as a consequence the trinomial market is not complete.

One of the ways to solve the problems in the incomplete market model in trinomial models is by assuming the embedded complete market model in the incomplete market model. At one period trinomial model will be obtained three embedded "completemarkets" from the original in complete market. Generally European option pricing trinomial model can be determined using formulation:

\[ V_0(\Theta) = \frac{1}{(1 + r)^n} \left[ \hat{p}_u \cdot C_u + \hat{p}_m \cdot C_m + \hat{p}_d \cdot C_d \right] \]

However, to simplify the process of calculating in European option pricing trinomial model is performed numerically. The process of European option pricing trinomial model numerically, firstly should be done is determine all the possible stock price, after that it determines the value of pay off, and the last step that needs to be done is to determine European option pricing at \( t=0 \) by using the backward induction algorithm. European option pricing process works backward from the period \((M-1)\) to period 0, where strike option for period \( M \) equal to the value of its pay off.

European option pricing trinomial models expressed in terms of the following equation:

\[ V_{i,j} = e^{-r \Delta t} \left[ \hat{p}_u \cdot C_{j+1,i+1} + \hat{p}_m \cdot C_{j+1,i} + \hat{p}_d \cdot C_{j+1,i-1} \right] \]

With \( n \) stated boundary interval an distated stock price level.

Parameters \( u \) and \( d \) are used in the trinomial model, is:

\[ u = e^{\sigma \sqrt{\Delta t}} \]

Because of \( u \cdot d = 1 \), it is obtained:

\[ d = e^{-\sigma \sqrt{\Delta t}} \]

\[ m = 1 \]

And for the parameters \( p_u \) and \( p_d \), obtained:

\[ p_u = \left( \frac{\exp \left( \frac{r \Delta t}{2} \right) - \exp \left( -\sigma \sqrt{\Delta t} \right)}{\exp \left( \sigma \sqrt{\Delta t} \right) - \exp \left( -\sigma \sqrt{\Delta t} \right)} \right) ^2 \]

\[ p_d = \left( \frac{\exp \left( -\sigma \sqrt{\Delta t} \right) - \exp \left( \frac{r \Delta t}{2} \right)}{\exp \left( \sigma \sqrt{\Delta t} \right) - \exp \left( -\sigma \sqrt{\Delta t} \right)} \right) ^2 \]

\[ p_m = 1 - p_u - p_d \]

C. Convergence

In this section we will study and illustrate the convergence of the binomial and trinomial option. The following will be presented simulating European option pricing of binomial and trinomial models with \( S_0 = 100 \), \( K = 110 \), \( T = 1 \), \( r = 0.05 \), \( \sigma = 0.3 \) for \( n = 100 \), \( n = 200 \), \( n = 350 \), and \( n = 400 \). In addition to the binomial and trinomial European option pricing models will be displayed graph of the binomial and trinomial trees models mentioned above which illustrates that stock prices might be.
Based on Figure 3, Figure 4, Figure 5, and Figure 6, it can be seen that although European option pricing Black-Scholes is an approximation of European option pricing binomial and trinomial models when $t \to \infty$, but it turns out that European option pricing convergence is not monotonic, it can be seen clearly from the chart European option pricing binomial and trinomial models that move up and down. Therefore, it is concluded that:
1. European option pricing with binomial and trinomial models is an approximation of Black-Scholes option pricing when $n \to \infty$.

2. Convergence is not monotonic.

The rearrangement movement of European option pricing binomial and trinomial models, resulting in the option price convergence is not monotonic, but the order convergence of European option pricing binomial and trinomial models can be determined. Basically European option pricing which obtained by using binomial and trinomial models will not be the same as European option pricing Black-Scholes. Therefore, there is a difference between European option pricing binomial and trinomial models with European option pricing Black-Scholes. The difference between the option price is referred as error. Error value of both the option price is defined as follows:

$$e_n = \left| c(t_0, S_0) - c_n(t_0, S_0) \right|$$  \hspace{1cm} (31)

By applying the central limit theorem in equation (31) is obtained that

$$\lim_{n \to \infty} e_n = 0$$  \hspace{1cm} (32)

This means that the option price is determined using binomial and trinomial models will converge towards the option price determined using Black-Scholes formula.

Basically, it is possible to determine in what order the option price convergence is obtained. This may be done to determine the exact upper boundary for the equation (31). Therefore, to explain the convergence order (order of convergence) as required under this definition.

**Definition 1**

Let $f : x \to \max \{x - K, 0\}$ be a European call option. A sequence of lattices converges with order $\rho > 0$ if there exists a constant $\kappa > 0$ such that

$$e_n \leq \frac{\kappa}{n^{\rho}}, \quad \forall n \in \mathbb{N}$$  \hspace{1cm} (33)

Furthermore, by applying the logarithm to the equation (33), be obtained

$$\log(e_n) \leq \log \left( \frac{\kappa}{n^{\rho}} \right)$$

where it is shown that the logarithm error as a function of $\log n$ will be located below a straight line with a slope (slope) $\rho$.

A lattice approach converges with order $\rho > 0$ if for all $S_0, K, r, \sigma, T$ the specified sequence of lattices converges with order $\rho > 0$ and denote this with $\mathcal{O} \left( \frac{1}{n^{\rho}} \right)$. Important to know that convergence of prices is implied by any order greater than 0. Higher order means “quicker” convergence. Thus the theoretical concept of order of convergence is not unique: a lattice approach with order $\rho$ has also order $\tilde{\rho} \leq \rho$. Order of convergence is very easy to observe in actual simulations; in figures we plot the error against the refinement $n$ on a log-log-scale. The following will be presented: simulation error of the European option of binomial and trinomial models with $S_0 = 100, K = 110, T = 1, r = 0.05, \sigma = 0.3$, for $n = 20, \ldots, 500$. 
In this simulation can be seen that European call option pricing binomial and trinomial models with values of the above parameters that converges with order 1. Additionally, trinomial model converge faster than binomial model. Other things that need to know that the value of \( K \) always changing based on the values of \( r, \sigma, S_0, T, \) and \( K \) are given. To obtain the value \( K \) in accordance with the values of \( r, \sigma, S_0, T, \) and \( K \) are given by way of trying to do any value \( K \) that might found. At simulation error of European options pricing binomial model with \( S_0 = 100, K = 110, T = 1, r = 0.05, \sigma = 0.3, \) for \( n = 20..., 500 \) earned values \( \Phi \) that found are 4. While the simulation error of European options pricing trinomial model with \( S_0 = 100, K = 110, T = 3, r = 0.05, \sigma = 0.3, \) for \( n = 20..., 500 \) earned value trinomial model of that found are 2.
3. CONCLUSIONS

Of the overall analysis and calculations have been done before it can be concluded as follows:

1. European option pricing binomial and trinomial models for increasingly larger \( n \) will converge to European option pricing Black-Scholes. Convergence European option pricing binomial and trinomial models are not monotonous. But the movement of trinomial option pricing model more stable than the movement of binomial option pricing model.

2. Order of convergence for binomial model is proposed by Cox-Ross-Rubinstein (1979) is one, beside that order of convergence order for trinomial model is proposed by Phelim Boyle (1986) is also one. Additionally, trinomial models converge faster than binomial model.

REFERENCES

1. INTRODUCTION

\( C^* \)-algebras have a very large class. In any case, one finds that general \( C^* \)-algebras show a variety of behavior and a plethora of characteristics. It made many researcher who have worked in the area find it is necessary to consider special classes of \( C^* \)-algebras. No doubtly, these classes are important for many reason: they provide the (counter) examples to test hypothesis and conjectures, they allow intermediate stages for proving general results, and they suggest concepts that are often important in studying more general \( C^* \)-algebras. The development of operator algebras in the last twenty years has been built on a careful research in these special classes.

In this thesis, we study \( C^* \)-algebras which are presented with graphs, called as graph algebras. It was Bratteli [6] who used graph as a tool to study inductive system of finite dimensional \( C^* \)-algebras and their limit (called as \( AF \)-algebras). Since that time many researchers have used graphs both as a tool to study large classes of \( C^* \)-algebras and as a source of an inexhaustible supply of examples. Somewhat later Cuntz and Krieger has introduced a very special class of \( C^* \)-algebras (named as Cuntz-Krieger algebras) generated by partial isometries ([7]). Briefly, if \( A \) is an \( n \times n \) matrix with \( A_{ij} = 0 \), (with non zero row or column), then the associated Cuntz-Krieger \( OA \) is defined to be the universal \( C^* \)-algebra generated by \( n \) partial isometries \( S_1, ..., S_n \) with orthogonal ranges that satisfy

\[
S_i^* S_i = \sum_{j=1}^{n} A(i, j) S_j S_j^* \quad \text{for} \quad 1 \leq i \leq n.
\]

In [9], Watatani recognised that Cuntz-Krieger algebras could be viewed as a \( C^* \)-algebras associated to finite directed graphs. This approach of viewing Cuntz-Krieger algebras as \( C^* \)-algebras associated to graphs had the advantage that it was more visual. However his works factually went mostly unnoticed, it was not until 1997 when Kumjian, Pask, and Raeburn and Renault rediscovered about this \( C^* \)-algebras associated to graphs. Graph algebras give an important role for development of the study \( C^* \)-algebras. Otherthan generalizing the Cuntz-Krieger algebras, there was also invention that many interesting class of \( C^* \)-algebras are either isomorphic to graph algebras or closely related to graph algebras. Other than that, their basic structure is well understood and
their invariants are readily computable. Furthermore, the graph not only determines the defining relations for the generators of the C*-algebra, but also many important properties of the C*-algebra may be translated into graph properties. Hence the graph provides a tool for visualizing many aspects of the associated C*-algebra. In addition, graph algebras are useful examples for much of the work that is currently being done in C*-algebra theory.

2. CONTENT OF KNOWLEDGE

2.1. C*-Algebras

Definition 2.1.1 A C*-algebra is a Banach ∗-algebra with the additional norm condition.

\[ \|A^* A\| = \|A\|^2 \quad \text{for all} \quad A \in \mathfrak{A}, \]

and this additional norm condition we call C*-condition.

Example 2.1.2. Let \( H \) is a Hilbert space. We define \( B(H) \) an algebra of bounded linear operators, it is C*-algebra with norm define as supremum norm and the involution is defined by the usual adjoint operation. This follows from the well known identity,

\[ \|A^* A\| = \sup_{|x| = |y| = 1} |(A^* A x, y)| = \sup_{|x| = |y| = 1} |(A x, A y)| = \|A\|^2. \]

In the special case that \( H = \mathbb{C}^n \), then \( B(H) \) is identified with the \( n \times n \) matrices \( (M_n) \). Some of the simplest, yet most important, classes of operators are the compact operators. We denote \( K(H) \) is the set of all compact operators on a separable Hilbert space. Furthermore, we can show that \( K(H) \) is a norm closed self-adjoint subalgebra of \( B(H) \), hence obviously it is also C*-algebra.

2.2. Graph Algebras

A directed graph \( E = (E^0, E^1, r, s) \) consists of countable sets of \( E^0 \) of vertices and \( E^1 \) of edges, and function \( r, s : E^1 \to E^0 \) identifying the range and source of each edge. We draw a graph by placing the vertices in a plane, and drawing a directed line from \( s(e) \) to \( r(e) \) for each edge \( e \in E^1 \).

The graph is row-finite if each vertex emits at most finitely many edges. We write \( E_n \) for the set of paths \( \mu_1 \mu_2 \ldots \mu_n \) of length \( |\mu| = n \), i.e. sequences of edges \( \mu_i \) such that \( r(\mu_i) = s(\mu_{i+1}) \) for \( 1 \leq i \leq n \). And write \( E^* := \cup_{n \geq 0} E_n \). We extend the range and source maps to \( E^* \) by setting \( r(\mu) = r(\mu 1) \) and \( s(\mu) = s(\mu |\mu|) \) for \( |\mu| > 1 \), and \( r(v) = v = s(v) \) for \( v \in E 0 \). If \( \mu \) and \( v \) are paths with \( s(\mu) = r(v) \), we write \( \mu v \) for the path \( \mu_1 \ldots \mu_i |v|_1 \ldots |v|_i \). And also the maps \( r, s \) extend to the set \( E^\infty \) of infinite paths \( \mu = \mu_1 \mu_2 \ldots \).

A loop is a path whose range and source are equal, and for given loop \( \alpha := a_1 a_2 \ldots a_n \) we say that \( \alpha \) is based at \( s(a_1) = r(a_n) \). We now introduce a new condition we called Condition (L): Every loop in \( E \) has an entry. That is, for every loop \( \alpha := a_1 \ldots a_n \) there exists \( e \in E 1 \) for which \( r(e) = r(a_i) \) for some \( i \) but \( e \neq a_i \). Look at the following example of graphs, it has one loop and also satisfies the condition (L).

\[ \begin{tikzpicture}
    \node (a) at (0,0) {e};
    \node (b) at (1,0) {v};
    \node (c) at (2,0) {f};
    \node (d) at (3,0) {w};
    \draw[->] (a) edge[bend left=20] (b);
    \draw[->] (b) edge[bend left=20] (a);
    \draw[->] (c) edge (d);
\end{tikzpicture} \]

\textbf{Figure 1}

2.2.1. C*-Algebras Associated to Graphs. Let \( E \) be a row-finite directed graph and \( H \) be a Hilbert space. A Cuntz-Krieger \( E \)-family \( (S, P) \) on \( H \) consists of a set \( \{ P_v : v \in E 0 \} \) of mutually orthogonal projections on \( H \) and a set \( \{ \text{Sel} e \in E 1 \} \) of partial isometries on \( H \) such that

- (CK1) \( S^*_e S_e = P_{s(e)} \) for all \( e \in E 1 \); and
- (CK2) \( P_v = \sum_{\{e \in E 1 | r(e) = v\}} S^*_e S_e \) whenever \( v \) is not a source.

Condition (CK1) and (CK2) are called the Cuntz-Krieger relations and condition (CK2) in particular is often called the Cuntz-Krieger relation at \( v \).
Theorem 2.2.1. For any row-finite directed graph $E$, there is a $C^*$-algebra $C^*(E)$ generated by a Cuntz-Krieger $E$-family $\{s, p\}$ such that for every Cuntz-Krieger $(T,Q)$ in a $C^*$-algebra $B$, there is a homomorphism $\phi T,Q$ of $C^*(E)$ into $B$ satisfying $\phi T,Q(\phi e) = Te$ for every $e \in E_1$ and $\phi T,Q(\phi v) = Qv$ for every $v \in E_0$.

$C^*(E)$ denoted the $C^*$-algebra of the graph $E$ or the Cuntz-Krieger algebra of $E$, and is generically described as a graph algebra.

2.2.2. Uniqueness Theorems of Graph Algebras.

a. The Gauge-Invariant Uniqueness Theorem. The gauge-invariant uniqueness theorem states that $C^*(E)$ is uniquely characterised by the existence of the canonical action of $T$ called the gauge-invariant action.

Theorem 2.2.2. Let $E$ be a row-finite directed graph, and suppose that $(T,Q)$ is a Cuntz-Krieger $E$-family in a $C^*$-algebra $B$ with each $Qv = 0$. If there is a continuous action $\beta : T \rightarrow \text{Aut} B$ such that $\beta(e) = zTe$ for every $e \in E_1$ and $\beta(Qv) = Qv$ for every $v \in E_0$, then $\pi T,Q$ is an isomorphism of $C^*(E)$ onto $C^*(T,Q)$.

b. The Cuntz-Krieger Uniqueness Theorem. The following result is essentially due to Cuntz-Krieger [7]. We apply this theorem to analyse the ideal in $C^*(E)$.

Theorem 2.2.3. Let $E$ be a row-finite directed graph in which every cycle has an entry, and $(T,Q)$ is a Cuntz-Krieger $E$-family in a $C^*$-algebra $B$ such that $Qv = 0$ for every $v \in E_0$. Then the homomorphism $\pi T,Q : C^*(E) \rightarrow B$ is an isomorphism of $C^*(E)$ onto $C^*(T,Q)$.

2.3. Simplicity and The Ideals. Recall that for $v, w \in E_0$, we write $v \leq w$ if there exist a path $\lambda \in E_*$ such that $s(\lambda) = w$, $r(\lambda) = v$. For subsets $M, N \subseteq E_0$ we write $M \subseteq N$ to mean that for each $v \in M$ there exists $w \in N$ such that $v \leq w$. This relation $\subseteq$ is a preorder. We say that a subset $H \subseteq E_0$ is hereditary if for $v \in H$ and $v \leq w$ then $w \in H$. A hereditary set $H$ is said to be saturated if $r^{-1}(v) \subseteq H$ and $\{s(v) \mid r(e) = v\} \subseteq H$ imply $v \in H$. The saturation of a hereditary set $H$ is the smallest saturated hereditary subset $H$ of $E_0$ containing $H$. We denote $E_0^H$ is the set of infinite paths $\lambda = \lambda_1\lambda_2...$, and $E_0^{\leq H}$ is the set obtained by adding to $E_0^{H}$ the finite paths which begin at sources.

The graph $E$ is cofinal if for every $\lambda \in E_0^{\leq H}$ and $v \in E_0$, there exists a vertex $w$ on $\lambda$ such that $v \leq w$. Look at the following graph $E$, we get the sets $\{u\}$ and $\{u, v\}$ are non-trivial hereditary subsets of $E_0$, but $\{v\}$ is not. The set $\{u, v\}$ is saturated, but $\{u\}$ is not because $s(e) \mid r(e) = v \subseteq \{u\}$.

![Figure 2](image)

Definition 2.4. We say $E$ satisfies Condition $(K)$ if for every vertex $v$, either there is no cycle based at $v$, or there are two distinct paths $\mu, v$ such that $s(\mu) = v = r(\mu)$, and $s(v) = v = r(\mu_i)$ for $i < |\mu|$, and $r(v_i) \neq v$ for $i < |v|$ (we call these distinct return paths).

Theorem 2.5. Suppose $E$ is a row-finite graph which satisfies Condition $(K)$. For $H \subseteq E_0$, let $I H$ be the ideal generated by $\{pv \mid v \in H\}$. Then $H \rightarrow I H$ is a bijection between the saturated hereditary subset of $E_0$ and the closed ideals in $C^*(E)$, with inverse given by $I T \rightarrow H$ is/isomorphism $\phi H = \phi E_0 \subseteq I T$. The quotient $C^*(E)/I H$ of $C^*(E)$ is isomorphic to $C^*(E\setminus H)$, and $C^*(E)/I H$ is isomorphic to the full corner $p H$ associated to the projection $p H$ defined in [2, Lemma 2.10].

Theorem 2.6. Suppose $E$ is a row-finite graph. Then $C^*(E)$ is simple if and only if every cycle in $E$ has an entry and $E$ is cofinal.

In years later, Kunjian and Pask defined a combinatorial object called a higher rank graph and described a way to associate a $C^*$-algebra to it [5]. If $\Lambda$ is a higher rank graph, then $C^*(\Lambda)$ is generated by a family of partial isometries satisfying relations similar to the Cuntz-Krieger relations that we have done on row-finite directed graphs. This $C^*$-algebras of $k$-graph is generalization of the $C^*$-algebras of row-finite directed graphs with no source.
REFERENCES


SOFTWARE FOR DETERMINING
THE BEST CELL PHONE PROVIDER AND PLAN

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ABSTRACT
Nowadays, cell phone providers compete to offer various cell phone plans. The wrong choice on cell phone provider and the plan will impact to the cost of cell phone uses. In this research, we develop a software for helping a customer to choose the best cell phone provider and plan. We classify cell phone plans according to the providers, and then we construct models of cell phone rates. To calculate the total cost of cell phone uses by the software, first, a user should choose a type of cell phone rate, then he should input the expectation of cell phone uses for calling, internet, and short message service (SMS). By using the software, we also can provide the optimal cell phone plan for our cell phone uses.

Keywords: Cell phone provider, Cell phone plan, Cell phone card, Cell phone rate, Cell phone provider rate model

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1. INTRODUCTION
To attract customers, cell phone providers offer a number of cell phone plans. This condition often makes customers confused to choose the best cell phone provider and plan. Pramuditya [5] stated that customers often moved up from one provider to others. This action is done in order to get a provider with the cheapest cost for their cell phone uses.

Moving from one provider to others is not the only way to save the cell phone cost. Each cell phone provider offers various cell phone plans, such as SMS package, internet package, and calling package, besides a regular package. If we know how much of our cell phone uses, we should be able to determine the best package for saving our money. In reality, our package choice is often not appropriate with the cell phone uses. As a result, the total cost of the cell phone uses continues to increase, Sidqia [4].

In this research, we develop a software to calculate the total cost of cell phone uses. The software will provide which cell phone provider and the plan that gives the minimum cost. The total cost is determined according to the type of cell phone rate, cell phone provider plan, as well as the cell phone uses.

2. RESEARCH METHOD
Indonesian cell phone providers offer various cell phone plans in order to obtain the large number of customers. The cell phone plans data for Indonesian providers can be obtained on provider’s website. According to Indonesian cell phone plans data, first, we classify the data into three types. Then we develop models for calculating the total cost of cell phone uses of each type. The complete explanation of each type can be obtained in Table 2.1.

The next step is to design an interface. The design of interface is illustrated on Figure 2.1.
Figure 2.1 Interface Design

Figure 2.1 shows that the inputs of the software are cell phone uses (as variables) and cell phone plan rates (as parameters). The output of the software is the total cost of cell phone uses. Data types for both the variables and the output are real, and data type of the parameters are two-dimensional array with real type. We use Delphi software as the programming language, because Delphi can facilitate the data needs and the interface design, see Marwati [2] and Madcom [1].

<p>| Table 2.1. Model of Type 1, Model of Type 2, Model of Type 3 (data per Mei 2013) |
|---------------------------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Package</th>
<th>Call</th>
<th>SMS</th>
<th>Internet</th>
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<td>GSM 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular</td>
<td>300/30 second</td>
<td>100/sms</td>
<td>2/kb</td>
</tr>
<tr>
<td>Call</td>
<td>2000/day</td>
<td>100/sms</td>
<td>2/kb</td>
</tr>
<tr>
<td>SMS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internet</td>
<td>300/30 detik</td>
<td>100/sms</td>
<td>1000/15mb</td>
</tr>
<tr>
<td>GSM 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular</td>
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</tr>
<tr>
<td>Call</td>
<td>1000/30 minutes to the same providers 00-06</td>
<td>125/sms</td>
<td>1/kb</td>
</tr>
<tr>
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<td>15/second</td>
<td>1250/980 sms to the same providers +20 sms to different providers</td>
<td>1/kb</td>
</tr>
<tr>
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<td>15/second</td>
<td>125/sms</td>
<td>1500/7mb</td>
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<td></td>
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<td></td>
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<tr>
<td>Regular</td>
<td>15/sec</td>
<td>90/sec</td>
<td>150/sms</td>
</tr>
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<td>Call</td>
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<td>30/sec</td>
<td>150/sms</td>
</tr>
<tr>
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<td>90/sec</td>
<td>2000/hari</td>
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<tr>
<td>Internet</td>
<td>15/sec</td>
<td>90/sec</td>
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<tr>
<td>CDMA 1</td>
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<td></td>
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</tr>
<tr>
<td>to the same</td>
<td>2/second (early 120)</td>
<td>150/sms</td>
<td>1000/100mb</td>
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<th>SMS Rate (local)</th>
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<td>800</td>
<td>150-300/sms</td>
<td>1000/100mb</td>
</tr>
<tr>
<td>&lt; 200 km</td>
<td>1545</td>
<td>150-300/sms</td>
<td>1000/10mb</td>
</tr>
<tr>
<td>&gt; 200 km</td>
<td>2727</td>
<td>150-300/sms</td>
<td>1000/10mb</td>
</tr>
</tbody>
</table>

CDMA 2

<table>
<thead>
<tr>
<th>Provider</th>
<th>Rate (international)</th>
</tr>
</thead>
<tbody>
<tr>
<td>to the same</td>
<td></td>
</tr>
<tr>
<td>provider in lokal</td>
<td>53.9/minute</td>
</tr>
<tr>
<td>area</td>
<td></td>
</tr>
<tr>
<td>to the same</td>
<td>375/minute</td>
</tr>
<tr>
<td>provider in</td>
<td></td>
</tr>
<tr>
<td>international area</td>
<td></td>
</tr>
</tbody>
</table>

All models can be used for a cell phone by call rate, SMS and internet. While model of TYPE 1 is used without considering both providers and distance (see Figure 2.2a), and model of TYPE 2 is used by considering the provider and no distance factor (see Figure 2.2b), model of TYPE 3 is used by considering both provider and the distance (see Figure 2.2c).

![Figure 2.2 Forms of Type 1, Type 2, dan Type 3](image)

The Algorithm and The Program

First we initialize the variables with zero. Then, we initialize the rate parameters according the online data from Indonesian providers. To anticipate end user which does not know the cell phone provider rate, we set the value rates as a default. But, the user can change the default value. The change can be done by replacing the default value on rate table. If we want to replace all default value, we can use deleted button. This button initialize all value on rate table by zero, and it will be done applying looping algorithm, see Munir [3].

The cell phone cost is calculated according both cell phone uses and cell phone provider plan. In the following paragraph, we calculate the total cell phone cost for model of type 1.

Let $x$, $y$, and $z$ be the uses of cell phone call (in minutes), SMS, and internet (in kbyte), respectively. If we symbolize the following parameters:

- regular cell phone call rate = $t_{rc}$, regular sms rate = $t_{rs}$, regular internet rate = $t_{ri}$, call package for cell phone call rate = $t_{ct}$, call package for sms rate = $t_{ts}$, call package for internet rate = $t_{ci}$, sms package for cell phone call rate = $t_{st}$, sms package for sms rate = $t_{ss}$, sms package for internet rate = $t_{si}$, internet package for cell phone call rate = $t_{it}$, internete package for sms rate = $t_{is}$, dan internet package for internet rate = $t_{ii}$, then

- cell phone cost for regular package = $f_{r} = 60t_{rc} + ty + zt_{ri}$
- cell phone cost for call package = $f_{c} = t_{ct} + ty + zt_{ci}$
- cell phone cost for sms package = $f_{s} = 60t_{st} + t_{ss} + zt_{si}$
- cell phone cost for internet package = $f_{i} = 60t_{it} + t_{is} + zt_{ii}$
The total cell phone cost for model of Type 2 is given in the following paragraph:

\[ f_2 = 60x_{t_{rn}} + 60x_{t_{rsb}} + y_{t_{rsf}} + y_{s_{rsb}} + z_{t_{rsi}} \]

The total cell phone cost for model of Type 3 is given in the following paragraph:

\[ f_3 = 60x_{t_{rn}} + 60x_{t_{rsb}} + y_{t_{rsf}} + y_{s_{rsb}} + z_{t_{rsi}} \]

3. SIMULATION RESULT

We use default rate on Table 3.1 for testing Type 1 model using our software. If we set cell phone call per-day is 10 minutes, sms uses per-day is 20, and internet uses per-day is 1000 kbyte, the total cell phone cost for regular package is Rp. 20000,-, the total cell phone cost for call package is Rp. 9000,-, and the total cell phone cost for internet package is Rp. 17000,-. The simulation result shows that the costumer should choose call package.
By changing some initial values, we get the simulation result on Table 3.2.

**Table 3.2 Simulation Result of TYPE 1**

<table>
<thead>
<tr>
<th>NO</th>
<th>Call</th>
<th>SMS</th>
<th>INTERNET</th>
<th>COST (PACKAGE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>REGULAR</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>900</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>1050</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1200</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>2250</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>2400</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>2550</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>20</td>
<td>1000</td>
<td>14000</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
<td>399</td>
<td>1995</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>400</td>
<td>2000</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>0</td>
<td>401</td>
<td>2005</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>16</td>
<td>679</td>
<td>8195</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>16</td>
<td>680</td>
<td>8200</td>
</tr>
<tr>
<td>19</td>
<td>2</td>
<td>16</td>
<td>681</td>
<td>8205</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>32</td>
<td>1792</td>
<td>18560</td>
</tr>
<tr>
<td>21</td>
<td>10</td>
<td>10</td>
<td>5000</td>
<td>38500</td>
</tr>
</tbody>
</table>

According the result on Table 3.2, we can conclude that for Type 1 model:

- Regular package is the cheapest cell phone cost for maximal six sms per-day (row 1).
- If the cell phone uses for sms are more than 7 sms, it will be better to choose sms package (row 1 and 2).
- If the cell phone uses per-day are 1 minute for calling and 0 – 7 for sms, the cheapest cell phone cost is for call package (row 3 and 4).
- If the cell phone uses per-day are 1 minute for calling and more than 8 for sms, the cheapest cell phone cost is for sms package (row 3, 4, 5 and 6).
- If the cell phone uses per-day are 1000 byte for internet, 5 minutes for calling and 20 for sms, the cheapest cell phone cost is for call package (row 13).
- If we only use internet for more than 400 kbyte, it will be better to choose internet package (row 14, 15 and 16).
- If the cell phone uses per-day are 680 byte for internet, 2 minutes for calling and 16 for sms, the cheapest cell phone cost is for call package (row 17, 18, and 19).
- Row 21 shows that if the cell phone uses per-day are 5000 byte for internet, 10 minutes for calling and 10 for sms, the cheapest cell phone cost is for internet package.

**Mathematical Analysis of Type 1 Model**

Mathematically, to calculate cost of the cell phone uses by value on Table 3.1 is given in the following paragraph:

Let $x$, $y$, $z$ be the cell phone call uses (minute), sms uses, and internet uses (kbyte) per-day, respectively. We calculate the total cell phone uses as the following equations:

- Regular package ($C_R$)

...
\[
\begin{align*}
  f_R &= 20.60x + 150y + 5z = 1200x + 150y + 5z \\
  f_T &= 1000 + 150y + 5z \\
  f_S &= 20.60x + 1000 + 5z = 1200x + 1000 + 5z \\
  f_I &= 20.60x + 150y + 2000 = 1200x + 150y + 2000
\end{align*}
\] (1)

- Call package \((f_T)\)
- Sms package \((f_S)\)
- Internet package \((f_I)\)

The cost of all packages will be in the same value if only if:

\[
f_R = f_T = f_S = f_I
\]

By substituting equation (1)-(4), we get:

\[
x = \frac{5}{6}; \quad y = \frac{20}{3}; \quad z = 400
\]

Equation (5) is upper bound of call phone uses for regular package per-day. It means that regular package is the best package for all call, sms, and internet uses, if the uses of them is llover than the value in equation (5), see rows 1, 14 and 15 on Table 3.2

By substituting equation (2) to equation (3), we get:

\[
\frac{x}{y} = \frac{1}{8}
\]

Equation (6) shows that the equilibrium value for both call package and sms package occur if the comparition between the cell phone call uses to the sms uses is 1/8, see rows 5, 17, and 18 on Table 3.2.

Substitute equation (2) to (4), we get:

\[
-1200x + 5z = 1000
\]

Finally, substitute equation (3) to (4), we get:

\[
-150y + 5z = 1000
\]

According the previous equations, we can determine the cheapest package for the cell phone uses per day, that are:

1. Regular package is the cheapest package, if the phone call uses per-day are not more than: 5/6 minutes for calling, 6 for sms, and 400 kb for internet.
2. Call package is the cheapest package, if it satisfies the following conditions:
   - \(x > \frac{5}{6}; \quad y > \frac{20}{3}; \quad z > 400\)
   - \(\frac{x}{y} < \frac{1}{8}\)
   - \(-1200x + 5z < 1000\)
3. Sms package is the cheapest package, if it satisfies the following conditions:
   - \(x > \frac{5}{6}; \quad y > 6; \quad z > 400\)
   - \(\frac{x}{y} < \frac{1}{8}\)
   - \(-150y + 5z < 1000\)
4. Internet package is the cheapest package, if it satisfies the following conditions:
   - \(x > \frac{5}{6}; \quad y > 20/3; \quad z > 400\)
   - \(-150y + 5z > 1000\)
   - \(-1200x + 5z > 1000\)

CONCLUSION

We construct three models for cell phone uses according various cell phone uses rate from some providers. We develop a software to calculate the total cell phone uses for two aims: 1) helping consumers to choose the appropriate cell phone provider. 2) optimizing the uses of cell phone package.

REFERENCES

SENSITIVITY ANALYSIS OF LINEAR PROGRAMMING MODEL WITH PARAMETER COEFFICIENTS OF THE OBJECTIVE FUNCTION IN THE FORM OF TRIANGULAR FUZZY NUMBERS

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Article Info

ABSTRACT

Sensitivity analysis is an analysis of the effects of changes in parameters of linear programming model having known optimal solutions. Change the parameters in question are the parameters of the objective function coefficients and parameter constraints, in form of crisp and fuzzy numbers. This study only addressed changes in the objective function coefficient parameters of a basic variable and a non-basic variable, especially the sensitivity analysis of linear programming model with parameter coefficients of the objective function in the form of triangular fuzzy numbers.

Keywords :
Sensitivity analysis, Parameters of the objective Function coefficients, Triangular fuzzy numbers.

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1. INTRODUCTION

Sensitivity Analysis is a basic tool for studying perturbations in optimization problems [1, 2]. We discuss how changes in an LP’s parameters effect the optimal solution. There are six types of changes in an LP’s parameters, change the optimal solution in classical linear programming [10]:

i. Changing the objective function coefficient of nonbasic variable
ii. Changing the objective function coefficient of basic variable
iii. Changing the right-hand side of constraint
iv. Changing the column of a nonbasic variable
v. Adding a new variable at activity
vi. Adding a new constraint.

That’s all, sensitivity analysis in LP with crisp parameters and soft constraint. Many application problems, modeled as mathematical programming problems may be formulated with uncertainty. The concept of fuzzy mathematical programming at general level was first proposed by Tanaka et al [4] in the framework of the fuzzy decision of Bellmann and Zadeh [7]. The first formulation of fuzzy linear programming (FLP) was proposed by Zimmermann [5], Maleki and Zaeimazad [8] used improving the Zimmermann method (IZM) for solving FLP. Concept of duality in fuzzy number linear programming problems proposed by Mahdavi-Amiri [9]. Kumar and Bhatia [1, 2] proposed a new method for solving sensitivity analysis for FLP problems, and then they used parameters are represented by interval-valued fully fuzzy numbers that proved by Jin-Shieh Su [6].

2. CONTENT OF STUDY

Preliminaries

There are several ways to define the fuzzynumbers; the most common is a subset fuzzy numbers in R that is normalization and connectively. Two of the most commonly used class is the trapezoidal fuzzy numbers and triangular fuzzy numbers according Susilo [3]. Fuzzy numbers which will be defined in this paper is a fuzzy triangular numbers. In addition to interval-valued fuzzy numbers defined interval (interval-
valued fuzzy numbers), and ranking fuzzy interval-valued numbers that have been formulated by Jin-Shieh Su [3] to obtain the solution of linear programming fuzzy numbers (FLP). First time, sensitivity analysis of FLP is proposed method by Bhatia and Kumar [1, 2]. This paper will examine the sensitivity analysis are proposed method by Kumar and Bhatia on changes in the objective function coefficient of basic variable and non-basic variable.

**Definition 1** Fuzzy point
\( \tilde{a} \) called a fuzzy point if the number of membership functions in \( \mathbb{R} \) is
\[
\mu_{\tilde{a}}(x) = \begin{cases} 
1 & x = a \\
0 & x \neq a 
\end{cases}
\]

**Definition 2** Fuzzy Triangular numbers
Let \( \tilde{A} \) is a fuzzy numbers in \( \mathbb{R} \) with membership function:
\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{b-a} & a \leq x \leq b \\
\frac{c-x}{c-b} & b \leq x \leq c \\
0 & \text{otherwise} 
\end{cases}
\]

written \( \tilde{A} = (a, b, c) \), \( \mu_{\tilde{A}}^L(x) = \frac{x-a}{b-a} \), \( a \leq x \leq b \) is called the membership function of the left, and
\[
\mu_{\tilde{A}}^U(x) = \frac{c-x}{c-b} \), \( b \leq x \leq c \) is called the membership function right.

**Definition 3** \( \tilde{B} \) called a level \( \lambda \) fuzzy number \( 0 < \lambda \leq 1 \), if its membership function is
\[
\mu_{\tilde{B}}(x) = \begin{cases} 
\lambda(x-a) & a \leq x \leq b \\
\lambda(c-x) & b \leq x \leq c \\
0 & \text{otherwise} 
\end{cases}
\]

written \( \tilde{B} = (a, b, c; \lambda) \).

**Definition 4** Let \( \tilde{A} = (a, b, c; \lambda) \) and \( \tilde{A}^U = (a, b, c; \rho) \) is level \( \lambda \) and \( \rho \) fuzzy numbers respectaly, with \( 0 < \lambda \leq \rho \leq 1 \) and \( p < a < b < c < r \), then an interval-valued fuzzy level \( (\lambda, \rho) \) \( \tilde{A} \) in \( \mathbb{R} \) is defined
\[
\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a, b, c; \lambda), (p, b, r; \rho)]
\]

The family of all level \( (\lambda, \rho) \) fuzzy number is defined
\[
F_{IN}(\lambda, \rho) = \{[(a, b, c; \lambda), (p, b, r; \rho)] | p < a < b < c < r, p, a, b, c, r \in \mathbb{R} \}
\]

**Definition 5** Let \( \tilde{A}, \tilde{B} \in F_{IN}(\lambda, \rho) \), \( k \in \mathbb{R} \)
\[
\tilde{A} \oplus \tilde{B} = [\tilde{A}^L \oplus \tilde{B}^L, \tilde{A}^U \oplus \tilde{B}^U]
\]
\[
\tilde{A} \otimes \tilde{B} = [\tilde{A}^L \otimes \tilde{B}^L, \tilde{A}^U \otimes \tilde{B}^U]
\]
\[
k\tilde{A} = [k\tilde{A}^L, k\tilde{A}^U]
\]

**Property 1** Let \( \tilde{A} = [(a, b, c; \lambda), (p, b, r; \rho)] \) and
\[
\tilde{B} = [(d, e, g; \lambda), (u, e, w; \rho)] \in F_{IN}(\lambda, \rho) \) then
(i) \( \tilde{A} \oplus \tilde{B} = [(a + d, b + e, c + g; \lambda), (p + u, b + e, r + w; \rho)] \)
(ii) \( \tilde{A} \otimes \tilde{B} = [(ad, be, cg; \lambda), (pu, be, rw; \rho)] \)
(iii) If \( k > 0 \), then \( k\tilde{A} = [(ka, kb, kc; \lambda), (kp, kb, kr; \rho)] \)
(iv) If \( k < 0 \), then \( k\tilde{A} = [(kc, kb, kr; \lambda), (ka, kb, kc; \rho)] \)
Definition 6 Let $\tilde{A} = [(a, b, c; \lambda), (p, b, r; \rho)] \in F_{IN}(\lambda, \rho)$, then $\tilde{A}$ ranking is defined $\mathcal{R}(\tilde{A}) = \frac{1}{\lambda} \int_{0}^{\lambda} d^* \left( [A^L(\alpha), A^U(\alpha)] \cup [A^L(\lambda), A^U(\alpha)], 0 \right) d \alpha + \frac{1}{\rho} \int_{0}^{\rho} d^* \left( [A^L(\alpha), A^U(\alpha)], 0 \right) d \alpha = \frac{1}{8} \left[ 6a + a + c + 4(p + r) + 3(2b - p - r) \right]$

Definition 7 Let $\tilde{A}, \tilde{B} \in F_{IN}(\lambda, \rho)$ then

$$\tilde{B} \lessdot \tilde{A} \in \mathcal{R}(\tilde{B}) \subset \mathcal{R}(\tilde{A})$$

Property 2 Let $\tilde{A}, \tilde{B}, \tilde{C} \in F_{IN}(\lambda, \rho)$ then

(i) $(F_{IN}(\lambda, \rho), \lessdot, \approx)$ satisfies the law of trichotomy
(ii) $(F_{IN}(\lambda, \rho), \lessdot, \approx)$ satisfies the following ordering relation
(iii) $\tilde{A} \approx \tilde{B}$ if and only if $\tilde{A} \lessdot \tilde{B}$ and $\tilde{B} \lessdot \tilde{A}$

Definition 8 Let $\tilde{A}_n, n = 1, 2, \ldots, \tilde{B} \in F_{IN}(\lambda, \rho)$. If $\tilde{A}_n \lessdot \tilde{B}$ then $\tilde{B} = \max_{n \in \{1, 2, \ldots\}} \tilde{A}_n$

Sensitivity Analysis for Fuzzy Linear Programming Problems

We denote

Maximize $\tilde{z} = \sum_{j=1}^{n} \tilde{c}_j x_j = (x_1, \tilde{c}_1) \oplus (x_2, \tilde{c}_2) \oplus \ldots \oplus (x_n, \tilde{c}_n)$
Subject to:
$\sum_{j=1}^{n} \tilde{a}_{kj} x_j \lessdot \tilde{b}_k \in [1, 2, \ldots, n]$
$x_j \geq 0 \quad j = 1, 2, \ldots, n$

Based on the definition of interval-valued fuzzy set and fuzzy ranking is proposed by Jin-Shieh Su [6] have shown a fuzzy linear programming problems

Maximize $\tilde{z}^* = \sum_{j=1}^{n} \tilde{c}_j^* x_j$
Subject to:
$\sum_{j=1}^{n} \tilde{a}_{kj} x_j \lessdot \tilde{b}_k \in [1, 2, \ldots, n]$ (FLP$_1$)

Sensitivity analysis of fuzzy linear programming problem is proposed method by Bhatia and Kumar [2] as follows:

Maximize $\tilde{z} = \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j = (\tilde{c}_1 \otimes \tilde{x}_1) \oplus (\tilde{c}_2 \otimes \tilde{x}_2) \oplus \ldots \oplus (\tilde{c}_n \otimes \tilde{x}_n)$
Subject to:
$\sum_{j=1}^{n} \tilde{a}_{kj} \otimes \tilde{x}_j \lessdot \tilde{b}_k \in [1, 2, \ldots, m]$
$\tilde{x}_j = [(m_j - \beta_{j2}, m_j, m_j + \beta_{j2}; \lambda), (m_j - \beta_{j1}, m_j, m_j + \beta_{j1}; 1)]$
$\tilde{c}_j = [(\beta_{j2} - \delta_{j2}, \beta_{j2}, \beta_{j2} + \delta_{j2}; \lambda), (\beta_{j3} - \delta_{j3}, \beta_{j3}, \beta_{j3} + \delta_{j3}; 1)]$
$\tilde{a}_{kj} = [(a_{kj} - \delta_{kj2}, a_{kj}, a_{kj} + \delta_{kj2}; \lambda), (a_{kj} - \delta_{kj1}, a_{kj}, a_{kj} + \delta_{kj1}; 1)]$
$\tilde{b}_k = [(b_k - \alpha_{k2}, b_k, b_k + \alpha_{k2}; \lambda), (b_k - \alpha_{k1}, b_k, b_k + \alpha_{k1}; 1)]$
$0 < \delta_{j2} < \delta_{j1} < c_j, 0 < \delta_{j3} < \delta_{j4}$
Thus, interval-valued fully fuzzy linear programming problems is

\[
\text{Maximize } \left( \frac{1}{16} \left( 16c_j + \delta_{j3} - \delta_{j2} + (4 - 3\lambda)(\delta_{j4} - \delta_{j1}) \right) m_j + \right)
\]

Subject to:

\[
\sum_{j=1}^{n} \left( \frac{1}{\beta_{j1}(4 - 3\lambda)(a_{kj} + \delta_{kj1}) + \beta_{j2}(\delta_{kj2} - a_{kj}) + \beta_{j3}(a_{kj} + \delta_{kj3}) + \beta_{j4}(4 - 3\lambda)(a_{kj} + \delta_{kj4}) \right) m_j \leq \left[ 16b_k + \alpha_{k_3} - \alpha_{k_2} \right]
\]

The steps method Sensitivity Analysis FLP

The steps of the proposed method are as follows:

**Step 1** We convert the chosen interval-valued fully linear programming problem into the crisp linear programming problem.

**Step 2** Solve the crisp linear programming problem to find the optimal solution \( \{m_j, \beta_{j1}, \beta_{j2}, \beta_{j3}, \beta_{j4}\} \).

**Step 3** Find the fuzzy optimal value of chosen interval-valued fully linear programming problem by substituting the values of fuzzy optimal solutions.

**Step 4** Applying the existing sensitivity analysis technique the optimal solution of crisp linear programming problem and discuss the effect of changing the cost coefficient objective function.

Maximize
\[
Z^* = C_{BV}^* X_{BV}^* + C_{NBV}^* X_{NBV}^*
\]

Subject to:

\[
B^* X_{BV}^* + N^* X_{NBV}^* \leq b^*
\]

\[
X_{BV}^*, X_{NBV}^* \geq 0
\]

**FLP_4**

\( X_{BV}^* \) = basic variables, \( X_{NBV}^* \) = non-basic variable

(i) The effect changing the cost coefficient basic variable

We compute what \( C_{BV}^*(B^*)^{-1} \) will be if it changing the cost coefficient basis variable \( c_j^* = c_j^* + \Delta \).

Then \( C_{BV}^*(\Delta) = ( \ldots, c_k^* + \Delta, \ldots ) \)

We can now compute

\[
\tilde{c}_j^* = c_{BV}^*(B^*)^{-1} a_j^* - c_j^* ; \tilde{c}_j^* \geq 0
\]
The effect changing the cost coefficient non-basic variable

Calculate all coefficients of the objective function value changes the non-basic variable to the formula:

\[ \bar{c}_j^* = c_{BV_j}(B^*)^{-1} \Delta c_j^* \]

\( B^* \) non-singular

Substituting the effect changing the cost coefficient fuzzy variable is as follows

\[ \tilde{c}_j = \left[ (c_j + \Delta - \delta_{j2}, c_j + \Delta, c_j + \Delta + \delta_{j3}; \tilde{\lambda}) \right] (c_j + \Delta - \delta_{j1}, c_j + \Delta, c_j + \Delta + \delta_{j4}; 1) \]

Numerical Illustrations

In this section, the proposed method is illustrated with the help of a numerical example.

Let the crisp linear programming problems

Maximize \( z = 20x_1 + 35x_2 \)

Subject to:

10\( x_1 + 15x_2 \leq 250 \)
25\( x_1 + 10x_2 \leq 300 \)
\( x_1, x_2 \geq 0 \)

Assuming

\( \delta_{11} = 7, \delta_{12} = 3, \delta_{13} = 8, \delta_{14} = 9, \delta_{21} = 5, \delta_{22} = 4, \delta_{23} = 6, \delta_{24} = 8, \delta_{111} = 5, \delta_{112} = 1, \delta_{113} = 2, \delta_{114} = 3, \delta_{211} = 4, \delta_{212} = 3, \delta_{213} = 9, \delta_{214} = 8, \delta_{121} = 7, \delta_{122} = 5, \delta_{123} = 4, \delta_{124} = 8, \delta_{221} = 4, \delta_{222} = 2, \delta_{223} = 2, \delta_{224} = 5, \alpha_{11} = 30, \alpha_{12} = 20, \alpha_{13} = 30, \alpha_{14} = 70, \alpha_{21} = 40, \alpha_{22} = 30, \alpha_{23} = 50, \alpha_{24} = 60, \lambda = 0.9, \) and \( p = 1 \).

Step 1

Maximize \[ \tilde{z} = \left[ (17,20,28; 9), (13,20,29; 1) \right] \otimes \tilde{x}_1 \otimes \left[ (31,35,41; 9), (30,35,43; 1) \right] \otimes \tilde{x}_2 \]

Subject to:

\[ \left[ (9,10,12; 9), (5,10,13; 1) \right] \otimes \tilde{x}_1 \otimes \left[ (12,15,18; 9), (11,15,23; 1) \right] \otimes \tilde{x}_2 \]
\[ \leq \left[ (230,250,280; 9), (220,250,320; 1) \right] \]
\[ \left[ (22,25,28; 9), (21,25,34; 1) \right] \otimes \tilde{x}_1 \otimes \left[ (8,10,12; 9), (6,10,15; 1) \right] \otimes \tilde{x}_2 \]
\[ \leq \left[ (270,300,350; 9), (260,300,360; 1) \right] \]

Step 2

Maximize \[ \left[ 20.475m_1 + 1.06\beta_{11} - 1.06\beta_{12} + 1.75\beta_{13} + 2.36\beta_{14} + \right] \]
\[ 35.37m_2 + 2.44\beta_{21} - 1.94\beta_{22} + 2.56\beta_{23} + 3.49\beta_{24} \]

Subject to:

\[ 158.45m_1 + 19.5\beta_{11} - 9\beta_{12} + 12\beta_{13} + 16.9\beta_{14} + 240.3m_2 + 28.6\beta_{21} - 10\beta_{22} + 19\beta_{23} + 29.9\beta_{24} \leq 253.875 \]
\[ 406.5m_1 + 37.7\beta_{11} - 22\beta_{12} + 28\beta_{13} + 44.2\beta_{14} + 161.3m_2 + 18.2\beta_{21} - 8\beta_{22} + 12\beta_{23} + 19.5\beta_{24} \leq 302.875 \]
Step 3

Maximum Solution

\[ m_1 = 0; \beta_{11} = \beta_{12} = \beta_{13} = \beta_{14} = 0 \]
\[ m_2 = 1.06; \beta_{22} = 3.54; \beta_{21} = \beta_{23} = \beta_{24} = 0 \]

Then,

\[ \tilde{x}_1 = [(0,0,0;9),(0,0,0;1)] \] variable non-basis
\[ \tilde{x}_2 = [(1,1,1;9),(1,1,4.54;1)] \] variable basis

\[ \tilde{z}_{maks} = [(17,20,28;9),(13,20,29;1)] \otimes \tilde{x}_1 \oplus [(31,35,41;9),(30,35,43;1)] \otimes \tilde{x}_2 \]

Thus,

\[ \tilde{z}_{maks} = [(3.1,3.5,4.1;9),(3.3,3.5,19.5;1)] \]

Maximize

\[ \tilde{z} = [(20,23,31;9),(16,23,32;1)] \otimes \tilde{x}_1 \oplus [(35,39,46;9),(35,40,48;1)] \otimes \tilde{x}_2 \]
Subject to:

\[ [(9,10,12;9),(5,10,13;1)] \otimes \tilde{x}_1 \oplus [(12,15,18;9),(11,15,23;1)] \otimes \tilde{x}_2 \]
\[ \leq [(230,250,280;9),(220,250,320;1)] \]
\[ [(22,25,28;9),(21,25,34;1)] \otimes \tilde{x}_1 \oplus [(8,10,12;9),(6,10,15;1)] \otimes \tilde{x}_2 \]
\[ \leq [(270,300,350;9),(260,300,360;1)] \]

Step 4

Maximize

\[ \begin{bmatrix} 23.475 m_1 + 1.3 \beta_{11} - 1.25 \beta_{12} + 1.94 \beta_{13} + 2.6 \beta_{14} \\ 39.37 m_2 + 2.76 \beta_{21} - 2.19 \beta_{22} + 2.81 \beta_{23} + 3.82 \beta_{24} \end{bmatrix} \]
Subject to:

\[ 158.45 m_1 + 19.5 \beta_{11} - 9 \beta_{12} + 12 \beta_{13} + 16.9 \beta_{14} + 240.3 m_2 + 28.6 \beta_{21} - 10 \beta_{22} + 19 \beta_{23} + 29.9 \beta_{24} \leq 253.875 \] (1)
\[ 406.5 m_1 + 37.7 \beta_{11} - 22 \beta_{12} + 28 \beta_{13} + 44.2 \beta_{14} + 161.3 m_2 + 18.2 \beta_{21} - 8 \beta_{22} + 12 \beta_{23} + 19.5 \beta_{24} \leq 302.875 \] (2)

Maximum Solution

\[ m_1 = 0; \beta_{11} = \beta_{12} = \beta_{13} = \beta_{14} = 0 \]
\[ m_2 = 1; \beta_{22} = 3.54; \beta_{21} = \beta_{23} = \beta_{24} = 0 \]

Maximize \( C_{BV} X_{BV}^* + C_{NBV} X_{NBV}^* \)
Subject to:

\[ B^* X_{BV}^* + N^* X_{NBV}^* \leq b^* \]
\[ X_{BV}^*, X_{NBV}^* \geq 0 \]

\[ C_{BV} = \begin{bmatrix} 39.37 & 3.82 \\ 240.3 & 29.9 \end{bmatrix} \]
\[ C_{NBV} = \begin{bmatrix} 23.475 & 1.3 & -1.25 & 1.94 & 2.6 & 2.76 & -2.19 & 2.81 \\ 161.3 & 19.5 \end{bmatrix} \]
The effect changing the cost coefficient basic variable

If \( C_{B V} = (39.37 + \Delta, 3.82) \), then \( c_{B V}^*(B^*)^{-1} = (-1 - 0.14\Delta, 1.97 - 0.22\Delta) \)

The coefficient of each non-basis variable in the new row 0 is as follows:

\[ \bar{c}_1^* = 38.1 + 1.8\Delta \geq 0 \]
\[ -1 - 0.14\Delta \geq 0 \]
\[ 1.97 + 0.22\Delta \geq 0 \]

By inequality (1), (2) and (3) obtained

\[ -5.56 \leq \Delta \leq 8.95 \]

\[ \bar{c}_2 = [(35 + \Delta, 39 + \Delta, 46 + \Delta; 9), (35 + \Delta, 40 + \Delta, 48 + \Delta; 1)] \]

(ii) The effect changing the cost coefficient non-basic variable

We can compute \( c_{B V}^*(B^*)^{-1} = (39.37, 3.82)(-0.14, 0.22, 1.18, -1.75) = (-1, 2) \)

Then, \( \bar{c}_1^* = (9.9, 25.4) - (23.48 + \Delta) = 17.4 - \Delta \geq 0 \)

\[ \bar{c}_1 = [(20 + \Delta, 23 + \Delta, 31 + \Delta; 9), (16 + \Delta, 23 + \Delta, 32 + \Delta; 1)] \]

3. CONCLUSIONS
Sensitivity analysis method of fuzzy linear programming problem is proposed by Bhatia and Kumar [2] is a new method that uses interval-value fuzzy linear programming. Until now, there is no other method more effective.

REFERENCES
2. MATHEMATICS EDUCATION
INFLUENCE OF PROBLEM POSING METHOD WITH MULTIMEDIA ON SPATIAL SENSE ABILITY AND MATHEMATICAL DISPOSITION OF THE TENTH GRADE STUDENTS OF VOCATIONAL SCHOOLS IN BOGOR

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* Universitas Pendidikan Indonesia
** SMK Negeri 3 Bogor

ABSTRACT

Problem posing method is one alternative to increase students’ spatial sense ability and mathematical disposition in learning geometry. In this method, students have a chance to construct their own understanding of mathematical concepts they learn by posing mathematical problems by themselves. The purpose of this experiment research is to investigate whether the achievement of students’ spatial sense and mathematical disposition in the experiment class is higher than the achievement in control class and also to examine whether there is any correlation between spatial sense ability and mathematical dispositions. Seventy two students were taken as samples out of all population the first grade students in SMK Negeri 3 Bogor. The experiment class is treated by problem posing method and the control class is given conventional learning method. From the calculation of the gain scores of spatial sense and mathematical disposition by using t test formula, the significance value obtained are 3.13 and 3.37 which are higher than the t table value 1.67. This means that the increase of spatial sense and mathematical disposition in the experiment class is higher than in the control class. From the calculation of the correlation test, the r value is 0.722 which is higher than the t table value that is 0.329 with the level of significance 95%, which means there is a significant relationship between spatial sense ability and mathematical disposition in learning geometry by using problem posing method with multimedia.

INTRODUCTION

Mathematics as one of the lessons that student must take at school has an important part in student’s reasoning quality process, so that students can improve their thinking ability such as logically, rationally and systematically thinking. Geometry as one of the subject which is can be use to attain that ability. Based on Sabandar (2002), in learning geometry students can form their systematical behavior, so they can understand the concept about relation between plane and space.

Basically, students are easier to understand geometry than the other subjects at mathematics. It is because student can found geometry’s form in their daily life, so geometry should be the easiest subject to be understood. But in reality, there are many students in Indonesia that have difficulties in understanding the geometry’s concepts. From the national test data, students who can answer the question about geometry is just 64.78% (puspendik, 2011).

One of the factors that caused the students’ difficulties in learning geometry is because geometry has an abstract characteristic. Kariadinata (2010) said in his experiment that there are many problems in geometry that need visual in problem solving, and generally students still have difficulties to construct geometrical space. This difficulty occurs because students’ mathematical spatial and abstract thinking abilities are still low.

One of the steps to improve students’ mathematical spatial sense ability is by giving them geometry learning with the right learning method. One of the important factors that can effect student achievement in mathematic is their mathematical disposition. Generally, National Council of Teachers of Mathematics (2000) define mathematical disposition as the way to think and act positively that can be seen in students’ interest and their selves confident in solving mathematical problems, and their want to reflect their thinking in mathematical learning process. Mathematical disposition can develop in interesting and challenging learning process, and it can motivate students to explore and develop their thinking and reasoning abilities. But in reality, there can be...
found the class that the students there have the mathematical disposition in low grade. It can be caused by teachers’ abilities, subject of the lesson, interaction pattern, and the method that used in mathematic learning.

One of the learning methods that can develop students’ mathematical disposition and spatial abilities is problem posing method. Problem posing method can be defined as one of the learning methods that give the students not only to get the subject and do the exercises, but also students can make and pose the mathematic problems and solve it by theirselves (NCTM,1989). This mathematic problem arrangement can based on new problem that taken from students’ experience or modified problem that already made by teacher. By formulate the problem theirselves, students can construct their mathematic understanding based on their experience.

Bruner (in Resnick & Ford, 1981) said that the best way to learn for student is to understand the mathematics’ concept, theorem, and principle by doing representation arrangement theirselves. This understanding development process is more important that the result, because the understanding about subject is more meaningful if the students do it by theirselves. So, by using problem posing method, students have the chance to get the meaningful learning experience in such interesting and challenging ways, so that they can improve their mathematic disposition and spatial abilities.

In order to make learning geometry more interesting and easier to understand by students, using multimedia as tool in learning process is important, especially for subject that difficult to understand by using ordinary method and media. Jacobs & Schade (in Munir, 1992) in their experiment said that people’s memories percentage just 1% from reading. People’s memories can be upgrade until 60% by using multimedia. So, in can be concluded that multimedia can make learning process more systematic, communicative, and interactive. The kind of multimedia that can be used for this experiment is by using slide show from power point, so the subject of geometry can be informed in more interesting and comprehensive way.

From the facts above, it can be assumed that learning process with problem posing method by using multimedia as power point can improve students’ mathematical disposition and spatial abilities. The proposes of this experiment are:

1. To acknowledge the development of students’ spatial sense ability who use problem posing method with power point in their learning process.
2. To acknowledge the development of students’ mathematical disposition ability who use problem posing method with power point in their learning process.
3. To acknowledge correlation between students’ spatial sense and mathematical disposition abilities.

EXPERIMENT’S METHOD

This experiment is quasi experiment by using pretest-protest group design. In this arrangement, experiment class and control class are determined from the population. Then, both of classes get the same pretest (O). Experiment class gets certain treatment variable which is problem posing method (X) in learning process, while Control class gets conventional method. After that, both of classes get the same protest (O). The arrangement pattern can be shown as:

<table>
<thead>
<tr>
<th>Pretest</th>
<th>Treatment</th>
<th>Protest</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>X</td>
<td>O</td>
</tr>
<tr>
<td>O</td>
<td>O</td>
<td></td>
</tr>
</tbody>
</table>

Note: O = Pretest and Protest
X = Problem posing method

This experiment consists of three steps, there are:

1. Preparation step that includes instrument making and developing, learning tools preparation, testing, instrument validation and remaking which have to be done by giving pretest and posttest about students’ spatial sense ability, including giving questionnaire about students’ mathematical disposition.
2. Implementation step that includes sample choices, giving pretest, implementation of problem posing method in learning process, and giving posttest after finishing treatment for experiment class.
3. Analysis step of the result of experiment’s data

Analysis of the result of experiment’s data is with statistic. Before testing difference of mean, we test the normality using Kolmogorov-Smirnov test in significance $\alpha = 0.05$ and homogeneity by using F Test to guarantee that data are from homogeny and has normal distribution. After that, calculate gain score of both of control and experiment classes.

Experiment’s Instrument

In this experiment, there are three instruments that are used such as:

1. Spatial sense ability test to measure students’ spatial sense ability. This test consists of five essay questions
2. Students’ response questionnaires about mathematical disposition. These questionnaires using Linkert behavior Scale to acknowledge the degree of students’ mathematical disposition before and after treatment in the experiment.

3. Observation manual to study the activities of students and teacher during learning process of both of control and experiment classes.

EXPERIMENT’S RESULT

Spatial Sense Ability Test

There is no difference between Mean of score of pretest of students’ spatial sense ability who use whether problem posing method or conventional method in learning process which are 14,72 and 15,25. While mean of score of posttest of students’ spatial sense ability who use problem posing method are higher than student who use conventional method which are 25,42 and 23,56.

The value of t-calculation from mean test N-gain is 3,13 and this value is bigger than the value of t-table = 1,67, so Ho is declined. This test shows that development of students’ spatial sense ability in experiment class is bigger and more significant than development of students’ spatial sense ability in control class.

Mathematical Disposition

From the result of data analysis, it can be conclude that development of students’ mathematical disposition ability in experiment class with multimedia is higher than development of students’ mathematical disposition in control class.

From the result of mean of score of pretest questionnaire of students’ mathematical disposition, we get t-calculation = 1,58 which is less than from t-table = 1,67. It means that mean of score of pretest questionnaire of students’ mathematical disposition in experiment class is same with mean of score of pretest questionnaire of students’ mathematical disposition in control class. While the value of significant of score of posttest questionnaire of experiment and control classes is 2,67, it means mean of score of pretest questionnaire of students’ mathematical disposition in experiment class is better than mean of score of pretest questionnaire of students’ mathematical disposition in control class.

From the calculation of difference test of two means with gain value in experiment and control classes, we get index gain value = 3,37, so Ho is declined. This experiment shows that development of students’ mathematical disposition ability in experiment class is higher and significant than development of students’ mathematical disposition in control class.

Observation of Students’ Activities during Learning Process

Based on mean of score from all of aspects of students’ activities that has observed, there is difference of activities development between students who learn using problem posing method and students who use conventional method. For example at indicator that student who does the exercise enthusiastic, mean of score in experiment class is 4,67 while mean score in control class is 2. It shows that development of student enthusiastic in doing the exercise in experiment class is higher than development of student enthusiastic in doing the exercise in control class. For indicator that students can make the summary of the subject and learning activity, mean of score in experiment class is 4 while mean score in control class is 2. It shows that students’ ability to conclude the summary in experiment class is higher than students’ ability to conclude the summary in control class.

Completion of Student

In this experiment, researcher determined the criteria minimum score of completion is 75. From data that are gained, it shows that in experiment class, sum of the total students who pass at spatial sense ability is 36 (100%). It means that students who learn with problem posing method success to reach the completion at spatial sense ability classically. While in control class, students who pass at spatial sense ability is 29 (80,59%). It means that conventional method cat not reach the completion at spatial sense ability classically yet.

1st Hypothesis Test

The hypothesis that is will be tested is:

\[ H_1 : \mu_1 > \mu_2 \]

Development of students’ spatial sense ability whose learn using problem posing method is higher than students who learn using conventional method.

From the result of analysis test by using t-test, we get that the value of t-calculation is 3,13 is higher than t-table which is 1,67 in significant degree \( \alpha = 0,05 \), so Ho is refused. It means that there is difference in students’ spatial sense ability between students who get problem posing method and students who get conventional method in learning process.

2nd Hypothesis Test

The hypothesis that is will be tested is:

\[ H_1 : \mu_1 > \mu_2 \]

Development of students’ mathematical disposition ability whose learn using problem posing method is higher than students who learn using conventional method.
From the analysis test, we get that the value of $t_{\text{calculation}}$ is 3.37 which is higher than $t_{\text{table}}$ for mathematical disposition ability. It means that there is difference in students’ mathematical disposition ability between students who get problem posing method and students who get conventional method in learning process.

3rd Hypothesis Test
We use correlation Product Moment ($r$) statistic to acknowledge that there is correlation between students’ spatial sense ability and students’ mathematical disposition who getting problem posing method in learning process.

The hypothesis that is will be tested is:

$H_0 : r = 0$

There is correlation between spatial sense ability and mathematical disposition ability.

After calculation the value of $r$, we get that $r_{\text{calculation}}$ is higher than $r_{\text{table}}$ in significant degree $\alpha = 0.05$. If the value of $f_{\text{calculation}}$ is less than the value of $f_{\text{table}}$, then $H_0$ is accepted.

From the result of correlation analysis, we get the value of $r_{\text{calculation}}$ 0.722 is higher than the value of $r_{\text{table}}$ 0.329 in significant degree $0.000$ which is less than 0.05. So $H_0$ is refused and $H_1$ is accepted. It means that the higher students’ spatial sense ability the higher students’ mathematical disposition ability. Coefficient of correlation between students’ spatial sense ability and students’ mathematical disposition ability is 0.722 which is high correlation based on interval $0.600 \leq r \leq 0.799$.

The experiment’s result about development of students’ spatial sense ability of students who use problem posing method with multimedia and students who use conventional method can be served as:

<table>
<thead>
<tr>
<th>The Ability that is tested</th>
<th>Problem Posing Method with Multimedia (n = 36)</th>
<th>Conventional Method (n = 36)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pretest</td>
<td>Posttest</td>
</tr>
<tr>
<td>Spatial Sense</td>
<td>$\bar{x}$</td>
<td>14.72</td>
</tr>
<tr>
<td></td>
<td>3.41</td>
<td>1.81</td>
</tr>
<tr>
<td>Mathematical disposition</td>
<td>$\bar{x}$</td>
<td>77.53</td>
</tr>
<tr>
<td></td>
<td>10.24</td>
<td>9.77</td>
</tr>
</tbody>
</table>

Discussion and Investigation
Spatial Sense Ability
The result data of pretest and posttest about spatial sense ability are tested and analyzed statistically. The result of students’ spatial sense ability of either experiment or control classes are low. The mean value of experiment class is 14.72 while control class is 15.25.

From the result of posttest in experiment class who use problem posing method with multimedia, we get mean score 25.42, so we can conclude that the attainment of students’ mathematical spatial ability is average. While the mean score of class who use conventional method is 23.56, so we can conclude that the attainment of students’ mathematical spatial ability is average.

From the calculation of $N_{\text{gain}}$ at class who use problem posing method, we get mean of gain score 0.70, so we conclude that development of students’ spatial sense ability is average. So at class who use conventional method, we get mean of gain score 0.56 and it means that development of students’ spatial sense ability is average too.

From the explanation of analysis data above, we can see that development of students’ spatial sense ability in experiment class is higher than in control class. It is because of in every learning process, students get the chance not only to do the exercise but also to make or pose the problem based on the related subject matter. Beside, students get reinforcement by using learning method based on multimedia that can help them to understand the giving subject matter, so that they are more ready and more confident to face and solve the mathematical problem that they pose before.

Students’ Mathematical Disposition
Based on analysis data of disposition’s questionnaire, we found that development of students’ mathematical disposition in experiment class is higher than in control class. Generally, we can conclude that students in experiment class give the positive response for problem posing method.

This development is effected by using problem posing method with multimedia in learning process. By using power point to show the subject matter, teacher can serve and package the subject matter in the best way. So student in class experiment give the positive response during learning process.

Correlation between Spatial Sense Ability and Mathematical Disposition
From the result of analysis data about correlation between students’ spatial sense ability and mathematical disposition, we conclude that there is correlation between developments of students’ spatial sense ability and mathematical disposition.
ability with development of students’ mathematical disposition in learning process which use problem posing method. It is because of in learning process with problem posing method, students found the new activity that challenge them to think creative and cognitively to arrange and pose mathematical problem. This interesting and challenging learning process effects improvement students’ motivation to learn mathematical. It also causes the development of students’ mathematical disposition in experiment class. Generally, we can conclude that students’ spatial sense ability in learn geometry subject effects the development of students’ mathematical disposition in experiment class with problem posing method.

Completion of Students’ Spatial Sense Ability

Learning process with problem posing method in experiment class have attain 100% in spatial sense ability with the lowest score 23 and the highest score 30 from the maximum score 30. While learning process with conventional method in control class have attain 80,58% with the lowest score 15 and the highest score is 28. From the data above, we can conclude that students’ percentage in attainment of minimum score of experiment class is higher that control class.

Difference of percentage of score attainment from both of classes is because of the advantage of problem posing method from the conventional one. Students get more chance to understand the subject matter by doing activity that they have planned before by their selves such as arrange the mathematical problem based on their daily life and the subject matter, pose it and solve it by their selves. So that students become more active in learning process.

While in control class who use conventional method, teacher becomes a center in learning process. Teacher gets the spotlight from the start until the end of learning process and students just have to pay attention on the teacher’s explanation and do the exercise. It makes students less motivate to prove or explore the subject matter, farther students become lazy and passive to learn the subject matter.

Summary

Based on the analysis and study above, we can conclude some of hypothesis of this experiment:

1. Development of students’ mathematical spatial sense ability which using problem posing method with multimedia is better than which use conventional method. Development of students’ mathematical spatial sense ability is average.
2. Development of students’ mathematical disposition ability which using problem posing method with multimedia is better than which use conventional method. Development of students’ mathematical disposition ability is average.
3. There is high correlation between mathematical spatial sense ability and mathematical disposition ability.

Suggestion

Based on the discoveries from this experiment, writer has some suggestions such as:

1. Mathematics teachers have to improve their ability in using technology so that it can develop the learning tools become more innovative and effective.
2. Implementation of problem posing method needs the right time and class management, so that we need to plan the learning activities effectively in order learning process by using problem posing method can be more optimal and efficient.

REFERENCES


THE APPLICATION OF PROBLEM BASED LEARNING MODEL
ONLINE TUTORIAL IN MATHEMATICS CURRICULUM ANALYSIS

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Universitas Terbuka

ABSTRACT

Analysis Mathematics Curriculum is a course that aims to make students able to analyze the school mathematics curriculum and its learning problems. Based on observations, the students do not have a good ability in terms of analyzing the curriculum. Problem Based Learning (PBL) indicated able to assist students in achieving the expected competencies. Through the issues presented through online tutorial based PBL indicated students able to achieve the expected competencies. The purpose of this study is 1) to determine how the application of PBL model online tutorial in mathematics curriculum analysis, 2) how the student responses on the implementation of PBL model online tutorial in mathematics curriculum analysis; 3) constraints faced in the implementation of PBL model online tutorial in mathematics curriculum analysis. The study was conducted in the tutorial online period 2013.1 with research subjects 40 people. Data were collected through observation and questionnaires were analyzed using descriptive qualitative method. Generally, the implementation of an online tutorial went well. While student responses through the statement "I convey independent opinion on the issues presented in the discussion" get the answers "strongly agree" that is the highest of 86%. The obstacles faced in the implementation of the tutorial include time and difficulty to devided into groups on discussion forums.

1. INTRODUCTION

Curriculum is a set of plans and arrangements regarding the purpose, material / content or learning materials, as well as the methods used to guide how the implementation of learning activities to achieve educational goals (PP No. 19, 2005). On the other hand, mathematics is a discipline that has distinctive properties compared with other sciences, because of different teaching and learning mathematics and science needed to understand the theory. Therefore, the mathematics curriculum is very important to facilitate the transfer of knowledge from teacher to students effectively and efficiently.

Relying on manual Education Unit Level Curriculum (SBC) elementary and secondary created by BSNP, schools were given the freedom to design, and implement curriculum float according to the situation, local conditions and the potential benefits that could be raised by the school. Teachers are expected to analyze the curriculum, because by analyzing the teacher will be able to easily map the material from the simplest to the most difficult level. Through the analysis of the curriculum is also where teachers can find materials that require a long time in their delivery and which require a short time. To study and analyze mathematics curriculum, graduate programs (S2) Mathematics Education, providing courses MPMT5204 Mathematics Curriculum Analysis.

Based on interviews with the tutor of this course, the student analyzes the curriculum is still weak. Tutor students added that weakness is evident from the discussions, assignments and answer student tutorials related to curriculum analysis article that has not reached the expected competencies.

One solution to guide students in analyzing the curriculum is the online tutorial through Problem Based Learning (PBL). PBL is an instructional model that begins with the provision of authentic problems that serve as the foundation for students to investigate. Savery (2006) Through the curriculum issues given students are expected are able to achieve competence. Article will discuss about 1) how the application of PBL models in an online tutorial course analysis of the mathematics curriculum, 2) how the student responses on the implementation of an online tutorial course analysis of PBL-based mathematics curriculum; 3) constraints encountered in the implementation of an online tutorial course analysis of PBL based mathematics curriculum.
Problem Based Learning (PBL)

According to Savery (2006), Problem Based Learning (PBL) is a learning-centered to learners that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem. The problem is as a basis for learners to learning critically and problem-solving skills training as well as getting the basic concepts of science learning. Good problems that must meet several important criteria include: A good problem should meet several important criteria such as:

a. Authentic problem. Problems that learners should be given to the problems associated with real problems.

b. It should occurred a problem or puzzle mystery. With the issues contain puzzles will certainly encourage learners to discuss.

c. The problem should be meaningful to learners according to developmental level.

d. The problem should be large enough so as to be able to meet the instructional objectives.

e. Good problem should benefit from the group.

An educator in a problem-based learning has to give power to the learners to take responsibility for learning. Moreover, an educator must be prepared to give some of his own authority in the classroom for the students. According to Kaufman (1998), the role of facilitators (educators) in a PBL will be effective if:

a. Stimulate communication educators and learners

b. Give full opportunity to the students to discover by themselves what is needed in a matter of learning from the teacher’s own knowledge.

c. Intervention students performed if the discussion the students are out of the specified path.

d. Serves as a resource for students as a determinant of the final conclusions.

e. Provide added value for learners who are looking for additional materials to study on other opportunities.

Each lesson surely has its advantages and disadvantages. According Hajriana (2010), Excess in PBL namely:

a. With PBL will occur meaningful learning. Students who learn to solve a problem then they will apply the knowledge they have or trying to learn the necessary knowledge. Learning can be more meaningful and can be expanded when students are faced with a situation in which the concept is applied.

b. In PBL situation, students integrate knowledge and skills and apply them simultaneously in a relevant context. That is, what they do according to the real situation that is no longer theoretical problems in the application of a concept or theory they will find once during the learning takes place.

c. PBL can improve the ability of critical thinking, fosters student initiative in work, internal motivation to learn, and to develop interpersonal relationships within the group work.

The lack of PBL are:

a. In PBL are needed preparations are very complex as learning tools, problems and concepts.

b. Difficulty of finding relevant problems to learning. Problems or issues that should be given according to the good criteria.

c. In PBL is time consuming.

Online tutorials

Online tutorial is tutorial service internet-based or Web-Based Tutorials (WBT), which is administered by each LPTK and can be attended by students through the Internet. The material provided in the form of text tutorials that can be accessed students wherever they are, without having to meet with a tutor. In this model, the tutor must prepare a tutorial script that allows the interaction between tutor and student. Online tutorials conducted through the Internet by using Moodle software.

Online tutorials Universitas Terbuka is the internet-based tutorial services offered by the Universitas Terbuka and attended by students through the Internet. This tutorial is done by tutors both in the center and at the Universitas Terbuka UPBJJ - UT by providing 8 times initiation including the gives 3 tasks to students participating in tutorials during the tutorial period.

Online tutorials conducted for 8 weeks beginning after the close of registration and registration period begins concurrently with course registration. Online tutorial activities consist of:

- Spread the material initiation of tutors to students as much as 8 times (1 initiation material per week),
- Provision of at least three tasks that must be done by students, and
- Question and Answer activity between tutors and students and among students.

Mathematics Curriculum Analysis

This course provides insight and experience to the students in analyzing the school mathematics curriculum and learning problems and tries to find an alternative solution based on theoretical studies based on the theories of education and relevant research results. This course includes the study of factors that influence the change in the curriculum, the school mathematics curriculum goal, the school mathematics curriculum...
Proceeding Internatioan Seminar on Mathematics, Science, and Computer Science Education

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2. RESEARCH METHOD

The study was conducted on 40 students taking the online tutorial course in mathematics curriculum analysis period 2013. During the period of online tutorials online tutorial based researchers applied the PBL. Arends in Trianto (2007) detailing the steps in the implementation of PBL teaching, there are 5 phases (stages) that needs to be done to implement PBL. Phases are: 1) Orient learners on the problem; 2) Organize students to learn; 3) To lead individual and group inquiry; 4) Developing and presenting the work; 5) Analyze and evaluate the problem-solving process.

Based on these definitions PBL models developed in the following online tutorials:

- **PHASE 1** Orienting students
  - Introduction
  - Explain the purpose and give motivation to the learner
  - Explain away the tutorial
  - Explain the material that will be discussed
  - Give the initiation material

- **PHASE 2** Organized students
  - Define the group
  - Prepare 3 forum discussion and 1 forum (if the question out of context)
  - Make 3 topic discussion in article

- **PHASE 3** Guide individually or group
  - Students discuss to solve the problem
  - Tutor as a facilitator
  - Tutor ask the learner to give references for each answer

- **PHASE 4** Develop and present scientific work
  - Ask to make a conclusion
  - Learner concluded if it is necessary
  - Construct thinking and activity
  - Giving the feedback

- **PHASE 5** Analyze and evaluate problem solving process


After studying this course, students are expected to select and formulate the mathematic problem learning and determine alternative solutions theoretically based on the analysis and assessment of aspects pertaining to the school mathematics curriculum. Competence of the course is broken down as follows. After studying this course, you are expected to explain:

1. The factors that influence the curriculum;
2. Curriculum objectives from perspective of the developed mathematical competence;
3. Parts of the mathematics curriculum especially in the potential to cause problems in learning;
4. Mathematical competence is developed in the mathematics curriculum;
5. Learning principles espoused in the mathematics curriculum;
6. Learning principles that fit the demands of the curriculum;
7. Principles of appropriate use of technology demands of the curriculum;
8. Principles of assessment as demanded by the curriculum;
9. Article analyzes the results of education / learning mathematics.
In this model, students are required to access the tutorial at least once a week, because they are obliged to engage in discussions. Tutors must be on increasing student motivation to follow this tutorial.

3. RESULTS

As has been revealed earlier that this study yielding about 1) to determine how the application of PBL model online tutorial in mathematics curriculum analysis, 2) how the student responses on the implementation of PBL model online tutorial in mathematics curriculum analysis; 3) constraints faced in the implementation of PBL model online tutorial in mathematics curriculum analysis

3.1. The application of PBL model online tutorial in mathematics curriculum analysis

Arend in Trianto (2007) detailing the steps in the implementation of PBL teaching, there are 5 phases (stages) that needs to be done to implement PBL. Phases are: 1) Orient learners on the problem, 2) Organize students to learn. 3) To lead individual and group inquiry; 4) Developing and presenting the work; 5) Analyze and evaluate the problem-solving process. Of these theories, models based online PBL tutorials are developed.

In phase orient students on the problem, the tutor explains the purpose and motivate students, explaining groove tutorial, explaining the material to be studied. In addition, tutors post material initiation. All of the activities at this stage were done on the first day. Still on the first day, researchers conducted a Phase organize learners. In this phase, Tutor students into three groups and three issues for being discussed by each group on the board. Furthermore, the third phase is to guide the investigation conducted on the first day to the sixth day. At this stage, students begin to discuss the topic of discussion has been prepared. In this activity, tutor just as the facilitators.

Hereinafter phase is to develop and present the work. Tutor asks student concludes the discussion and other students can add an incomplete conclusion. This activity is carried out on the sixth day. Analyzing and evaluating the problem-solving process is the final phase of this model. During this phase the tutor asks students to reconstruct the thinking and activities that have been carried out during the process of their learning activities. Feedback is done in this phase is done on the seventh day.

The implementation conducted several discussions regarding the content. In the first discussion forum, students discuss about the definition of curriculum. Because it is still early in the discussion, the researcher does not give a problem but ask students to find information about the definition of curriculum and give opinions on the current curriculum. Students are able to use existing information to identify whether something is reasonably large. In this case the student is able to identify what is meant by the curriculum and are able to identify how the application of each school curricula. Students can review the curriculum and sought understanding the differences and similarities of the entire curriculum ever, is and will be applicable in Indonesia. Students are also able to identify similarities and differences in the position of curriculum and curriculum development process based approaches and curriculum development models that could be applied in accordance with the conditions in which the curriculum implemented.

In the subsequent discussion forums, question given about learning math. Given problem is about the different ways to teach congruent and geometry in elementary, junior high and high school. The differences in this teaching are to be known because the difference in the third stage of thinking are in the different levels, so the treatment must be different. In this case the student has managed to find the equation of a thing when he is faced with two or more types of things. They can also find similarities and differences of a thing when he is faced with two or more types of things. Some students have connected with the solving mathematical learning theory. For example, for elementary students, students connect with according to Piaget's theory of cognitive development. According to him, in an attempt to understand the child's natural surroundings are no longer too to rely on information sourced from the five senses, as children begin to have the ability to distinguish what is visible to the eye with the real one. So, for elementary school students with the deductive approach is a thought process that is based on the statements of a general nature to the things that are special to a particular logic. In addition, the teacher shows and explains with 2 (two) real props shaped something that has the same shape and size.

On the development of junior high school students, college students solving problems connecting with ArjooT.V stating that the cognitive aspects include functions such intellectual understanding, knowledge and thinking skills. In addition, students connect with Jean Piaget problem solving. Cognitive development of children at the time was in Junior High School (SMP), at the stage of "formal operations stage", which is the fourth and final stage of cognitive stages. So for junior high school students, with an inductive approach which presents a number of special circumstances can be concluded to be a principle or rule. As for the development of senior high school students the delivery is almost equal to junior high school students. With inductive approach which presents a number of special circumstances can be concluded to be a principle or rule. In practice, teachers provide two planes in the same size; the teacher directs the two shapes to find a
concept that congruent with properties and geometry with congruent plane property. Then the students draw their own conclusions what the two are congruent up.

When students were asked to analyze the article deals with the problems of mathematics curriculum, in this case the students are asked to find relevant articles and analyze it. In this case the students are required to search for journal articles and then analyze the article. Students should make a statement and gather information that supports the statement, organize information, and provide an explanation for the direct statement in the form of new information.

In the sixth discussion, students are faced with the problem of learning mathematics in the classroom in the form of cases. The first case is when the teacher explains about "Two integers and the sum is 84. What is the greatest product that can occur with both these integers?". In the second case is when the teacher teaches about "ball area". These cases were able to perform critical thinking skills for making decision category. At first the students identify the strengths and weaknesses of teachers when teaching then decide the appropriate strategy to teach the material in the case.

Still in the same discussion, through a case study given to the students were asked to create a lesson plan to teach the material related to the case presented. The Hot issues discussed today is the curriculum of 2013. Pros and cons regarding the application of the curriculum is still being debated. Researchers raised the topic to determine the extent of students' ability to criticize the curriculum. Ibrahim (2000) who argued scales participants are encouraged to work together on a common task and they have to coordinate its efforts to complete the task. In other words, collaboration between learners can to achieve the learning objectives so that students understand the concepts they are learning.

The following are examples of student opinion regarding the curriculum in 2013, namely 1) Curriculum 2013 should be used as an upgrade human resources in the face Era globalisation; 2) Curriculum in 2013 is expected to reduce the pressures and challenging for students both elementary -and middle- level; 3) Curriculum 2013, to do a variety of early preparation; 4) Implementation of Curriculum 2013 in June 2013, teachers will have an important role; 5) Birth of the New Curriculum in 2013 should be more emphasis on attitude or behavior students better; 6) Curriculum 2013 must designed such that students are active in each of the learning material.

3.2. Student Response Results

The response was immediate disclosure of individual methods in a more honest approach and assessment obtained actual condition (Suhanto, 2012). Through questionnaires that are presented, the statement “I convey independent opinion on the issues presented in the discussion of” answers “strongly agree” that is the highest of 86%. In the discussion forum and assignments is much interact student to issue an opinion on a given issue. Through PBL students are expected to be independent in their own opinion, their own sourcing without the help of others. This is consistent with the expression Tambourish (2012) that PBL centered on learner pedagogy, the focus of this study was active, collaborative, and knowledge of learners through engagement with real-world problems or cases.

The percentage of “agree” is the highest for the statement "I am an active role determines reference. "and"I actively participated as a discussion on the issue presented " respectively by 70 % and 60 %.

In this case the tutor always asks the student opinion to argue, but if the opinion is referring to the opinions of others, then the students were asked to search for references. Most of the students have been involved actively in the discussion forum. Tutor only as a facilitator in the implementation of discussion. According to Savery (2006) Problem Based Learning (PBL) is learning -centered learners that empower learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem.

The statement did not agree to get high percentage in the statement “does not correspond to the formation of discussion topics that I am interested in " by 58 %, " I do not discuss all of the information that had been obtained " by 57 %, and " I'm not asking questions " . While the statement is the most strongly disagree “I do not participate actively “. From the results obtained, it appears that students are already quite active following this tutorial. They feel has actively participated and followed this tutorial properly. This is consistent with the revelation that PBL is a learning motivating, challenging, and fun (Norman, 1992) which is done through a process to resolve the problem (Barrows, 1997). Savery (2006) PBL methods to develop specific skills, including the ability to think critically and solve problems such as analyzing complex, real-world problems, to find, evaluate, use resources appropriately, to work together, demonstrate effective communication skills, and using the knowledge and intellectual skills so that learners can continue to be motivated to learn.

Student Opinions about "I am very happy to follow the tutorial online math curriculum analysis "is achieved with 76 % strongly agreed “and 24 % agreed. It can be seen that students feel happy to follow online tutorials based online tutorial PBL based. In addition, students are also happy to discuss the problems faced in accordance with their daily life as a teacher. And analyze curriculum to guide them in teaching.
3.3. Obstacles encountered in the implementation of an online tutorial

When done the research, there are several constraints which are:

a. Tutor time constraints

Obligations tutor in this tutorial is to always access online tutorials and a facilitator in the discussion forum. However, due to busy tutor, tutors cannot access the tutorial every day. So the tutor cannot fully perform its obligations under this PBL based online tutorial. Nevertheless, students still follow tutorial with good and active discussion.

According to Kauffman (1998), the role of facilitators (educators) in a problem-based learning will be effective if: 1) Stimulate communication educators and learners; 2) Provide full opportunities for learners to discover for themselves what is needed in a matter of learning to impose knowledge teachers themselves; 3) Intervention learners performed, if the discussion the students are out of the specified path; 4) Serves as a resource to learners as a determinant of the final conclusion; 5) Provide added value for students who seek additional materials to study on other opportunities.

b. Difficulty of dividing into groups on the discussion forums

In the organizing phase learners organized to learn. Tutor students form groups, where each group will select and solve different problems. In this case the tutor sends a message to students about the grouping. But there are some students who are involved in discussions of other groups. This is the input to the tutor so that for the next tutorial, more active tutor to organize students through group discussions.

Each lesson certainly has its advantages and disadvantages. According Hajriana (2010), PBL advantages: 1) the problem-based learning will occur meaningful learning. Learners who learn to solve a problem then they will apply the knowledge they have or seek to know the necessary knowledge. Learning can be more meaningful and can be expanded when students are faced with a situation in which the concept is applied. 2) In the situation of problem-based learning, students integrate knowledge and skills and apply them simultaneously in a relevant context. That is, what they do according to the real situation that is no longer theoretical problems in the application of a concept or theory they will find during the learning takes place. 3) Problem-based learning can improve the ability of critical thinking, fosters student initiative in work, internal motivation to learn, and to develop interpersonal relationships within the group work. Besides the lack of problem-based learning, namely: 1) In the problem-based learning necessary preparations are very complex as learning tools, problems and concepts; 2) Difficulty in finding relevant to learning problems. Problems or issues that must be given in accordance with the criteria a good problem; 3) in problem-based learning requires a fairly long time.

Not only limited time for tutor, but also not all students are actively discussing and timely task. This is in accordance with the statement Zabit (2010) which states that PBL is very difficult to implement. According to Todd in (Zabit, 2010) The trouble is, (i) the role of educators, teacher inability to understand the material and the rules/processes in PBL makes learning implementation is not going well (ii) the role of the learner, the process of learning through PBL contrary to their study habit, so sometimes they reject this new approach, (iii) reaction of colleagues, associated with individuals or small groups who are around educators and learners. Usually they do not support and defend the traditional teaching methods. They believe that PBL is just a waste of time and ineffective. (iv) Team members, educators need to be aware of very difficult to develop a strategy to support each other and unify the group (v) develops process skills, in this case the ability to solve problems is difficult for many teachers so that they give emphasis to student difficulty in developing and practicing important processes, so that the PBL failed to do.

4. CONCLUSION

In general, the implementation of PBL in the online tutorial course curriculum analysis goes well. This is pretty good evident from the student response. 75% of students believe strongly agree with the statement “I am very happy to follow the tutorial online math curriculum analysis”. While the obstacles encountered in the implementation of this tutorial is limited time tutor, difficulty dividing a group of students on a discussion forum. Addition possessed limited time tutor, not all students are actively discussing and timely task. Implementation and the constraints encountered, is expected through this tutorial online tutorials quality became better. So that students become more comfortable and motivated to follow the tutorial. If the students are comfortable with the learning support services provided so competencies that are expected to be easily achieved.
REFERENCES

Information and communication technology (ICT) is a set of tools and other technologies that are used for the manufacture, storage, management and communication of information. ICT is used in education to support the teaching and learning process in collaboration with student activity. This study focused on the development of learning tools in the form of Lesson Plan and the Student Worksheet to improve the creativity of students in the learning process. Form creativity of students presented in the form of their work in completing the Student Worksheet. In Student Worksheet has also been on the characters that have been written in the indicator in the Lesson Plan. This research is using a learning-based Learning Management System (LMS) with Moodle application. Through the application of Moodle LMS with the characters that appear include: discipline, responsibility, self-contained. The research was conducted in the Department of Mathematics Education at Teachers’ Training College (IKIP PGRI) Semarang Calculus course for first year students. A questionnaire has been administrated for collecting the data. The results of this research are: learning using ICT, which is characterized by the development of learning tools and enhanced student’s creativity, as well as advantages and disadvantages of learning using ICT.

1. INTRODUCTION
Learning in Teachers’ Training College Calculus PGRI Semarang current study focuses on the development of cognitive abilities but rather over ride character education. Results of research that has been conducted in Teachers’ Training College PGRI Semarang conclude that e-learning is used in an effective web-based learning. Previously, Ariyanto [1], also conducted a study on the material Geometry, Geometry conclusion that learning using multimedia such as instructional video is very effective to increase the activity and motivation of learners. Character in the learning achieved include improving student discipline and creativity that has a positive effect on student learning outcomes.

Mathematics education requires a new paradigm made innovations and integrated learning, including using ICT media. As one example of the results of research Rosenberg [2], states that e-learning that uses Internet technology to transmit a series of solutions to improve the knowledge and skills of students buzzing. This is supported by the Ritz [6] states that the useful application of technology to aid learning and increase knowledge. This can be done by integrating technology into science and math. The research of Manuela Pacchier, Brigitte Maier [3] showed that when the concept of a material science or applied already obtained expertise in student learning will refer to tap advance, whereas when the independent learning skills already acquired refer students to online learning. While Prayito [5] in his research concluded that e-learning has been implemented which provide good impact is that it can complete the learning outcomes of students and foster active learners.

On the other hand character education through a planned effort by planting system behavior values (character) to resident education, which includes knowledge, awareness and volition, and actions will shape the whole person[7]. Since one of the goals of character education by Su’ud, et al [7] is to develop students’
ability to be self-sufficient, human, creative, responsible and insightful nationality. The results that have been done [5] that explores the character education lesson study shows that through the character of students, among others, discipline, responsibility and able to work together to increase. Results of this study indicate that the use of models of learning can increase and develop character.

The main problem of this research is to design a learning model based on e-learning as what is appropriate for students in order to assist students in exercising their classroom learning. The purpose of this study is to develop a learning tool to improve student academic achievement through instructional design and visual presentation of instructional design-based learning. In particular, the purpose of this study was generated design-based learning e-learning. The products with the learning tool of syllabi, lesson plans and teaching materials, learning models and the implementation of learning based e-learning.

2. RESEARCH METHOD

Preparation of design and the learning is done in a laboratory scale. Activities to be carried out include:

a. Conducting Needs Analysis
Needs analysis was conducted to determine Calculus-based Learning Design E-learning as to what is appropriate to foster student creativity and character PGRI Semarang Teachers’ Training College. Needs analysis was conducted by observation, interviews with the lecturer of the course of Calculus, and reviewing the results of previous studies as well as literature from books, papers, and articles.

b. Compile Draft Learning Design
At this stage of planning to make the draft Design Learning Calculus-based e-learning to foster creativity and character, preparing materials and material sources.

c. Validate Draft Learning Design
Results of drafting Calculus-based learning design e-learning to foster creativity and character first tested the validity of the experts involved 6 people consisting of 2 people Calculus matter expert, 2 expert evaluation and learning and 2 multimedia expert. Validation is intended to anticipate user error. Matter experts provide an assessment of the content of the material, learning experts assessing aspects of learning, while the multimedia experts provide an assessment of the aspects of the display and programming aspects. Data validation results matter experts, learning experts and multimedia specialists consider to revise the calculus-based learning design e-learning to foster creativity and character.

d. Revised Draft Learning Design
Validas by a team of experts, instructional design instructional materials and devices that have been validated to be repaired if there are discrepancies or errors in the draft, then revise the draft research design the learning according to the records and input from expert validation. The results of this revision to the students and then tested on a small scale the individual trials.

3. RESULT AND ANALYSIS

Development carried out in this study is in the form of lesson learning, learning-based Student’s Worksheet and e-learning. In this research, E-learning using learning management system (LMS).

3.1. Lesson Plan
The validator obtained from the assessment feedback, corrections, and suggestions are used as consideration in doing repair or revision lesson plan. Discussions with the validator while other revisions can be seen in Table 1. From Table 1, Lesson Plan general improvement lies in how the character and creativity appear explicitly in the learning process. Thus it can be used lesson plan clearly and can be used by anyone who uses it.

<table>
<thead>
<tr>
<th>Indicators of cognitive load on the Lesson Plan, Affective and Psychomotor Character appears on each indicator Creativity in Lesson Plan had entered in the indicator</th>
<th>After Validation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indicators on the RPP enough on Cognitive and affective. Desirable traits must not appear all Creativity is more explicitly visible in the indicator</td>
<td>Adapted to the material Customizable characters denego material</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Revision lesson plan
3.2. Student Worksheet

The validator obtained from the assessment feedback, corrections, and suggestions are used as consideration in doing repair or revision Student Worksheet. Some errors and suggestions validator can be seen in Table 2. In the Student Worksheet improvement is more on how the questions presented in it can be a guide for students, so that students become creative mindset and not monotonous. In relation to questions about the Student Worksheet presented contextually so that students will be many different perspectives, but remains in critical thinking to find solutions of each of each matter.

Table 2. Revision Student Worksheet

<table>
<thead>
<tr>
<th>No.</th>
<th>Before Validation</th>
<th>After Validation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student’s Worksheet contains questions guided exercises</td>
<td>The questions and exercises should be building concept problem presented contextually</td>
<td>Student creativity can flourish</td>
</tr>
<tr>
<td>2</td>
<td>About the matter presented in the abstract</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.3. Advantages and disadvantages of E-learning

Design of e-learning instructional media as in Figure 1 below brings benefits to students that students can access lecture material anytime and anywhere as long as it has a connection to the internet. Students can do their job without having to follow lectures and to collect the duties well. With the use of e-learning in the student can work independently so the ideas can be written in the work of Student’s Worksheet. Students can interact with the professor candidly and freely regardless of others’ opinions. Thus students will increase the sense of responsibility towards himself in following the learning process.

Figure 1. Design E-learning Calculus I
Weaknesses of learning by using e-learning or ICT in general, the learning process ridak appear. Because that leads into the ICT-based learning ore-learning is the result rather than the process. Communication between students and students and even students with the professor could be very limited. So emotional, or learning discussions that appear if the lecture material alone, do not touch on things like how the social nature of communication in working together is not visible in the process.

4. CONCLUSIONS

Design learning tools such as lesson plans and Student’s Worksheet in ICT-based learning in e-learning opportunities for students to be able to think creative in exploring the information lectures. Creativity is still within the framework of critical thinking is to look for a solution of the problem issues in the lecture Calculus I. So that characters such as self, responsibility, confidence will remain in the student mindset. With learning support learning e-learning more open to ideas.

REFERENCES

PROBLEM-BASED LEARNING APPROACH USING DYNAMIC GEOMETRY SOFTWARE TO ENHANCE MATHEMATICS CRITICAL AND CREATIVE THINKING ABILITIES

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ABSTRACT
Thinking critically and creatively in mathematics are important ability for each student in learning. To enhance this ability, it should be given with the learning approach that allow students to make observations and exploration to build their own knowledge. Technology application in learning as an effort to improve students’ thinking. Cabri 3D, a dynamic geometry software (DGS), can be used as tools to assist in mathematics learning with problem-based learning approach. A study was designed to examine problem-based learning approach using dynamic geometry software in the learning of 3D geometry object. A total of 60 students studying in tenth grade participated in the research. The instruments were consist of mathematics critical thinking test, mathematics creative thinking test, mathematical attitude scale and students' worksheet based on dynamic geometry software. The study found that problem-based learning approach performs better influence on attaining and gaining mathematics critical and creative thinking abilities, and mathematical attitude than conventional learning.

Keyword:
Problem-based learning
Critical thinking
Creative thinking
Mathematical attitude

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1. INTRODUCTION
Developing mathematics thinking skills to be the focus of mathematics educators in the classroom. In the research of Riedesel [9] found that generally the children learned math only and did not learn to think mathematically, and lack of using what they have known in advance for learning process. Mathematics educators' efforts can be seen from a little student who love math of each class [38]. Learning mathematics is closely related to activity and learning processes as well as the characteristics of mathematical thinking as a science and human activity, that is, a pattern of thinking, organizing pattern of logical proof that using the term is defined carefully, clearly, and accurately [19].

Thinking allows someone to represent the world as a model and provide a treatment effectively in accordance with the goals, plans, and desires. Thinking involves the mind of manipulation to information, such as when someone formed a concept, engage in problem solving, perform reasoning, and decision making. Thinking is a high-order cognitive functions and analytical thought processes that become a part of cognitive psychology [27]. In the context of learning, developing thinking skills aimed at a few things, such as (1) receive training in thinking to make decisions and solve problems wisely, (2) apply knowledge, experience and skills to think either inside or outside the school, (3) generate ideas or creative and innovative creation, (4) overcome thinking in a hurry, and a narrow, (5) improve cognitive and affective aspects, and so on intellectual development, and (6) to be open to receive and give opinions, make judgments based on the reason and evidence, and dare to give views and criticism[31].

Mathematics teachers have been figuring out how to give the teaching of critical thinking in favor of establishing environments that allow for developing the critical and creative thinking through discussion and
interaction in classroom. Patterns of thinking in mathematical activities are instead of lower-order thinking and higher-order thinking. Garrison, et al [7] stated that if the critical thinking is developed, a person will tend to seek the truth, open-minded and tolerant of new ideas, able to analyze problems well, think systematically, inquisitive, mature in thinking, and to think critically independently. Meanwhile, according to Learning and Teaching Scotland [20] when a student develop the creative thinking skills, it would generate a lot of ideas, making a lot of connections, has a lot of perspective about something, creating and imagination, and result orientation.

Critical thinking is necessary in the process of learning that involves active control over the cognitive processes that take place [16]. Critical thinking is higher level thinking, which is reasonable, reflective, responsible, and skillful thinking [33], the ability to control the mind, to consciously examine the elements of the reasoning [34], to make decisions with something that believes about the truth and will be done [32], to help formulate or solve, decision load, or fulfill a desire to understand [13]. There are two characteristics in critical thinking, firstly, learn how to ask, when to ask, and what the question is, and secondly, to learn how to reason, when to use reasoning, and what method of reasoning used [3]. And someone who thinks critically then accustomed to ask the right questions, incorporating relevant information, efficiently and creatively organize the information, have a reasonable sense, consistent and trustworthy in the conclusions. Meanwhile, Fawcett [26] observed that a student using critical thinking will (a) select the significant words and phrases in any statement that is important and ask that they be carefully defined; (b) require evidence supporting conclusions he/she is pressed to accept; (c) analyze that evidence and distinguish fact from assumption; (d) recognize stated and unstated assumptions essential to the conclusion; (e) evaluate these assumptions, accepting some and rejecting others; (f) evaluate the argument, accepting or rejecting the conclusion; (g) Constantly reexamine the assumptions which are behind his/her beliefs and actions.

On the development of critical thinking in math class can be done by doing activities such as comparing, making contradiction, induction, generalization, sort, classify, prove, relate, analyze, evaluate, and create patterns, sequenced on an ongoing basis [27]. In order to build critical thinking in learning, the students needed to face with new contradictory problems, so he/she would construct his mind seeking truth and reason [36]. And the influence factors in developing critical thinking abilities were the interaction between teachers and students. And Glazer [12] using three indicators to assess student’s critical thinking: (a) proof, to prove a statement; (b) require evidence supporting conclusions he/she is pressed to accept; (c) analyze that evidence and distinguish fact from assumption; (d) recognize stated and unstated assumptions essential to the conclusion; (e) evaluate these assumptions, accepting some and rejecting others; (f) evaluate the argument, accepting or rejecting the conclusion; (g) Constantly reexamine the assumptions which are behind his/her beliefs and actions.

Thus, critical thinking can be defined as the process and also the ability to understand the concepts, apply, synthesize, solve problems, demonstrate and evaluate the information obtained. It requires an assessment through a variety of criteria such as clarity, precision, accuracy, reliability, applied ability, and other evidence that supports the argument to make a decision. For example, when students are facing the problem, then they will try to understand and found to detect the presence of things that necessary for the purposes of settlement.

Example 1: For secondary students: Geometry (Prove)

![Figure 1. The Parallel Planes](image)

The M's plane parallel to the N's plane and the P's plane. Line segment \( \overline{AC} \) on the M's plane, \( \overline{BD} \) on the P's plane, the point of E and G on the N's plane. Prove that \( \frac{AE}{EB} = \frac{CG}{GD} \) !

Teacher should encourage students to think creative in learning activities, with giving many challenge problems. Puccio and Muddoch argued that creative thinking contains aspects of cognitive skills and metacognition, that is, identify problems, make a question, identify data that relevant and not, productive, producing many different ideas or new idea, dare to take a position, act quickly, act or think that something as a part of complex whole, utilizing other people's ways thinking critically [4]. Creative thinking can led to the
acquisition of new insights, new approaches, new perspectives, or new ways to understand something [6], the ability to generate ideas or new ways to produce a product [23], or Sharp stated novelty, productivity, and impact or advantage [21], the ability to solve problems and/or to develop thinking in structures, taking account of the peculiar logical-deductive nature of the discipline, and of the fitness of the generated concepts to integrate into the core of what is important in mathematics [8]. Treffinger & Isakson stated creative thinking was creating and expressing a meaningful connection, that the process to perceive gaps, paradoxes, challenges, problems, opportunities, and then thinking about the possibilities with a different perspective, variation possibilities and non-routine, and elaborating alternatives [40].

Creative thinking as the process of understanding a problem, find possible solutions, interesting hypotheses, test and evaluate, and communicate the results to others. The results of creativity include new ideas, different perspectives, solve problems chain, and combine ideas that incorporated into the four components or indicators, such as: (a) fluency, have many of the ideas in variety of categories, (b) Flexibility, have a variety of ideas, (c) originality, have new ideas for solving problems; (d) Elaboration, able to develop ideas to solve the problem in detailed [10]. And some of the strategy to develop creative thinking ability, such as: (a) Redefine the problem, (b) Questioning and analyzing assumptions; (c) Selling creative ideas; (d) Generating ideas; (e) recognizing the two sides of knowledge; (f) Identifying and overcoming barriers; (g) Taking risks wisely; (h) Tolerate ambiguity; (i) Establish self-reliability (skills); (j) Finding true interest; (k) Delaying gratification; (l) Creating creativity model [28]. And from some of the experts stated above, it can be concluded that the creative thinking in mathematics as the ability to solve mathematical problems which includes these components: fluency, flexibility, originality and elaboration. Assessment toward students' creative ability in mathematics is important.

Creative thinking in mathematics can be suggested as orientation or disposition of math instruction, including the discovery and problem-solving tasks. This activity may bring students to develop more creative approach in mathematics. Task activities can be used by teachers to improve students' skills relating to the creativity dimensions. To measure the students' creative thinking ability can be done by exploring the work of students who represent the creative thinking process [25], or based on what students communicated, verbally or in writing, in the form of student work, problem solving, answer verbally for teacher questions [6]. Some experts have developed an instrument to measure the mathematics creative thinking abilities, such as Balka and Torrance, Balka developed instruments Creative Mathematical Ability Test (CAMT) and Torrance developed instruments Torrance Tests of Creative Thinking (TTCT). Both of the instruments contained math problems based on real problem that easy look by student in their environment [11]. Jensen developed by giving the task to create a set of questions or creating questions based on the information from the questions given, that presented in the form of narrative, charts, or diagrams [15]. Balka, Torrance, and Jensen's models above is often called problem posing tasks or problems finding or production divergent. The tests measure three aspects of mathematics creative thinking abilities, namely fluency, flexibility, and novelty. Fluency aspects related to the number of relevant questions, Flexibility related to the number of kinds or types of questions, and novelty relates to the uniqueness or how a particular type of question rarely.

**Example 1: For secondary students: 3D Geometry (Novelty)**

![Figure 2. Cube with planes](image)

ABCD, EFGH is a cube with the length of lines are $x$ cm. If the length of $CP$ equal to $\frac{3}{4}$ from the length of $CG$.

- length of $CS$ equal to $\frac{1}{4}$ from the length of $CG$, and the length of $CR$ is $\frac{1}{4}$

*From the length of $CG$. What can you show with this cube?*

Teaching 3D geometry in classroom is expected to provide attitude and systematic habits for students to be able to give an idea for making relationships between the geometry's construction. Because it is necessary to provide adequate opportunities and media for students to observe, explore, try and find the principles of geometry through informal activities and then forward to the formal activities and apply it [18]. The concepts and high level skills have a linkage between elements which difficult to be taught through book only, and the mathematics learning will be faster if the learning activities in the classroom using to computers utilized effectively [41]. One of dynamics software 3D geometry is Cabri3D that can be used to help students and
teachers to overcome some of these difficulties and making the learning of 3D geometry easier and more appealing. This software provide facilities for exploration, investigation, interpretation and solve math problems with interactive [5], more easily to draw 3D geometry than on the board [14], for learning 3D geometry in high school can improve comprehension skills and creativity, students' skills in discussions with peers and teachers, develop imagination and special visualization, more understand to correlate in theory and application, efficient in learning, increase confidence to contribute in the group [1].

Another important for student in learning activity is learning approach by teacher. Problem-Based Learning Model is one model of learning that can be done by involving the students in the group, supporting student activities more dominant, and the role of the teacher as a facilitator. Students investigate it yourself, find the problem, and then solve the problem under direction from facilitators (teachers). Problem-Based Learning (PBL) application in class could improve student’s critical thinking skills. This learning would help students to process information in their minds and arranged knowledge in their world and environment [37], to engage the learner in investigative problem solving that integrates the skills and concepts from a variety of content areas (Moffit in [30]). Stepien stated PBL could change the pattern of the traditional teaching and learning process in which a process that inspired the students more actively as the process of assimilation and accommodation section for the knowledge to help students to become professionals in a particular field. On traditional learning approaches tend to be less effective in the process of being done, and more difficult to choose which material should be given to the students [22].

Characteristics of problem-based learning approach consists of three things, namely the students solve problems according to their ability, the problem is ill-structured, meaning a lack of necessary information and loading issues unresolved, complex problem through inquiry and investigations, requires a reason to be resolved, and be resolved with more than one way [39]. Lee and Tan [24] revealed several disadvantages in PBL such as: (a) The learning required in more time; (b) Constraints on teacher factors that difficult to change the orientation from teachers-centered to students-centered, (c) Difficult to design problems that meet the standards of Problem-Based Learning.

2. RESEARCH METHOD
This research was conducted with quasi-experiments. Researchers will collect quantitative data to measure mathematics critical and creative thinking, and qualitative data from mathematics attitude. Quantitative research methods were used to compare the two classes with different treatment. The first class used problem-based learning approach using dynamic geometry software (experiment class) and other classes using conventional learning strategies (control class). Qualitative data used to gather information about students' mathematics attitude. The study involved 60 grade-10 students from moderate level school in West Bandung. Subjects are not randomized, but the researchers receive potluck the subject [35]. In the early stages of this research is to establish a sample of each school ranks high, medium, low. Then from each school were taken two classes as classes and the experimental and control classes.

Development of teaching materials was prepared by creating a lesson plan (RPP) that begun with reviewing the subject of geometry in mathematics textbooks used in the school, student worksheet (LKS), and problems in the RPP based on the subject of 3D geometry. The instruments used in this study were a test and non-test forms. The instruments were mathematics critical thinking ability test, mathematics creative thinking ability tests, mathematics attitude consist of student’s opinion toward problem-based learning approach assisted by dynamic software program, and observation, and interview. The validity instrument used Product Moment Pearson formula, reliability instrument used Alpha formula, and gain with Hake formula [29].
ABCD.EFGH is a cube with the length of lines are 5 cm. If the point of P, Q and S are the middle point of $\overline{BC}$, $\overline{CD}$ dan $\overline{CG}$.

a. What is the distance from the point of E to $\triangle PQS$?
b. If the point of N is the middle point of $\overline{PQ}$, what is the distance from the point of E to N?

ABC is an equilateral triangle. The surface ABC has lines AB, AC, BC 6 cm. Point O is the center point of the surface ABC. If the distance between point B to point O is 3.5 cm and the distance from point T to O is 6 cm. What is the distance between point C and $\triangle TAB$?

Figure 4. student worksheet (LKS)

3. RESULT AND ANALYSIS

Pretest was conducted to determine the ability of the beginning and the post-test to determine the capability after the learning process. The result of normality and homogeneity test showed no difference in the experimental class and the control class in the ability of students at the beginning and some differences at the end. It indicated the learning process in the experimental class had a positive effect on students’ abilities. The results in normalized gain test for mathematics critical and creative thinking ability students in the experimental class and the control class showed increased ability in the experimental class better than in conventional teaching.

In the experimental class, for critical thinking, the student’s ability of Prove, Generalization, Problem solving showed in moderate categories. Its means that students thinking can be influenced by learning approach. Compared with the control class that conventional learning has a lack of impact to improve critical thinking.

Table 1. N-Gain Result

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Gain average categories</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
</tr>
<tr>
<td>Critical thinking</td>
<td></td>
</tr>
<tr>
<td>Prove</td>
<td>0,508</td>
</tr>
<tr>
<td>Generalization</td>
<td>0,690</td>
</tr>
<tr>
<td>Problem solving</td>
<td>0,510</td>
</tr>
<tr>
<td>Creative thinking</td>
<td></td>
</tr>
<tr>
<td>Fluency</td>
<td>0,607</td>
</tr>
<tr>
<td>Flexibility</td>
<td>0,767</td>
</tr>
<tr>
<td>Originality</td>
<td>0,938</td>
</tr>
<tr>
<td>Elaboration</td>
<td>0,342</td>
</tr>
</tbody>
</table>

In the experimental class, for creative thinking, the student’s ability of flexibility and originality have a high categories better than the ability of fluency and elaboration in moderate categories. Students argued that using dynamic geometry software (DGS) like Cabri 3D give more chance to be actively for exploration, investigation, interpretation and solve math problems. Meanwhile, in the control class showed the enhancement of students’ mathematics creative thinking ability categorized as high on originality, moderate on fluency and low category on flexibility and elaboration. In conventional approach, students can be improve more better in originality ability, because they have more easy to solve problems with own ways than other ability. Problem-solving ability showed more better in experimental class because supported by computer program so students could explore the forms of 3D geometry without worrying wrongdoing. This exploration would improve the knowledge and experienced for students in solving problems. Such Shannon [2] stated that solved a problem in mathematics actually created some problems again, so students need an ability to know exactly what to do next.

In the mathematics creative thinking abilities’ students’ improvement influence and flexibility, were very influenced from the computer program which can be easier for students to observe and explore the twisting,
shifting, enlarge, shrink and create variations of 3D geometry object with length and size of the angle that automatically changed as desired. So that students can freely generate a lot of opinions, methods, and solutions in different ways. Students' thinking skills in elaboration and originality ability increased influenced by training with the questions at student worksheet that could provoke students' creativity by adding parts of objects that facilitate for problem resolution. This process was repeated with the students together in a group effort in responding to the issues presented, thus construct an imagination that allows students to obtain an unprecedented resolution. Dahan [17] suggested that the use of computer program provides a tool for users to develop a variety of ideas and imagination in constructing geometric shapes.

The problems presented to students in class discussions and activities that may affect the growth of self-confidence of students to do their own discovery in resolving the problems, quite an effect on increasing the mathematical critical thinking skills and creativity for students. With the discussion on learning allows students to interact between friends of the class to express their opinions, asking, responding to other people’s opinions, explain and clarify their own thinking to solve problems, so the ability to think critically and creatively in mathematics will increase.

Correlation test result showed Pearson correlation coefficients for mathematics critical and creative thinking ability students in the experimental class at moderate and the control class obtained at low category.

<table>
<thead>
<tr>
<th>Mathematical Ability</th>
<th>Group</th>
<th>Correlation Coefficient</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Critical and Creative Thinking</td>
<td>Experimental</td>
<td>0.549</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>0.310</td>
<td>Low</td>
</tr>
</tbody>
</table>

Correlation values obtained indicate the magnitude of correlation between students’ mathematics critical and creative thinking ability in the experimental class show that students who have top rankings on tests of mathematics critical thinking skills can be predicted to have good rankings on tests of mathematics creative ability and vice versa.

Giving students the attitude scale in this study based on the affective’s attitude that aims to determine the students’ attitude towards learning mathematics in general, toward Problem-Based Learning approach, towards computer-assisted learning, and toward mathematics critical and creative thinking. The attitude scale consisted of 25 statements, divided into 14 positive statements and 11 negative statements. Generally, students expressed a positive attitude towards problem-based learning. Students expressed feel that the student worksheet (LKS) help them to learn math, enjoys working in groups, proud to be involved in discussions in class, and considers math lessons very interesting. Students’ attitudes toward learning 3D geometry assisted DGS, showed preference for the use of Cabri 3D in helping to resolve the problem and approval to learn 3D geometry with DGS. It can be concluded that the students' attitude has given a positive impact towards learning mathematics assisted DGS, like mathematics using a computer, and easy to understand the subject of 3D geometry. The students’ attitude showed preference for the problems from teacher and sincerity in solving the problems. The students had a positive attitude towards the problems of mathematics critical and creative thinking. Most of the students were challenged with problems that were given, be creative, and be sure always had a way to solve such questions.

Students generally responded positively to problem-based learning assisted computer in 3D geometry subject. From 30 students, as many as 63.3% of students consistently revealed that through their lessons, learned math so fun and not boring considered, 76.6% of students consistently demonstrated seriousness in learning, 60% of students agreed activities consistent with the discussion groups and presentations made so interesting in math, 83.3% of students felt helpful to understand the 3D geometry object with assisted by a computer, 63.3% consistently argued that problem-based questions made students feel many challenges to be developing ideas, creativity appeared in search completion and could express their opinions in the discussion, and 86.7% of students who expressed more pleasure from learning as given and studied like this helps them to get used to express thoughts through discussion does, argue, ask questions, and discover new knowledge that was not previously been unthinkable. Students argue this makes students enjoy learning cooperate in solving the problems.

4. CONCLUSION

Based on the findings and discussion the study derived some conclusion as follow.

Students taught by problem-based learning approach assisted with dynamic geometry software got better grade and better gain on mathematics critical and creative thinking ability than that of students taught by conventional teaching. However both abilities were classified as medium. Nevertheless, there was no difference of students’ grade and students’ gain on mathematical critical and creative thinking ability between students taught by problem based learning approach assisted with computer program and students taught by conventional teaching. Both abilities were classified as medium.

In problem-based learning approach, there was a high correlation between mathematical critical and creative thinking abilities. However the mathematical creative thinking tasks tended more difficult than
mathematical critical thinking tasks. Moreover, students performed positive opinion toward problem-based learning approach assisted with computer program and toward tasks on mathematical critical and creative thinking.

Based on those conclusion, researcher proposed some suggestion as follows. Mathematics teachers were suggested to implement problem-based learning approach and mathematics software in the classroom as an alternative teaching approach for improving students mathematics critical and creative thinking and other high mathematical abilities as well. In order to students manipulate the program smoothly and obtain a deeper understanding of 3D geometry, it is hoped that students are offered addition exercises about important features of the software. Moreover, the software is suitable for teaching geometry. So it is appropriated for each school to develop this mathematics software through intensive training for mathematics teachers in order to maximize their teaching-learning process.

REFERENCES

DESIGN AND EFFECTIVENESS OF INTERNATIONAL STANDARD MATHEMATICS LEARNING PACKAGES

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ABSTRACT

This study aimed at supporting; (1) Indonesian government policy, according to law No. 20 Year 2003 on National Education System, especially on International Standard Education, and (2) International Class Program, State University of Makassar commenced in 2007. The specific targets of this research were to produce and find the effectiveness of: (1) Student book; (2) Student Worksheet and (3) Lesson Plan, in supporting English for Mathematics course in Mathematics Education and to know their effectiveness. The modification and adaptation of R & D model of Four-D (Thiagarajan, 1974) was used to conduct the study consisting of four phases; Define, Design, Develop, and Disseminate. The references used as Standard Packages refer to: (1) Indonesian Qualification Framework based on President Regulation No. 20 year 2012, (2) English as a language, in which the packages were made, (3) the validators were native speakers, (4) the structure of the learning packages which can lead students’ paradigm, insight, and knowledge to worldwide things. The research finds that the development of: (1) Student Book emphasizes on three facets; the clarity of the content structure, language, and problem solving, (2) Student Worksheet emphasizes on three primary aspects; task direction, order of the task, and language, (3) Lesson Plan emphasizes on four primary aspects: formulation of basic competency, time allocation, learning material, and its structure. The learning packages are effective to improving students’ achievement, specifically shown by the increase of students’ score from pretest to posttest as 8.16 of an ideal score 100.

Keywords : Design and Effectiveness, Learning Packages, International Standard.

Introduction

International standard education has been organized by Indonesian government in The National Education System Number 20 Year 2003. The regulation states that the central government and/or the local government establish at least one school in each educational level to be developed into an International Standard School (ISS). An ISS is a school that satisfies all standards of national education and is enriched by contents referring to educational standards from at least one country that is a member of Organization for Economic Co-operation and Development (OECD) and/or from other developed countries with certain advantages in the educational field.

One of the obstacles to the Indonesian government in the attainment of international standard education is that there has not been international standard learning packages including their effectiveness. The intended learning packages, which has not been internationally standardized is not only in Mathematics and Natural Science domain, but also for other lessons in both schools and colleges.

Rationalizations of international standard education through Bilingual Approach are; (1) sociocultural issues which encompasses (a) symbolic and psychological importance for individual’s identity and (b) practical value for intercultural communication, (2) economic which consists of (a) globalization economy and economic development (b) limited number of global/regional lingua francas (esp. English as an international language), and (3) cognitive which comprehends (a) additive bilingualism and (b) superior language learning ability and intercultural sensitivity. (Baker, 2006)

The idea of Baker (2006) about Bilingual Approach is relevant to and even has similarity to the
implementation of international standard education in Indonesia by considering that; (1) the method of bilingual approach is applied to a study program in which English is not its main discipline, (2) the majority of students who are taught using bilingual approach still has low achievement in English language, and (3) the use of Bilingual Approach in learning is highly likely to create more intensive interaction than that of monolingual approach.

Problem Statements
1. How to design International standard mathematics learning packages for English for Mathematics course?
2. How far is the effectiveness of international standard mathematics learning packages for English for Mathematics course which have been developed?

Literature review
Several countries have applied bilingual approach for instance; (a) Australia which combines (German/English; Franch/English; Greck/English; Arabic/English; Hebrarw/English; Indigenous Language/English and Japanese/English), (b) Thailand, specifically Sarasas Extra School, which combines (English/Thai), and (c) Japan, which combines (Japan/English) in the learning process in Katoh Gakuen (Baker, 2006).

Definition of International Standard Education in Indonesian Version
There are several definitions about International Standard Education in Indonesia. Hasibuan (2008) relates international standard education to dual degree. It is stated from the result of an interview (Darina, 2006) that one of the distinctions or the differences between ordinary campus and international standard campus is the facilities that they have. Besides that, Made (2008) defines it as a college which either undertakes curriculum in which English is a medium of instruction, has library with the collection of books from foreign countries, or establishes cooperation with foreign educational college. In compliance with Made, Azizy (2008) states that International standard education is an education of which the regulations used are satisfying the international standards. In addition, it has several criteria in the form of international standard (Furqan, 2008). Upu (2008a; 2008b; 2008c), suggests some criteria about international standard education that as follows:
1) The facilities and infrastructures encompass; (a) learning facilities and learning packages based on ICT; (b) library based software (more preferable); (c) the availability of proper class room; multimedia; clinic; sport facilities, and (d) contemporary supporting learning facilities.
2) The leaders of the faculties; (a) satisfy minimum master degree graduated from High College which is minimum B accredited in either domestic or graduated from High College in abroad; (b) are capable of speaking English actively; (c) have wide conception and are able to establish international network, possess managerial competency, and have a good entrepreneur leadership; and (d) are capable of applying several principles of Educators, Manager, Advisor, Supervisor, Leader, Innovator, and Motivator.
3) The Educators; (a) capable of facilitating a learning based ICT; (b) capable of upholding a learning with English as a medium of instruction; (c) have master degree or doctoral degree as their education qualification in B accredited college either in home or abroad; and (d) innovative and creative in organizing learning.
4) The management of the education is multi-cultural; establishes conjunction “sister faculties” to international standard faculties of OECD countries or those of developed nation; free of illegal narcotics, smoke, and violence, and occasionally follows international scientific, mathematics, and technological events.
5) The process of learning is oriented on (a) spiritual intelligence; (b) emotional and social intelligence; (c) intellectual intelligence; (d) kinesthetic intelligence; (e) being competitive, active, and creative intellectual; (f) soft skills; and (g) the use of assessment based authenticity.

Several issues of International Standard Education in Indonesia
1) Low ability of oral English
Lecture’s oral English ability, especially pertaining to the subjects is low. Moreover, the motivation and the potency of the lectures in following several courses in abroad is also low. Furthermore, in the opinion of many lectures, as well as Mathematics, English is generally considered difficult to learn and to teach.
2) The discrimination and the exclusiveness among students
International standard education is likely to evoke great distinction among students whose
parents are able to fund their children and those whose parents are not able (Upu, 2009a). International standard education needs international standard facilities requiring fund, reachable for parents’ students in international standard education, which is higher than regular standard facilities. Since international standard facilities are inherently prepared for students in multicultural and multiethnic environment and those from various countries. As a result, those students in international standard education may feel better than others.

3) Misconception toward Free fund education (fully subsidized education)

A failure to interpret the free fund education (fully subsidized education) concept from some Indonesian people is a challenge in undertaking international standard education. Whether it is realized or not, that the people, in one side, consider that free fund education (fully subsidized education) is a process in acquiring knowledge where all kinds of needs of the education management is free of charge. However, in the other side, to implement high quality education, it needs high cost which is, indeed, funded by students’ parents especially for those who are wealthy. Education is the responsibility of every person, not only for parents, whose children study at school. The responsibility of education management is confided by National Education System Constitution.

4) Obscurity of the standard of international standard education

To maximize the quality of international standard education in Indonesia, the government needs to define standards, so that they can be used as a Explicit Reference or the Basic Law. When the standard decree of the International Standard Education is not immediately enacted, every education level and education institution may haphazardly acclaim itself as an International Standard Education. The intended standards cover input, processes, and output. Moreover, it may include some operational standards, especially of labor, curriculum in Indonesia, fund, and other necessary standards. Recently, the majority of people measure the standard of international standard education by only seeing the use of medium of instruction in teaching that is English and other languages supported by Information Communication and Technology (ICT). In fact, several other standards, including curriculum and partners help are occasionally neglected.

5) Partners Help

Partnership program between educational colleges in Indonesia and one of or more educational colleges in developed nations is inevitable. Educational college in developed nation has notable experience in creating the goals of international standard education. One of the critical matters related to this issue is how the developed nations manage their education resources to satisfy the standards of international standard education settled by an authorized board. Consequently, Indonesian government needs to adapt the criteria of education institution from OECD countries or developed nation to be made as a companion.

6) Model of Learning

By considering the ability of each lectures, which is very heterogenous in terms of their English proficiency, it needs a reference for model of lecturing. Concerning to this situation, the education college in Indonesia should gain experience from other countries which successfully implement the reference. Subsequent to stipulating a model for one kind of learning, it firms up the approach, method, and strategy of learning based on each level of learning.

Framework for Research Thinking

The framework for thinking of this research is described in a fishbone diagram below:
Research Method
This research type was Research and Development aiming at developing Student book, Student worksheet, and Lesson Plan. Next, they were studied on how far their effectiveness in a learning is.

Subject and the time of Research
The subject of this research was students in the first semester on “sarjana” program in International standard Class program, majoring in mathematics education. They have heterogeneous background in terms of mother tongue, language ability, mathematics ability, sociocultural, and parents’ education level.

The Instrument and Data Collection

**Learning Packages Validation Sheet**
The validators were asked to write down the appropriate score by giving a sign (√) on an appropriate column. Furthermore, the validators were asked to give general conclusion by using either of the categories: very good, good, fair enough, insufficient, very insufficient.

1) Observation Sheet of Students’ activity
The observed students’ activities in this sheet include paying attention to lecturer’s explanation or other students’ argument, discussing learning material with other students, solving problems, discussing with the lecturer, and doing irrelevant activities, i.e. doing activities unrelated to classroom activities.

2) Observation Sheet of Lecturer’s ability in managing learning
This kind of instrument is made to obtain data, related to the ability of a lecturer in managing as one of data, which support the effectiveness of a learning packages.

3) Observation Sheet of learning packages accomplishment
This kind of instrument is made to obtain data in the field about the learning packages practicality. The process of data collection was done by observers using the observation sheet as a guide for them to observe the accomplishment of certain aspect or the component of the learning packages when lecture hold his or her learning based on given guidelines.

4) Students’ Response Questionnaire
The intended data of students’ response toward the activity of field test are students’ response toward the aspect of learning encompassing the learning topic, Student book, student worksheet, the...
situation in classroom, the lecturer’s method in learning, the lecturer’s performance.

The Technique of Data Analysis

The results of Learning packages validation

Score in the form of validation from experts and practitioners for each learning package was analyzed by considering advices and comments from the validators. The result of the analysis were used as a guide to revise the learning packages. The process of the analysis consecutively were; (1) recapitulating the assessment from the experts, (2) finding the average of experts’ assessment for each criterion, (3) finding the average of each aspect, (4) finding the total of the average ($\bar{X}$), (5) defining the category of the validity for each criterion ($\text{AI}_i$) or the average of aspect ($\overline{A_i}$) or the total of the average ($\overline{X}$) by consulting to the predetermined categories of validation.

The used criterion to determine the validity degree of the learning packages was the value of ($\overline{X}$) for the whole aspects which was, at a minimum, included in quite valid category, and the value of ($\overline{A_i}$) for each aspect included, at a minimum, in valid category. Otherwise, it was necessary to make revision based on the validator’s advice and to evaluate several aspects, especially for the low grade aspects. Subsequently, the process of validation was repeated and the data were analyzed afterward. That kind of cyclic process was undertaken until the learning packages was, at a minimum, included in valid category.

The research results indicate that the validity degree of the learning packages; (a) students’ book has $\overline{V} = 4.1$, of a category reference as valid is (3.5 ≤ $\overline{V}$ < 4.5), (b) students’ worksheet has $\overline{V} = 4.1$ of a category reference as valid is (3.5 ≤ $\overline{V}$ < 4.5), (c) lesson plan has $\overline{V} = 4.3$ of a category reference as valid is (3.5 ≤ $\overline{V}$ < 4.5). Thus, all those packages are valid.

Data analysis of students’ activities

Analysis of the results of observation toward students activities comprised: (1) the frequency of the average of each activity category for each meeting was conducted by summing up the frequency of the intended activity category divided by the number of observed students, (2) the percentage of each category of students activities for each meeting was conducted by means of dividing the frequency of the average of each category of students’ activities (point 1) for each meeting by the maximum sum of the frequency of observations within those meetings multiplied by 100%, and 3) the average of each category of students’ activities for all meetings was figured out by dividing the sum of percentages for each of students’ activity categories in all meetings by the number of meetings.

The analysis results show that all categories are in a tolerance interval of ideal time that is allocated during the instruction takes place. It is particular for categories of 1, 4, 6, and 7, they must be satisfied. The average value of category 1: to listen to/to look at lecturer’s explanation actively is 45,33 of a tolerance interval 37 – 47 minutes. The average value of category 4: to carry out problems is 8,67 a tolerance interval 0 - 8 minutes. The average value of category 6: to present/to address questions in front of all groups is 10 of a tolerance interval 5 – 15 minutes, and the average value of category 7: to respond their friends’ answers or to tell opinion/idea is 12 of a tolerance interval 5 – 15 minutes.

Data analysis of observation to lecturer’s activities

The observation was undertaken by trained observer in order to be able to operate the observation sheet appropriately.

The analysis result indicates that the totally average of value of lecturer’s activities is $\overline{V} = 4.6$ of a category reference as valid is (4.5 ≤ $\overline{V}$ ≤ 5). Hence, if it is viewed from overall aspects lecturer’s activities, then it is categorized as effective in learning activities.

Data Analysis of Practicability of Learning Packages

The activities administered were as the following: (1) to recapitulate the results of observation to the practicability of learning packages, (2) to find the mode of observation to each activity, (3) to find the mode of each observation aspect for meetings, and (4) to determine the practicability categories of each aspect of overall aspects by conforming the mode of each aspect of the fixed $\overline{X}$ categories;

The analysis results of observations wholly show that the utilized learning packages are “available”. The learning packages are also practicable or utilized with good enough and even it can be stated as perfect enough on the basis of the obtained mode value.

Data analysis of students’ responses

The activities conducted were: (1) to figure out the number of students that give positive response corresponding to the asked aspect, and then to figure out its percentage, (2) to determine category for
students’ positive responses by conforming its percentage with fixed criteria, (3) if the analysis results show that the students’ responses have not been positive, then it is undertaken revision to the packages being developed.

The analysis results show that all categories are in a tolerance interval of ideal time that is allocated during the instruction takes place. It is particular for category 1, 4, 6, and 7, they must be satisfied. The average value of category 1: to listen to/to look at lecturer’s explanation actively is 45.33 of a tolerance interval 37 – 47 minutes. The average value of category 4: to carry out problem is 8.67 of a tolerance interval 0 – 8 minutes. The average value of category 6: to present/to address answers in front of all groups is 10 of a tolerance interval 5 – 15 minutes, and the average value of category 7: to respond their friends’ answers or to tell opinion/idea is 12 of a tolerance interval 5 – 15 minutes.

The Research Implementation System

![Diagram of the Research Implementation System]

Notes: - Implementation
- : Cycles
: Kinds of Activity
: Activity Result

Figure 1. The Modification and adaptation of Learning Packages Development Model Four-D
Thiagarajan, et al.
The criteria used as international standard learning packages: (1) Indonesia Qualification Framework of Indonesia, President Regulation No. 20 Tahun 2012, (2) learning packages is written in English, (3) the validators of learning packages are Native Speakers, (4) the structure of the learning packages content can lead students’ mind, insight, and knowledge to worldwide things.

The procedure, Product, and Try-out of Learning Packages
The procedure of a learning packages development in this research serially takes following phases:

Define Phase
Define phase aims at determining and defining the necessary conditions in a learning. The activities in this phase are front end analysis, students analysis, learning material analysis, task analysis, and the specification of learning goals.

Front end analysis
Based on review to English for Mathematics course in the department of mathematics, state university of Makassar, the fundamental problem in fact that is needed to attempt the solution is the way of presenting lecturing (Upu, 2012). The lecturing currently tends to be lack of providing enough opportunity to students to develop their ideas. As a result, students become passive, lazy to ask question, moreover to express their ideas. In addition, the lecturing process is dominated by lecturer, meanwhile students only already listen to and copy what the lecturer tells.

The analysis and the study of students
Students’ prior knowledge is heavily influenced by thinking way that they brought from secondary school. Meanwhile, the used language by those following English for Mathematics course tends to have not been formal and their ability to analyze mathematical sentence is still low. Both of them influence their ability to understand mathematics. Whereas concerning students’ cognitive development, this domain inclines to be heterogen in terms of English ability. Meanwhile, their inclination to make group and discuss each others to develop their social-culture (Upu, 2009b, 2010a, 2010b) is more homogen.

The analysis and the study of learning material
The material or content analysis is intended to identify, elaborate, and organize systematically the main material that students will learn. The material is organized hierarchically and choosen in the light of basic competence and indicators. Analysis of learning or lecturing material that is provided currently, is less able to build students’ insight, mind set, and commitment globally.

The analysis and the study of learning tasks
The tasks that lecturer gives to students do not vary. They depend on lecturer’s desire. In general, quiz in the classroom is given more frequently than homework is. In terms of understanding to learning material, quiz in the classroom is more able to build ability to understand students’ material than homework is. Hence, a great deal of learning goals refer to quiz.

The analysis and the study of learning’s goals
Learning goals have not described an appropriate hierarchy, either it is according to language or viewed from the mathematical content or material. Therefore, the learning packages, particularly student book and student worksheet must be organized, such that those both situations can be overcome.

Design Phase
The kinds of activities of this phase are the choice of media of the learning, learning format, the design of learning packages, and the choice of learning outcome test.

The choice of Learning Source
This kind of choice aims at determining the appropriate learning source to provide international standar learning material. The learning source is also adapted based on the analysis of the basic competency, tasks, and the campus’ facilities.

The choice of selingkung model
The choice of selingkung model is based on the importance of understanding to support students’
international insight in terms of; learning material, learning model, approach, method, and strategy as well as learning source that will be used and developed.

The product of the development of learning packages and the experiment
The initial design of learning packages for English for Mathematics is done to produce Prototype-I consisting of Student Book, Student Worksheet, and Lesson Plan. The prototype I is developed to produce Prototype-2.

Development Phase
The phase of development aims at producing prototype-2 of learning packages which is in the form of the revision of prototype-1. The advice from experts and practitioners and also the learning activity analysis plays important role in this phase.

Expert and practitioner validation of learning
Expert and practitioner validation is aiming at evaluating the content and the language of the learning packages. Its assessment covers several indicators: a) selingkung model, b) language, c) illustration, and d) learning content. Each following indicator consists of several sub-indicators namely.

a) Selingkung model, which consists of: the explicitness of learning material arrangement, the numbering of topics, the attractiveness of topics, the balance between the text and the illustration, the kind and the size of the font, the space arrangement, the appropriateness between the physical size and the students,
b) Language which encompasses the veracity of the grammar, the suitability between the sentence and the students’ cognitive development and literacy ability, guideline to refer to other reading sources, the veracity of each terminology definition, the simplicity of the sentence, and the clarity of the guidelines.
c) Illustration which comprehends supporting illustration aiming at specifically describing concept and the connectivity between the illustration and the concept, the clarity, the understanding, the use of local context, and the gender deviation
d) Content which includes the correctness of the content, the composition of the content, the suitability of the content to the curriculum, the inclusion of some important related information, the connection between the content and the previous learning material, the appropriateness of the content to the students’ mindset, the inclusion of exercises related to the concept, and the stereotype which is not partially focused on several issues (ethnic, gender, religion, and social status).

The recommendation from experts and practitioners and also the analysis output of prototype-I are used to obtain prototype-2. Its validation includes: (1) the content of the learning packages questioning is the content of the learning packages suitable to the learning material and the measured learning goals, (2) language: (a) the sentences of the learning packages use well and correct sentence, (b) the sentences of the learning packages are not ambiguous.

After the prototype-1 is assessed and revised based on the suggestions from the experts and the practitioners and also based on a reflective thinking, the prototype can be resulted.

Restricted field experiment of learning packages development
Restricted field is obtained to get direct description from the students and the observers about their responds, reactions, and comments as revision process of prototype-2 aiming at preparing prototype-3. The activities undertaken in the restricted field experiment are: (1) experimenting the prototype-2, that is a suitable instrument satisfying validity criteria. The kind of experiment is the implementation of learning in classroom using valid learning packages and instrument, (2) analysing the result of the field experiment, and (3) revising the learning packages and the instrument based on the result of the experiment to produce Prototype-3.

The main aim of the learning packages implementation in this phase is to know the explicitness, the perusal, and the appropriateness between the planned time/duration in the lesson plan and time/duration in its implementation. The output of the experiment is used for the completion of the learning packages (Prototype).

In this phase, research team observes the students’ activity, the ability of the lecturer in managing the learning, and the accomplishment of the learning packages. The development phase (develop) produces prototype-3, and that prototype is then disseminated to produce the final prototype.

Dissemination Phase
Dissemination phase is performed by undertaking socialization of the finalized learning packages. The socialization can be done through seminar, workshop, and general course which may involve lectures, students, teachers, and educators.
CONCLUSIONS

Conclusion:
1) The development of international standard learning packages of English for Mathematics course accentuates: a) Students Book encompassing three main aspects namely the aspect veracity of the standard of the competency, the structure, the content, the used language, and the problem solving. b) Student Worksheet including three primary facets namely the direction how to work with the Student Worksheet, the order which should be followed to work with it, and also the language it uses, and c) Lesson Plan which emphasizes four main facets namely the basic competency, time allocation, the learning material, and the structure of the learning content for each meeting.
2) Generally, international standard learning packages is effective to improve students’ achievement for students attending English for Mathematics course, specifically shown by the increase of students’ score from pretest to postest as 8.16 of an ideal score 100.

Suggestions:
1) It is suggested for lecturers, especially for lecturers majoring in Mathematics Education, to mind several important issues in developing international standard learning packages, therefore the learning packages can be made as a rudimentary reference to teach their own subjects.
2) It is recommended for students majoring in Mathematics Education which attends English for Mathematics course to simultaneously integrate the use of learning packages.

REFERENCES
HYPNOTEACHING: A WAY TO DECREASE MATHEMATICS ANXIETY

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ABSTRACT

Most of students assume that Mathematics is a phobia, because for them, the content of Mathematics is difficult, confusing and hard to be understood. These emotional condition called mathematics anxiety can be the cause of impediments to mathematics achievement. Certainly that affective factors can negatively impact mathematical performance and lead to avoidance of mathematics and mathematics-related fields. To overcome this problem, we need a method which can create a comfort and pleasant situation in learning, namely Hypnoteaching. Hypnoteaching is not a strange or unusual thing that related to mystical and magical. Hypnoteaching that applied in classroom is not meant hypnotizing students that make all of students fall asleep then suddenly classroom become silent because all students unconscious. Hypnoteaching is scientific and appear in every situation and condition in teaching and learning activities. This article talk about how Hypnoteaching decrease student’s Mathematics Anxiety. The six didactic principle of Hypnoteaching, the six-step plan to unlock student’s master mind, how to create Mathematics teacher’s charm in teaching, and how to construct the power of the student’s unconscious mind are the focuses of discussion in this article.

1. INTRODUCTION

Many students feel a tension in learning mathematics because for them, the content of Mathematics is difficult, confusing and hard to be understood. They assume that mathematics is a phobia. These emotional condition called mathematics anxiety. The consequences of being anxious toward mathematics are the avoidance of mathematics and the decline in mathematics achievement. The effect of mathematics anxiety is debilitating mathematical performance. It also involves interference in manipulation and solving the mathematical problems in a wide variety of ordinary life and academic situations. Many students who suffer on mathematics anxiety have a little confidence in their ability to do mathematics. The outcome is low self-esteem and fear of failure. It causes problems for processing the incoming information as well as the previous learned information for mathematical problem solving. Such as students tend to avoid mathematics whenever and wherever possible. Certainly, correlation between mathematics anxiety and academic performance is negatively significant. Students who have a high level of mathematics anxiety have lower levels of mathematics achievement. It also noted that math’s anxiety is a serious constraint. Performance in mathematical tasks and reduction in anxiety is associated with improvement in student’s achievement.

To overcome this problem, we need a method which can create a comfort and pleasant situation in learning, namely Hypnoteaching. Hypnoteaching is a variety of innovative learning methods. Hypnoteaching can foster a sense of student’s affection to both mathematics and mathematics teachers. If
the students love with the teacher or the content of mathematics, the student certainly will do anything for his beloved mathematics such as study hard, doing homework, doing the task with pleasure and passion. Hypnoteaching also make students enjoy mathematics learning and looking forward to mathematics learning.

Hypnoteaching is a blend of teaching which involves the conscious mind and the unconscious mind. Jaya said that Hypnoteaching is a blend of two words, Hypnosis and Teaching [1]. Therefore, some of the literatures don’t mention Hypnoteaching, but Hypnosis in Teaching.

Hypnoteaching is not a strange or unusual thing that related to mystical and magical. Hypnoteaching is scientific and appear in every situation and condition in teaching and learning activities so that students can easily receive information and understand the lessons that taught by teacher. Almatin said that: "Hypnosis is a technique that is effective, fast and efficient way to deliver information to the unconscious mind”[2].

Hypnoteaching that applied in classroom is not meant hypnotizing students that make all of students fall asleep then suddenly classroom become silent because all students unconscious. Almatin said that: "Hypnosis Learning is a learning that is designed to create a comfortable and enjoyable situations in a controlled environment to be able to get into the unconscious mind of the child”[2]. Furthermore Hakim said that: "Emotions and the student's unconscious mind can easily record and imitate every word and of teacher's action in the classroom”[3].

One of the Hypnoteaching’s advantage is the student’s willing to do every instruction from teacher with pleasure, such as doing exercise, homework, obey the school rules, etc. It means that teacher empower students, not deceive. This is corresponding with the statement of Abdullah (2006:44): "Learners are living creature that has potential that can be developed, so that should be empowered not deceived”[4].

2. LITERATURE REVIEW

2.1. Mathematics Anxiety

Mathematics Anxiety is a emotional condition in studying mathematics. Karimi and Venkatesan said that any learners have already experience mathematics anxiety in our schools consequently that is the outcome of low self-esteem and the fear of failure. Psychological indicators of mathematics anxiety include such things as feelings of tension, fear and apprehension, low self confidence, a negative mind set towards mathematics learning, feeling threatened, failing to reach potential, and a temporary reduction in working memory[5].

Cavanagh and Sparrow said that there are two dimention of mathematics anxiety: horizontal dimension and vertical dimention [6]. The horizontal dimension shows three types of anxiety: anxiety when being taught mathematics, anxiety when mathematics knowledge is being assessed, and anxiety when mathematics is required. The vertical dimension illustrates levels of anxiety. Extreme anxiety is indicated by somatic (physical and body) factors such as heart palpitations. Cognitive (mental processes) factors indicate high anxiety; such factors here are confusion, and one’s mind going blank. Low anxiety factors are generally attitudinal, shown by lack of confidence. It is also suggested that the indicators are cumulative, that is a person showing extreme anxiety will also exhibit those of low anxiety. These lower indicators could be less obvious due to the over-shadowing of the extreme indicator.

2.2. History of Hypnoteaching

Hypnoteaching has found by John Gruzelier, a psychologist from Imperial College in London. He did a research using FMRI, a tool to determine the brain activities. He found that people who are hypnotized, his brain activities increased especially on the part of the brain that affect the high-level thinking processes which associated with the people's behavior. He said that human able to do everything that can’t be imagine. Therefore hypnosis greatly impact in motivating and improving human performance. In teaching and learning activities, hypnosis can motivate students, improve concentration, self-confidence, discipline and organization. These skills can be improved by hypnoteaching.

Furthermore, hypnoteaching pioneer in Indonesia is Novian Triwidia Jaya, an education consultant. He has studied Hypnoteaching since 2001 through autodidact brain program and characters through training, either directly or remotely. Hypnoteaching is a blend of teaching which involves the conscious mind and the unconscious mind. Jaya said that Hypnoteaching is a blend of two words, Hypnosis and Teaching[1].

Hypnosis comes from the word "Hypnos" which is the name of the Greek god of sleep. The word "hypnosis" was first introduced by James Braid, a famous doctor in the UK who lived in 1795-1860. Prior to James Braid, hypnosis known as Mesmerism / Magnetism. Some definitions of hypnosis are:

a. Hypnosis is a technique or a practice in influencing others to enter into a trance hypnosis
b. Hypnosis is a condition where attention becomes centralized so that the level of suggestibility (power received suggestions) increase highly.
c. Hypnosis is the art of communication to influence someone to change his level of consciousness. This is achieved by decrease the brain waves from Beta to Alpha and Theta.
d. Hypnosis is the art of communication to explore the unconscious.
e. Hypnosis is a condition of increased consciousness.

The field of hypnosis are:
a. Stage hypnosis, is the hypnosis that serves as a means of entertainment and usually performed on a stage or in public.
b. Self-Hypnosis (otohypnosis), is hypnosis that serves as a means for ourself in order to get into the personal unconscious for therapeutic purposes and self-development.
c. Forensic hypnotism, is the hypnosis that serves as a means for stringing back memories of crime victims or witnesses in the trial.
d. Experimental hypnotism, is the hypnosis that serves as a means to conduct experimental studies.
e. Hypnotherapy (Medical Hypnotism), is the hypnosis that serves as a means of hypnotic treatment in the medical world.
f. Hypnoparenting, is the hypnosis that serves as a means for educating and caring for children in the family.
g. Hypnoteaching, is the hypnosis that serves as a means for educating and teaching students in education.

The U.S. Departement of Education, Human Division said that: “Hypnosis is the by-pass of the critical factor of the conscious mind followed by the establishment of acceptable selective thinking”. According to author’s interviews with Mr. Syahrul Komara Wednesday, March 30, 2011, he said that: “Hypnoteaching is a development of hypnosys and NLP from the USA.” NLP (Neuro Linguistic Programming) was developed by John Grinder and Richard Bandler in the 1970s. NLP is a technique that can build a closeness.

Hypnosis in education is not mean to deprive students of consciousness. They still conscious, but actually has been hypnotized by teacher’s suggestion through word or teacher’s attitudes. Hypnosis is a technique that is effective, fast and efficient way to deliver information to the unconscious mind. Hypnoteaching is a learning method which can create a comfort and pleasant situation in a controlled environment with a blend of conscious mind and unconscious mind so that the child's subject matter can easily get into the unconscious minds of students.

Hypnoteaching bring student’s unconscious mind from Beta to Alpha and Theta. Alpha and Theta condition is a condition when the best students receive instruction from teachers to make learning activities become more effective. Here are some types of brain waves as measured by a device called EEG:
a. Beta
   Beta is very conscious of the wave conditions between 12-25 rounds per second. Beta condition is a condition when we feel alert, analytical and very critical. In this condition, the conscious mind has a 100% role in doing the thinking.
b. Alpha
   Alpha is a relaxed state, with waves between 7-12 rounds per second. In this condition, the brain undergoes a reduction in stance of alert, analytical and critical. Brain began to open an input. In this condition, the conscious mind only contribute 25% for thinking. This condition is usually achieved when happy, relaxed, imagine and before going bed.
c. Theta
   Theta is a very relaxed condition between the conscious and the wave between 4-7 rounds per second. In this condition, the brain is very open to input, because the conscious mind does not work anymore. The subconscious mind remains active and the 5 senses are still active so that they can receive input. The subconscious mind can not distinguish between right and wrong, the brain only works on command.
d. Delta
   Delta is the sleep conditions with waves 0.5 to 4 rounds per second. In this condition, all the inputs can not work, because the 5 senses is no longer active. However, the subconscious mind remains active but not able to accept input.

Junior high school student aged approximately 13-15 years. At this age, students' brain waves are in Beta condition, the condition is very critical for the input. Therefore, hypnoteaching can be applied to bring student’s consciousness from Beta to Alpha or Theta in learning process to be more effective.

2.3. Hypnoteaching as Learning Methodes
It is known that the fundamental things that must be considered in the learning process is the selection and the using of appropriate learning techniques. Learning techniques that still has not been varied, it means that teacher should be implement an innovation in learning techniques in order to avoid boredom for students. In addition, innovative learning techniques are expected to cover the deficiencies of learning techniques that have been applied in order to remind the student learning outcomes.

One variation of innovative learning techniques is Hypnoteaching. Hypnoteaching is a unique way in teaching, creative as well as imaginative. Before the teaching-learning process takes place, students are conditioned to be ready to learn. All the preparations in learning is maximal. Students learn in a fresh condition. Student’s emotional and psychological did not escape from teacher’s attention, teacher made learning activities in fun atmosphere. In addition, teachers are also required in a steady state both emotionally and psychologically. Teachers also have to fresh and ready to teach, because the teacher will transmit a remarkable learning’s virus that will be transmitted to all students in the classroom.

Hypnoteaching is not a strange or unusual thing that related to mystical and magical. Hypnoteaching is scientific and appear in every situation and condition in teaching and learning activities so that students can easily receive information or understand the lessons taught by the teacher. Almatin said that: "Hypnosis is a technique that is effective, fast and efficient to bring information into the subconscious mind"[2].

Hypnoteaching is not a method of learning, but rather to a dynamic learning process to achieve the best results in learning. With hypnoteaching, perceptions changed quickly due directly to the subconscious mind of educators and students. So that the process can proceed smoothly in teaching and learning activities.

Hypnoteaching that applied in classroom is not meant hypnotizing students that make all of students fall asleep then suddenly classroom become silent because all students unconscious. Almatin said that: "Hypnosis Learning is a learning that is designed to create a comfortable and enjoyable situations in a controlled environment, to be able to get into the subconscious mind of the child"[2]. Furthermore Hakim said that: "Emotions and the subconscious mind students can easily record and imitate every word and of teacher's action in the classroom"[3]. Then Abdullah said that: "In the scenario of learning, teacher rather dominate the learning activities in imparting knowledge what is conveyed by the teacher so that students have to carry out the practice of teachers working on the command"[4].

Actually, teaching is to provide information to the conscious and the subconscious mind to understand a value and a new understanding as addition to the existing understanding or replace a rudimentary understanding. Sometimes, an information is difficult to be understood by the student's mind due to other thoughts that interfere with the absorption process during an information to the child's subconscious mind. In this case, the subconscious mind stores various kinds of human long-term memory, either entire information derived from empirical experience (experience that is felt directly by the students) as well as information derived from inductive experience (experience gained from speech, writings and impressions obtained from sources outside himself). Jaya said that: "It turns our minds are filled by the subconscious mind"[1]. Then MC Gregor states: "hegemony subconscious mind is so great and really master one's mind as much as 88%. The conscious mind leaving only about 12% of total mastery "(in [1]). So to maximize the potential of the subconscious mind, there will be an increase in extraordinary intelligence in students.

Smart describes human consciousness quadrant as follows:

![Quadrant of Human Consciousness](image)

*Figure 1. Quadrant of Human Consciousness*[7]
a. Quadrant I called consious competence is a condition in which a person can do things with a clear conscience. This condition is common and often occurs in life, because in this condition a person can do something consiously.

b. Quadrant II called unconscious competence is the highest position of the quadrant of one's consciousness. Because someone could unknown for doing something, whether it is in a state of spontaneous or in a state of being hypnotized. This condition is expected for students at the time of learning, that is, without realizing students could know and understand about the material that being taught.

c. Quadrant III called un-consious incompetence is the lowest position or the position of the bottom quadrant of human consciousness. Therefore, in this condition a person is not conscious and can not do something.

d. Quadrant IV called consious incompetence is a condition in which a person with a clear conscience can not do anything.

Information coming through the senses is not directly absorbed by the subconscious mind. This process requires the analysis of the conscious mind that has formed critical area or areas which aims to filter all information that coming from various sources. It happened because the subconscious mind is central and suggestive. Therefore, it is logic to go to the subconscious mind that stores human long-term memory, the information must be sealed by a partition wall called the critical area (CA) or the reticular activating system (RAS). CA or RAS is a temporary shelter before information is actually delivered to the subconscious mind.

Critical areas are needed in daily life as a filter of information for selecting things that harm and conflict with ourselves. Critical Area is often needed as a bulwark / protector, for example, to anticipate someone against fraud and the like. However, sometimes critical area also selects the whole thing is not desired by someone. That include giving the notion that math is hard, math is boring, I hate math, and so on.

To address the critical areas that are overactive in children, hypnoteaching is a way to disable the critical areas and the rest on students. Thus, information is needed on the student's minds that can be easily absorbed and stored in the student's subconscious mind. Hakim said that: "the best and fastest method penetrates CA is using hypnosis"[3]. It must be remembered that this technique is not a magical, mystical or use certain spirits. However, Hypnoteaching using persuasive communication techniques and emphasized on the selection of the pattern language to the giver information, namely teachers. This suggests that the body and the brain is a unity that can not be separated.

One of the hypnoteaching's benefits is to make students want to execute teacher’s commands gladly, such as working exercises independently, doing homework, obey the school rules and so on. It means that in this case the teachers empower students, not deceive. This is consistent with the statement of Abdullah said that: "Learners are living creature that has potential that can be developed, so that should be empowered not deceived"[4].

To empower learners, Hypnoteaching create a meaningful learning for students. Meaningful learning means bring things that are known to the students in the learning process. This is correspond with learning theory of Ausubel. Ausubel said that: "Learning meaningful arise when students try to connect new knowledge with their knowledge"[in [8]]. Student’s experienced that are often appear in daily life is also used in Hypnoteaching learning methods. Hypnoteaching connect daily life in learning content so that can make learning meaningful in order to improve student’s achievement.

Hypnoteaching also apply the didactic principle in learning mathematic. The didactic principle derived from the Greek "didasekei" which means teaching and "didaktikos" which means good at teaching. Thus, didactic teaching is a science that provides the principles on ways to express mathematics in particular so that it can be controlled and owned by the students. It is part of a didactic or pedagogical knowledge to educate children. There are 6 didactic principles that applied in Hypnoteaching:

a. Apperception Principle

Hypnoteaching connect new knowledge with student’s knowledge who already possessed in his mind. It aims to create a meaningful learning in teaching and learning activities. This is corresponding Ausubel Theory.

b. Demonstration Principle

Teachers who implement Hypnoteaching, in addition using pictures or diagrams, teacher should also use a model or a real object in order to help students’ reasoning. This is correspond with Bruner Theory.

c. Motivation Principle

Motivation is definitely very important in Hypnoteaching. Hypnoteaching foster motivation with penalties, give a gift or reward, praise and foster a sense of success. Motivation of the teacher's role as a
suggestion that can enter into the subconscious mind deeply so that the child can develop a sense of love and interest in Mathematics.

d. Self Principle
Hypnoteaching can foster self honesty in children. Such as work on the problems or tasks without cheating, collect homework on time, go to class on time and so on, so that Hypnoteaching implant soft skills indirectly in order to make children’s behaviour become better.

e. Correlation Principle
Hypnoteaching connect the subject that will be taught with other subjects and linking relationships between subject with other subjects in daily life. Abdullah said that: "According to classical education expert John Dewey, learners will learn best when they learn the new subject relate with the subject they have already known, and the learning process will be productive if the students are involve actively in the learning process"[4].

f. Principle of Evaluation
Evaluation of learning that use Hypnoteaching is comprehensive, continuous, goal-oriented, objective, open, meaningful and educational.

2.4. Create Mathematics Teacher’s Charm in Teaching
Setiawan said that there are six way to create Mathematics teacher’s charm in teaching [9],

1. Motivate Yourself
One’s success depends on one’s intention to strive and work hard to achieve success. Do something that is believed to be able to develop the self quality, Include hypnoteaching.

2. Pacing
Pacing means equating the position, gestures, language, and brain waves between teachers and students. The basic principle is "likely human, or rather assembled / interact with the like / have a lot in common", such as status, age, hobby, needs, and others - others. Commonalities among some people, will emit the same brain waves, then every message delivered from the one on the other will be accepted and understood very well. It is the same with many students, if they hate the process of learning, the brain waves means not equivalent to the student teacher. Teachers and students are not “click”.

The ways to pacing the students are as follows:

a. Equate the position of the students who are in teaching activities and feel things experienced by the students in the present.

b. Use language that matches with the language that is often used by students.

c. Perform movements and facial expressions appropriate to the subject that being taught.

d. Relate subject matter to daily life.

e. Follow the latest news as a source of knowledge

3. Leading
After doing pacing, the students will feel comfortable with the teacher. Leading means to lead or drive after you do the pacing therefore most of students follow teacher’s saying or assign with pleasure.

4. Use Positive Words
The use of positive word is according to the work of the subconscious mind that is not willing to accept the negative words because negative words can perceived conflict. For examples are: "I want you to never ever imagined a rabbit wearing a hat. I repeat again that you are not allowed at all to imagine a rabbit wearing a hat. That there is even more to imagine a rabbit wearing a hat, but the initial instruction is not be imagined ".

5. Give Praise
Praise is the reward that can increase self-esteem. Compliments is one way to establish students's self-concept. So give a sincere compliment to the students. Especially when he managed to do or achieve. Slightest form his achievements, still give a compliment. Including when he managed to make positive changes to itself, though it may still be below the standard of his friends, teacher should still give a compliment. With honors, a person will be motivated to perform better than ever.

6. Modeling
Modeling is the process of giving the speech and behavior role model through consistent. It is very necessary and be one of the key hypnoteaching. There needs to be confidence in the students. It means that being consistent with the speech teacher is an example of a nice figure for students as a teacher.

2.5. Construct the Power of the Student’s Unconscious Mind
In the teaching-learning process, students' ability to concentrate is different every age level. The older a student is, the more longer the span of their ability to maintain concentration. However, eventhough they are at the same age level, their ability to concentrate is still differ, can be 1 minute, 2
minutes or even more. The students could be bored, sleepy or fuss with the other students. The outbreak of the concentration of a student will be contagious to other students. The impact is the classroom become unmanageable.

Hypnoteaching provide a rapid way, happy, enjoyable and effective for students. To restore the original concentration with several techniques, here is how to construct the power of the student’s unconscious mind (in [1]):

1. Yelling
Yelling or screaming is used to restore the concentration of students into the lesson by shouting something together. The words of yelling replied by shouting and should have been agreed since the beginning of the lesson. Teachers who see students begin distracted, can use this technique to restore the concentration of students.

2. Reward and Penalty
Every individual has the motivation to do something and all the motivation that is based only on two driving gift (reward) and punishment/sanctions (penalties). There is someone who is motivated and eager to do something as a gift. But on the other hand, there are also moved by fear of punishment. Both driving motivation is what is used to make the classroom more active and fun in learning. However, in this case the punishment is punishment which is something that is not scary for students.

3. Emotional Clock
Emotional clock is means to regulate emotions of students. Emotion in every student always change every second and every student has a different emotion. The younger age, the changing of students emotions is more rapidly. Therefore it is necessary to use emotional clock so that students are in the same emotions.

Each student will not know whether in which condition. They can be in a variety emotions, such as: want to learn, want to tell, bored, want to rest, want to sleep, and other desires. Therefore student’s emotions need to be directed to create an effective teaching and learning process then students would still feel excited and happy in learning.

2.6. Learning Steps of Hypnoteaching


Step 1: Mind
In this case Mind means to create a pleasant situation in learning. Classroom environment, such as teachers, classrooms, environment outside the classroom should be in synergy to give peace to student’s mind.

Step 2: Acquiring the Fact
Acquiring the Fact means to obtain the facts. In the learning process, students need examples and facts in daily life. For example, why students should learn chemistry can be explained by the fact that the soap, shampoo, detergent, etc is the result of a chemical reaction. So does our body composition.

Step 3: Search Out the Meaning
Search Out the Meaning means to find meaning. Teachers must be able to provide a realistic analogy and explanation to every intent and purpose of any subject matter.

Step 4: Trigger the Memory
A teacher should be able to raise the student's curiosity by provide a more interesting detailed explanation. Questions can be made more attractive such as a chain question game. It triggers every student to ask and triggers memory for recall or call the information into the memory of the mind of students.

Step 5: Exhibit
Exhibit means to demonstrate. A teacher must be able to provide real examples by practicing the problems that can help students to understand and explore the subject matter because in a learning process, students should be able to demonstrate what they learn is. A test or exercises are a reflection from student’s reliability in demonstrate what they has learned.

Step 6: Reflect
Reflect means to reflect on lessons have learned. A teacher should conclude and reflect the new subject matter described. It's easier for students to remember and understand the subject matter that had just acquired. Teachers also have to pay attention the reflections of students when students do not understand what has been taught by teachers.

2.7. The Advantages and Disadvantages of Hypnoteaching

Although Hypnoteaching is relatively new, it is believed that hypnoteaching have advantages and disadvantages. The advantages of Hypnoteaching are:
a. Teaching and learning process is more dynamic and there is a good interaction between teacher and students.
b. Students can develop their talents and interests.
c. The teaching and learning process provide a lot of skills.
d. The learning process is more diverse.
e. Students can easily master the material, because they more motivated to learn.
f. The learning process is active.
g. Monitoring the students is more intensive.
h. Students are more imaginative and creative thinking.
i. Students learn honestly.
j. Students can absorbed the lesson more quickly and more lasting, because students do not memorize but reasoning.
k. Students’s attention to the lesson will be fully absorb.

Meanwhile, the disadvantage of hypnoteaching are:
a. Teacher don’t have much time to give attention to one of the students in the number of students in a classroom.
b. Teachers have to learn Hypnoteaching before implement it in the classroom.
   Not all teachers master hypnoteaching.

3. RESEARCH METHOD
This article describe how Hypnoteaching decrease student’s mathematics anxiety. Research design in this study is to seek and to answer the question “is the use of hypnoteaching can decrease student’s mathematics anxiety?”. In this study, I try to decrease mathematics anxiety by implementing hypnoteaching in the classroom by combining all aspects of hypnoteaching in teaching and learning activities in mathematics class. The subject matter that would be choosen is two variable system of linear equations (in Indonesia called SPLDV: Sistem Persamaan Linier Dua Variabel). The research framework to implement hypnoteaching in the classroom is describe below:

Yelling
Yelling that will be used is describe in the table below:

<table>
<thead>
<tr>
<th>TEACHER</th>
<th>STUDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi</td>
<td>Yes</td>
</tr>
<tr>
<td>Hello</td>
<td>Hi</td>
</tr>
<tr>
<td>What’s up champ?</td>
<td>Spirit, Learn, Fight</td>
</tr>
</tbody>
</table>

Reward and Penalty
Reward and Penalty that will be used by researchers is reward and penalty in the form of stars of expression. If a student gets the reward, then teacher will give stars with smiling expression. Conversely, if the student gets a penalty, then the teacher will give star with sad expressions. The type of Reward and Penalty that will be used in the research is:

<table>
<thead>
<tr>
<th>Reward</th>
<th>Penalty</th>
</tr>
</thead>
</table>

The criteria of Reward and Penalty are:

<table>
<thead>
<tr>
<th>No.</th>
<th>Reward/Penalty</th>
<th>Criteria</th>
<th>Nilai</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>😊</td>
<td>Given to student who competence and discipline</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>😊</td>
<td>Given to student who competence but not discipline</td>
<td>1</td>
</tr>
<tr>
<td>3.</td>
<td>😞</td>
<td>Given to student who is not competence but discipline</td>
<td>-1</td>
</tr>
<tr>
<td>4.</td>
<td>😞</td>
<td>Given to student who is not both competence and discipline</td>
<td>-2</td>
</tr>
</tbody>
</table>

In the end of learning, all reward and penalty collect then counted as a contribution assessment for
Emotional Clock

Researchers will create an emotional clock by using cardboard which is then colored to attract the student’s attention. This emotional clock set in front of the class so that all students can see it clearly. The body of emotional clock describe in figure below:

![Figure 2. Emotional Clock](image)

a. Silent Clock indicate that students have to silent and concentrate for studying.
b. Discussion Clock indicate that students have to discuss the topic of subject that given by teacher.
c. Free Clock indicate that students free their emotions, such as laughing, talking and sighed in a certain time but do not interfere with other classes.
d. Button Clock indicate that students have to activated their learning condition that assisted by the teacher. Teacher makes a black circle dot on the board and ask students see that black dot while breathing with ease. It aims to enable the student subconscious mind to stay focused on learning.

Learning Steps

Step 1: Mind (create a pleasant situation in learning)
Teacher can create a pleasant situation in learning by using suggestion words and action such as:

a. By the time the teacher walk into the classroom, teachers should not create a sense of tension in the minds of students, such as the voice of teacher’s footsteps should be avoided.
b. When the teacher enters the classroom, the teacher must be smile friendly with wide eyes opened and looking into each pair eyes of the students then greeted with warmth and enthusiasm.
c. Teacher should be saying hello to all of students and ask about them. It is better if teacher seasoned with a bit of humor to relieve the student’s tension and anxious.
d. Before starting the lesson, teacher raise student’s spirit through brain gym by using the music in Alpha or Tetha condition. Here researcher choose a music in Alpha condition: Meet Me Half Way from Black Eyed Peas album with the brain gymnastic movements that had been prepared by the teacher.
e. At the beginning of learning activities, teachers motivate students, tell the purpose of studying and the benefits of SPLDV in daily life.
f. Teacher inform the relate between SPLDV and the previous lesson.

Step 2: Acquiring the Fact (Getting the Facts)
Teacher explains why students should learn SPLDV by giving examples and facts in daily life.

Teacher explains that SPLDV often happens in life, especially in buying and selling. For example, the following sample questions:

Upin buy 2 2B pencils and 4 puzzle book at Koperasi Sekolah for Rp 10,000. Then Ipin also buy 3 2B pencils and 1 puzzle book at Koperasi Sekolah for Rp 10,000. If Mail want to buy 3 2B pencils and 12 puzzle book at Koperasi Sekolah, what is the price have to be paid by Mail?

In the above example problems, teachers are using the names of cartoon characters that are currently viewed in Television. In addition, teachers also use "2B pencils", "puzzle book" and "Koperasi Sekolah". Furthermore, teachers should really bring 2B pencils and books into the classroom and then the complete above equation system. In this case, the price of a 2B pencil’s and a book’s price is same or equal with the price of a 2B pencil and the price of a book at Koperasi Sekolah in fact. This is what Acquiring the fact is, using facts and real events that often occur in daily life. It aims to make students become interested in the subject.
Step 3: Search out the Meaning (Finding the Real Meaning)

Through examples and facts in Step 2, indirectly students have discovered what the meaning of two variable linear equations, two variables systems of linear equations, substitution and elimination. The teacher guide students discover what exactly SPLDV mean, understanding Substitution and Elimination in mathematics.

The example in Step 2, students may find that the price of 1 2B pencil and 3 puzzle book can be exchanged with the price of 1 2B pencils and 3 puzzle books. It implies that students have found and use the substitution method in solving the example problems.

Step 4: trigger the Memory

After SPLDV explained through examples and facts, teachers bring those examples and facts into models of Mathematics. The example in step 3 above is made to the mathematical modelling:

Suppose \( p = \) 2B pencils and puzzle books = \( b \) so that:

- 2 pencil 2B and 4 puzzle books is \( 2p + 4b \)
- 3 2B pencils and 1 puzzle book is \( 3p + 1b \)
- 3 2B pencils and 12 puzzlebooks are \( 3p + 12b \)

Math models are:

\[
\begin{align*}
2p + 4b &= 9000 \\
3p + 1b &= 9000 \\
3p + 12b &= 9000
\end{align*}
\]

Then the teacher complete these equations mathematically.

Step 5: Exhibit (Demonstrate)

Teacher gives many problems and let the students solve it in front of the class. Teacher try to get as much as possible all students worked in front of the class.

Step 6: Reflect

Teachers guide students to conclude and reflect the lesson described then give tasks or homework for students. All steps from step 1 to step 6 was conducted using the words and actions suggestion.

4. CONCLUSIONS

Hypnoteaching is a variety of innovative learning methods which can create a comfort and pleasant situation in a controlled environment by blending the conscious and the unconscious mind in teaching so that students can easily receive information and understand the lessons that taught by teacher. Hypnoteaching is scientific and appear in every situation and condition in teaching and learning activities. Therefore, I’m sure that hypnoteaching can decrease student’s mathematics anxiety by implementing it in the classroom. I recommend to apply hypnoteaching in the next study by using the framework for research that I have proposed.

Hypnoteaching method is vary rare and almost never being implemented in the classroom because most of teacher never heard Hypnoteaching and of course never apply it in the classroom. Thus, Hypnoteaching is very important and although Hypnoteaching is relatively new, I believe that if hypnoteaching apply in mathematics class, student’s mathematics anxiety will decrease in every dimension of mathematics anxiety. Therefore mathematics is not a phobia anymore.

REFERENCES

EFFECTIVENESS OF THE DIFFERENTIAL CALCULUS LECTURE BY USING TEACHING MATERIALS BASED ON OPEN ENDED APPROACH

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STKIP PGRI Pontianak

ABSTRACT

The objective of this research is to describe the effectiveness of the differential calculus lecture by using teaching materials based on open ended approach to improve students’ mathematical representation ability. The developing of teaching materials referred to the model of 4-D teaching material development. The description of the effectiveness of the lecture was taken from the third stage of the 4-D model. The results indicate that (1) the average of students’ mathematical representation ability is in medium level, (2) the students’ activities during the learning process are active; and (3) the students’ responses to teaching material of differential calculus based on open ended approach are quite good. Based on the data analysis, it is concluded that, the differential calculus lecture by using teaching materials based on open ended approach is effective.

Keywords: Effectiveness of lecture, Teaching materials, Open ended approach

1. INTRODUCTION

Kemp, et al [1] suggests that in order to measure the effectiveness of learning begins with asking the question: what has been achieved by students? To answer this question can be seen from the number of students who successfully achieve the learning objectives in a given time. Furthermore, Suherman, et al [2] states that the interest influences the process of students’ learning outcomes, if students are not interested in learning anything then it cannot be expected that they can study well, on the contrary, if students learn according to their interests, it can be expected the results will be better. Eggen and Kauchak [3] says that the effectiveness of learning characterized by the involvement of the student in learning, especially in the organization and the discovery of information. Therefore, the more active students in the learning process, the more effective the learning implemented.

Based on some opinions above, it is concluded that the effectiveness of learning according to Kemp emphasizes on students’ learning outcomes. Meanwhile, according to Suherman, it emphasizes on student interests, and according to Eggen and Kauchak, it emphasizes on students' learning activities. In this study, the authors combine some opinion above so that the effectiveness of the lecture is based on the students’ ability in doing the tests of differential calculus which are the tests of mathematical representation ability, the learning activities of students during the lecture, and the students’ responses to the use of teaching materials based on open-ended approach in lectures.

Differential calculus is one of the subjects studied in the mathematics education study program. In studying differential calculus, it cannot be separated from mathematical representation. Mathematical representations are expressions of mathematical ideas presented by the students as a model or as a substitute form of situation that is used to find the solution of problems being faced by the students as a result of their mind interpretation. A problem can be represented through images, tables, graphs, words (verbal), or mathematical symbols. In calculus, students are required to have good mathematical representation ability so that they can solve problems related to mathematical concepts involving calculus. Many problems in calculus are more easily solved if they are represented in a visual form first.

Representation ability is one components of a standard process in the Principles and Standards for School Mathematics in addition to the ability of problem solving, reasoning, communication, and connections (NCTM, [4]). It contains several reasons. According to Jones [5], there are three reasons why representation is
one of the standard processes, namely: (1) the fluency in doing translation among different representation types is the basic skill that the students need to develop a concept and mathematical thinking; (2) the mathematical ideas presented by a teacher through various representations will give an enormous influence on students in learning mathematics, and (3) the students need practice in building their own representation so that the students have the ability and understanding of good flexible concept that can be used in solving the problem.

Some forms of mathematical representation, such as verbal, images, numeric, algebraic symbols, tables, diagrams, and charts are inseparable parts of mathematics lesson. However, generally in learning mathematics, mathematical representations are studied or taught only as a supplement in solving mathematical problems. As essential component of learning, mathematical representation ability should be trained in the process of learning mathematics. This causes the ability of students and students’ representation is limited.

NCTM [4] states that students’ mathematical representation ability is very limited, so that when the students solve the problems, their ways to solve the problem tend to see the connection of essential elements in the problems dominated by symbolic representation, without paying attention to other forms of representation. Several studies have also revealed the weaknesses of student’s mathematical representation ability. The difficulties encountered among others are: the students’ difficulties in bridging representations and flexibly moving from one representation to other representations (Yerushalmy, [6]). Ferrini, et al [7] says that in studying calculus, students often feel satisfied with the different results and representations, and do not always consider that the results are inconsistent, and even contradictory. Similarly, Sfard [8], Zachariades, et al [9], Hong, et al [10], states that students have minimal ability in bridging representations without understanding the common thread among the concept ideas of represented materials.

The difficulties were also encountered by the writers during teaching differential calculus at STKIP PGRI Pontianak. Most students were still having difficulties in using various forms of mathematical representation to explain mathematical ideas and to solve mathematical problems. They also were still difficult in translating between forms of mathematical representation. This condition certainly needs to be solved, considering that the students of STKIP are becoming mathematics teachers that should be able to develop the ability of representation to their students.

The selection of appropriate learning approach will support the development of the representation ability. One of alternative approaches to learning mathematics that is expected to improve the students’ mathematical representation ability is open ended approach. Sawada [11] says that in the open-ended approach, teachers give situation problems to the students in which solutions or answers to the problems can be obtained in various ways. The teachers or tutors then use the differences of approaches or ways used by learners to provide experience to the students in discovering or investigating something new by combining knowledge, skills, and various mathematics methods or ways which have been learned by the students previously. By various ways of solution and answer, it gives students a lot of experience in interpreting the problems and it may also generate different ideas in solving the problems (Silver, [12]). This will certainly open the possibility for the students to use a variety of representations to find solutions to their problems, and it can help students solve the problems creatively, so that through learning by using open ended approach, it is expected to improve students’ mathematical representation ability.

To carry out the lecture by using open ended approach, lecturers or teachers need teaching materials which are based on the approach. Therefore, this study was preceded by developing teaching materials based on open ended approach to differential calculus lecture. The teaching materials will also consider the ability to be developed, that is the mathematical representation ability. Furthermore, teaching materials that have been developed are tested in the class lecture to obtain information about the effectiveness of differential calculus lecture using the materials. In addition, the effectiveness of lecture is observed from the improvement of mathematical representation ability, students’ activities during differential calculus lecture, and students’ responses to differential calculus teaching materials based on open-ended approach which has been arranged.

2. RESEARCH METHOD

The objective of this research is to describe the effectiveness of differential calculus lecture by using teaching materials based on open ended approach. This study was preceded by the development of the differential calculus teaching materials with the development of 4-D model (four D model) which refers to four stages namely to define, design, develop, and disseminate (Thiagarajan, et al [13]). However, for the purpose of this research, the development stages of teaching materials used were only up to and including the development stage (the third stage). The description of the effectiveness of the lecture was taken from the trial stage which is the third stage of the 4-D model.

This study was conducted in January to May, 2013; it began with designing teaching materials, validating them, and trying them out to the lectures in the class. Trials of lecture were conducted on 4 March to 27 May 2013. The trials were conducted to Mathematics Education Study Program of STKIP PGRI Pontianak. The subjects this research were the second semester students of Mathematics Education Study Program of STKIP PGRI Pontianak, B class, who were taking differential calculus lecture in the academic
year of 2012/2013. The research subjects are 48 students; 16 male students and 32 female students. To assess the effectiveness of differential calculus lecture using the teaching materials, measurement techniques, observation, and indirect communication techniques are used. The research instruments are the mathematical representation ability test, students’ activity observation sheets, and students’ questionnaire responses.

The data of students’ mathematical representation ability are analyzed to see the improvement occurred after the students obtain differential calculus lecture by using teaching materials based on open ended approach. The analysis of the improvement is conducted by using normalized gain formula, namely:

\[
\text{gain} = \frac{\text{posttest score} - \text{pretest score}}{\text{maximum possible score} - \text{pretest score}}
\]

Then the gain calculation results are interpreted by using the classification of Hake [14], that can be seen in Table 1.

<table>
<thead>
<tr>
<th>Magnitude of g</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>g &gt; 0.7</td>
<td>High</td>
</tr>
<tr>
<td>0.3 &lt; g ≤ 0.7</td>
<td>Moderate</td>
</tr>
<tr>
<td>g ≤ 0.3</td>
<td>Low</td>
</tr>
</tbody>
</table>

The data of students’ activities are analyzed by using percentage analysis. The percentage achievement of students’ activities for each item of activities is calculated by the formula:

\[
\text{percentage} = \frac{\sum_{i=1}^{10} \chi_i}{\sum N} \times 100
\]

The average of overall students’ activities is calculated by adding ten activity achievements for each item of activities, and then it is divided by ten. Students’ activities are said to be active if the percentage of students’ achievement activities is more than or equal to 50%.

The data of the students’ responses to the compiled teaching materials are analyzed with the following steps:
1. Calculating the scores of the answers given by the students to each statement item in the students’ questionnaire responses with the provisions for questions with positive criteria value for 1 (strongly disagree), 2 (disagree), 3 (agree), and 4 (strongly agree). For statements with negative criteria value for 1 (strongly disagree), 2 (disagree), 3 (agree), and 4 (strongly disagree).
2. Calculating the composite average score of positive and negative criteria for each condition, then determining its category with the provisions of average score:
   1.00 to 1.49 = student’s response is not good,
   1.50 to 2.49 = student’s response is poor
   2.50 to 3.49 = student’s response is quite good, and
   3.50 to 4.00 = student’s response is good

The criteria of lecture effectiveness criteria refer to the merging of criteria proposed by: Kemp, Suherman, and Egen and Kauchak, who states that the lecture is said to be effective if (1) the increase of the average of students’ mathematical representation ability is classified as moderate or high (by using the formula of normalized gain), (2) the average of active students’ activities is at least 50%, and (3) the responses of students to differential calculus teaching materials based on open ended approach are quite good or good.

3. RESULT AND ANALYSIS
3.1. Students’ Mathematical Representation Ability
Description of data on the test results of students’ mathematical representation ability after attending lectures by using differential calculus teaching materials based on open ended approach is presented in Table 2 as follows:

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>5.22</td>
<td>4.74</td>
</tr>
<tr>
<td>Postest</td>
<td>40.03</td>
<td>6.22</td>
</tr>
<tr>
<td>N-Gain</td>
<td>0.37</td>
<td></td>
</tr>
</tbody>
</table>
From Table 2 it is seen that the initial ability of students’ mathematical representation of differential calculus is very low, with an average pretest score of 5.22 out of a maximum score of 100. After obtaining the lecture by using differential calculus teaching materials based on open ended approach, there is an increase of mathematical representation ability of students, with the posttest score of 40.03. Calculation of the average increase in the mathematical representation ability of students using normalized gain formula, it is obtained the average increase of 0.37 which is categorized as moderate.

The result illustrates that the differential calculus lecture using the open ended approach can improve the students’ mathematical representation ability, although the result has not yet been maximum because the average score is still very low at 40.03 from maximum score 100. This happens because in the open ended approach, students are given the opportunity to resolve a problem with various answers and ways/methods, so that it would appear diverse representation of the problem.

Open problems given to the students are not only oriented to get the answers or final results, but they more emphasis on how students arrive at the answers, students can develop methods, or different ways to solve the problems. It provides opportunities for students to undertake a greater elaboration, so that students can develop their mathematical thinking, as well as assisting the development of students’ creative activity in using a variety of representations in problem solving.

The results are consistent with Inprasitha’s [15] research, who found that by applying open ended approach, students had more opportunities to do something, to think, to play an active role, to do something original, and to draw appropriate conclusions with their own ways. The study also concluded that the learning ability of students who obtained learning with open approach was better than the conventional class.

The results are also in line with Dewanto’s [16] research on learning with problem-based learning approach (BBM) which concluded that through non-routine problems, including an open issue, could improve students’ mathematical representation ability.

Mathematical representation ability of students as measured in the test consists of three indicators, namely: (1) students can use visual representations (graphs) to explain mathematical ideas and (2) students can do the translation from symbolic representations to visual representations (graphs), and use the visual representations to solve the problems. The details of the average score for each indicator of the tests are presented in Table 3 as follows:

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Pretest Average</th>
<th>Pos test Average</th>
<th>N-Gain</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ind 1</td>
<td>1.25</td>
<td>29.32</td>
<td>0.28</td>
<td>Low</td>
</tr>
<tr>
<td>Ind 2</td>
<td>11.58</td>
<td>57.17</td>
<td>0.50</td>
<td>Moderate</td>
</tr>
</tbody>
</table>

Table 3 shows that the increase in the mathematical representation ability of students in the first indicators is classified low. Meanwhile, in the second indicator, it is classified moderate.

The increase of the students’ mathematical representation ability in the indicator of using a visual representation (graph) to explain mathematical ideas (first indicator) is low. Many students do the mistake in observing the graph and estimating the derivative of a function shown by the graph at a given point. This shows the weakness of the students’ visual representation ability. The weakness of students’ understanding about the derivative definition also causes students difficult in completing the test items at the first indicator. Student should have understood that the derivative concept obtained from a tangent curve, therefore, in estimating the derivative of a function, they can draw a tangent line at a given point on the graph and then use the slope of the tangent line in searching its derivatives. This certainly still needs the improvement of lecture process so that the lecture can obtain maximum results. In the lecture process, it still needs more practice in observing the graph and finding as well as explaining mathematical ideas associated with the graph.

### 3.2. Students’ Activities

The description of students’ activities during the differential calculus lectures using teaching materials based on open ended approach is presented in Table 4.

Table 4 shows that the average of students’ activities during the differential calculus lecture using teaching materials based on open ended approach for all activities is 61.5%.

Students’ activities during the differential calculus lecture using teaching materials based on open ended approach are quite active, it is seen from the average activity of active student at 61.5%. This is in line with the Nohda’s [17] opinion who says that learning by using open ended approach can help the development of creative activities of students. Through this approach, each student has the freedom to solve problems according to their ability and interest, students with higher abilities can perform a variety of mathematics activities, and students with lower abilities can still enjoy mathematics activities according to their own abilities.
Table 4. Students’ Activities during the Lecture

<table>
<thead>
<tr>
<th>No.</th>
<th>Students’ Activities</th>
<th>Percentage of Students’ activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Responding to lecturer’s questions about their prerequisite knowledge.</td>
<td>47%</td>
</tr>
<tr>
<td>2.</td>
<td>Listening to lecturer’s explanations about the materials and the uses of lecture materials</td>
<td>97%</td>
</tr>
<tr>
<td>3.</td>
<td>Reading the open problems related to the lecture materials will be studied as including in the teaching materials and paying attention to lecturer’s explanations related to the problems.</td>
<td>100%</td>
</tr>
<tr>
<td>4.</td>
<td>Resolving open problem contained in lecture material individually and followed by a group discussion.</td>
<td>89%</td>
</tr>
<tr>
<td>5.</td>
<td>Asking for certain parts which have not been understood by them.</td>
<td>38%</td>
</tr>
<tr>
<td>6.</td>
<td>Some students from different groups (representative of each group) to write down the results of his work on the board and explain them to his friends.</td>
<td>7%</td>
</tr>
<tr>
<td>7.</td>
<td>Students from other groups provide feedback (questions or comments) to work results of the rendering group.</td>
<td>27%</td>
</tr>
<tr>
<td>8.</td>
<td>Asking the lecturer and answering the questions given by the lecturer.</td>
<td>14%</td>
</tr>
<tr>
<td>9.</td>
<td>Summarizing the materials they have learned.</td>
<td>96%</td>
</tr>
<tr>
<td>10.</td>
<td>Listening/recording their duties and information about the materials to be studied at the next meeting.</td>
<td>100%</td>
</tr>
</tbody>
</table>

Average 61.5%

Some items of students’ activities during the lecture are still quite passive. This can be seen in Table 4 that in some items of the activities are less than 50% of students who perform these activities. Students still look fear in asking and answering questions of the lecturer, only 14% of students who undertake the activities. Similarly, only 7% of students who display their work on the board and explain to their friends in the activities of class discussion. Even only 27% of students who their responses. These show that students who dare to express their differences of opinion in resolving open problem are still low. This may be due to students are not familiar with oral activities in lecture. Obviously in this case, the reinforcing from the lecturer is still necessary so that the students are more willing to express their opinions, such as by providing stimulus in the form of simple questions, and gradually motivating the students to give answers or different ways in resolving the open problem.

3.3. Students’ Response

The description of data of the students’ response results after obtaining differential calculus lecture based teaching materials using open ended approach is presented in Table 5.

Table 5. Students’ Responses to the Teaching Materials

<table>
<thead>
<tr>
<th>No</th>
<th>Indicator</th>
<th>Variable</th>
<th>Score Average</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Attention</td>
<td>Motivation and learning pleasure.</td>
<td>3.03</td>
<td>quite good</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Understanding the teaching material and thinking more critically</td>
<td>2.84</td>
<td>quite good</td>
</tr>
<tr>
<td>2.</td>
<td>Confidence</td>
<td>Eliminating misconception</td>
<td>2.82</td>
<td>quite good</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Taking the important ideas and remembering them.</td>
<td>2.31</td>
<td>Poorly</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Number of tasks and the difficulty level of the task.</td>
<td>2.30</td>
<td>Poorly</td>
</tr>
<tr>
<td>3.</td>
<td>Satisfaction</td>
<td>Confidence in learning and test.</td>
<td>3.08</td>
<td>quite good</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Satisfaction with the content, quality of writing and drawing</td>
<td>2.92</td>
<td>quite good</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TOTAL</td>
<td>2.80</td>
<td>quite good</td>
</tr>
</tbody>
</table>

Table 5 shows that the responses of students to differential calculus teaching materials based on open ended approach overall are quite good.
The overall results of processing the data on student responses to the differential calculus teaching materials based on open ended approach quite good. By using the materials, it makes the students more motivated and enjoy learning, the students feel more easier to understand the materials and make them think more critically. The students also feel quite satisfied with the content, quality of writing, and images of teaching materials. The lowest average score on the aspects of belief is some students still feel that the information in the teaching materials is too much so that it is difficult for them to take important ideas and to remember them. Most students also states that the tasks in the teaching materials are too much and too hard, however, learning by using the teaching materials makes them confident in learning and in solving the tests.

4. CONCLUSIONS
Based on the data analysis, generally it can be concluded that the differential calculus lecture by using teaching materials based on open ended approach is effective. In detail, it can be summed up as follows:
1. The average increase of students’ mathematical representation ability using normalized gain formula of 0.37 is categorized moderate.
2. He average of students’ activities during the differential calculus lecture using teaching materials based on open ended approach is 61.5%. The average score of students’ responses to teaching materials of differential calculus based on open ended approach of 3.33 is categorized quite good.

REFERENCES
THE DEVELOPMENT OF ANALYTIC GEOMETRY TEACHING MATERIAL BASED ON SOLO TAXONOMY (STRUCTURE OF THE OBSERVED LEARNING OUTCOME) AS AN EFFORT TO ENHANCE STUDENTS’ MATHEMATICAL COMPETENCE

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ABSTRACT

This result is the development study of analytic geometry teaching material which is designed to enhance students’ understanding in mathematics. This teaching material is based on SOLO taxonomy. The learning stages in this model include uni structural stage, multi structural, relational, and abstract. This development study covers: identification and the development of learning material model structure, media development to provide teaching material and the implementation of teaching material with media which has already been chosen, along with evaluation and dissemination teaching material and its learning. The developed instruments in this research are adjusted with the necessity based on research developmental. At the beginning of this research, the researcher conducts a deep theoretical and empirical examination to elaborate analytic geometry syllabus according to curriculum structure and courses dissemination which is exist in UPI curriculum 2010. The next step is to arrange teaching material which is provided in form of printing media with computer programe. Afterwards, teaching material which is produced is to be tested in a definite manner by means of instruction experimentsto be examined the effectiveness and its influence toward students’ competence. The result of this study is became the consideration material in aiming and completes the model of teaching material before it is produced to be disseminated.

Keywords: Solo taxonomy, Mathematics competence.

1. INTRODUCTION

Learning mathematics is so important, so that the result of mathematics’ credit hours at school could be said is quite much. Mathematics education majors as a producer of teachers, so in the process of learning the models, methods, and the approaches it is should be chose precisely so the students could understand the materials well. In a lot of subjects, there are some students who say that they do not understand about the subjects. Then, some different models, methods, and approaches have been tested, for each subject. It is expected, that later on it could be found a proper way to teach particular material, so that all of the students could say that they are already understood the material and when they become a teacher, they could convey the materials easily to their students.

Analytic geometry is a basic science which needs to be mastered widely and deeply by the students, particularly a teacher to be, because analytic geometry is so much found in mathematics at school. Analytic geometry is also a compulsory subject that must be signed by a –teacher to be- students at the second semester with the allocation three credit semesters. In the study of analytic geometry at the previous year, expository method is used and the achievement of the final test score is still low. Beside that thing, there are a lot of students who comment that they are not really understood. Because of that, it is needed some efforts to develop a better, more interesting, motivate, and fun learning model.
One of the options is to use a model of mathematics learning with using SOLO (Structure of the Observed Learning Outcome) taxonomy. The superiority from SOLO taxonomy is that the learning is started from a simple to an abstract form with paying attention to the students’ response. So it is expected that the students could understand the concept of analytic geometry easier. Thus, learning mathematics with SOLO taxonomy is reasonable to become a reference of analytic geometry teaching material development at the level of university.

SOLO taxonomy is an important taxonomy to understand the connection between study quality and mental development stage. Biggs and Collis (Alagmulai, 2006) stated that each cognitive stage lies the same response structure and it is increasing from a simple to an abstract form. Beside that, according to Collis (Romberg, 1982) he said that based on the quality of students’ response model, structure of learning result (SOLO stage) is categorized in to five stages, they are: pre structural, uni structural, multi structural, relational, and abstract stage. Biggs in Ipurangi (no time information) decided SOLO as “a framework for understanding”. While Romberg (1982) stated that SOLO is concerned to the reasoning.

According to Biggs and Collis (Alagmulai, 2006) generally, a response model on each SOLO stage is marked by five abilities, they are: (1) pre structural, refuses to give response, (2) uni structural, able to take conclusion based on one connection, data or information in a concrete manner, (3) multi structural, able to take conclusion based on two or more connection, data or information, but still in a separate manner, (4) relational, could think deductively and makes conclusion based on two or more connection, data or information in an integrated manner, (5) abstract, could think inductively or deductively and could arrange the general principles or hypotheses based on the given information.

Here is one of the examples of students’ response based on each stage of their learning result quality for a question which is equivalent with what Biggs and Collis (Firdaus, 2004) have made. Find the score of \( p \) from these questions;

\[
(112 : 56) \times 7 = (112 \times 7) : (p \times 7)
\]

1. Pre structural response
   “I haven’t done it yet” or “I don’t want to answer this question”.
   Those responses showed that the student do not want to answer the question.
2. Uni structural response
   “56, because 56 is not exist on those two different sides yet”.
   This kind of answer showed that the student is only using one part of the data to answer the question.
3. Multi structural response
   \[
   \begin{align*}
   2 \times 7 & \Rightarrow 784 : (p \times 7) \\
   = 14 & \Rightarrow 784 : ? = 14, \text{ so } ? = 56 \\
   \text{So that } p = 56 \times 7 = 392 
   \end{align*}
   
   This response showed a combination from arithmath to decrease the complexity and focus on “\( p \)”. Even though, the students look like that they are not able to solve the connection in the whole thought yet.
4. Relational response
   \[
   \begin{align*}
   2 \times 7 & \Rightarrow 784 : (p \times 7) \\
   = 14 & \Rightarrow 784 : (p \times 7) = 14 \\
   784 : 56 = 14 & \Rightarrow \text{So that } 56 = p \times 7 \\
   \text{So } p = 8 
   \end{align*}
   
   This answer showed that the students are able to answer the question with using arithmetic arrangement and they could take the relation in a statement in to thought and success in answer the question.
5. Abstract response
   \[
   \frac{b \times c}{d} = \left( \frac{b \times c}{d} \right) \text{ where } d = p \times 7
   \]
   The pattern which is suggested is same as the characteristics in times operation on spherical numeral which is distributif. So that, it is could be answered as this way:

\[
\begin{align*}
\frac{112}{56} \times 7 & = \frac{112 \times 7}{56} \\
56 & = p \times 7, \quad \text{So } p = 8
\end{align*}
\]

That response shows the characteristics which is focused on the connection between operational and numeral.

If we design that question in form of multiple choice, with five answers option so the question could be designed as below:
(112 \div 56) \times 7 = (112 \times 7) : (q \times 7), is:

a. 784  
b. 112  
c. 56  
d. 8  
e. 4

Look at the deception option on b and c is a typical choices for students who are in a place of uni structural response, so a multiple choice question is also could be used as diagnostic instrument. The analysis of students response will classify the development stage of students study result on the uni structural, multi cultural, relational and abstract stage.

Mathematics competence which is introduced by Kilpatrick, Swafford, and Findel (2001) is, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Conceptual understanding competence understands the mathematics concepts, operations, and relations in various representations. This competence is in accordance with understanding mathematics concept, it is explaining the relation between concepts and communicate it in various representations. Procedural fluency competence is capable to use procedures in a flexible, accurate, efficient and precise manner. Strategic competence is an ability to formulate, represent, and solving mathematics problems. This competence is included in to solving problems stage. Adaptive reasoning competence is using reasoning on a pattern, arranges the generalization or proof in justifies mathematics statement. Productive disposition competence corresponds attitude changes in to a more positive direction on the using of mathematics in life, so that the students become tough and confidence in solving the problems.

Generally, problems formulation in this research is “How to develop teaching material of analytic geometry based on the model of mathematics study based on SOLO taxonomy as an effort to increase students’ mathematics competence.

2. RESEARCH METHOD

Research method which is going to use is to follow a set of developmental research that will be conducts through thought experiments and instruction experimentation. Here are is somewhat like below.
### 3. RESULT AND ANALYSIS

Based on the sequence of material and concept map of lecture material of Analytic Geometry could be divided into eight topics, they are: (a) Coordinate System and Area Vector, (b) Straight Line Equation, (c) Circle Equation, (d) Parabola, Ellips, and Hyperbola Equation, (e) Space Coordinate System, (f) Flat Field, (g) Straight Line in Space, and (h) Globe, Ellipsoida, and Paraboloida Equation.

Developing teaching material could be conducted through the concrete thing first, because the reality in the classroom there are some students who having difficulty in understanding the material, even though the students have already classified into concrete formal stage. Beside that, it is needed to do a deep review about the topic which is included in to teaching material. Based on that repersonalization, furthermore it is arranged the sequence of teaching material which is expected it could improve the students mathematics competence. From that sequence of teaching material and learning model which is used, then it is developing the teaching material and a detail student’s assignments that is suited with the learning stage according to SOLO taxonomy which are unistructural, multistructural, relational, and abstract. Those teaching materials and students’ assignments are served in form of Handout, Students Worksheet, and Powerpoint media.
A problem appears in the implementation of Analytic Geometry learning which is according to SOLO taxonomy is that some students find a difficulty in abstract stage, so that the students should be given concrete example again. That problem is considered as a material to improve the teaching material of Analytic Geometry.

From the result of end semester test, it is achieved the average score of 67.11 with the standard deviation of 19.42, the minimum score which is achieved by the students is 24, while the maximum score is 100. From the result of average score in semester test, it is seen that the mathematics competence which is achieved by the students through the learning through SOLO taxonomy is classified as enough.

4. CONCLUSIONS
Here are the conclusions from this research:
1. Based on the sequence of the material and concept map of Analytic Geometry lecture, it could be divided into eight topics, they are: (a) Coordinate System and Area Vector, (b) Straight Line Equation, (c) Circle Equation, (d) Parabola, Ellips, and Hyperbola Equation, (e) Space Coordinate System, (f) Flat Field, (g) Straight Line in Space, and (h) Globe, Ellipsoida, and Paraboloida Equation.
2. The students’ assignments which are developed based on the arrangement and concept map by following the stages on SOLO taxonomy supports the students to get involve actively in each stage, so that the student competence in Analytic Geometry subject is increasing.

REFERENCES
THE MODIFIED LEARNING OF MODEL-ELICITING ACTIVITIES (MEAS) TO ENHANCEMENT OF STUDENTS’ STATISTICAL THINKING ABILITY AND STATISTICAL DISPOSITIONS

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ABSTRACT

This paper is a result of research that aims to enhance of students’ statistical dispositions and statistical thinking ability through a modified of MEAs learning by integrating Didactical Design Research into teaching materials. Research was conducted to all students S1 of mathematics education who follow courses at the Basic Statistics at even semester 2011/2012 in a state university in Bandung using quasi-experimental methods to Split-Plot Nested Design. The results showed that: in the early-mid-semester and at the middle-end of the semester enhance of statistical dispositions and statistical thinking ability that students acquire learning that modified MEAs significantly higher than students who receive conventional learning.

Keywords: Statistical thinking, Statistical disposition, The modified MEAs learning.

1. INTRODUCTION

At this time the experts are busy talking about students’ statistical thinking ability and statistical dispositions. The results of research Martadiputra and Tapilouw (2011) indicates that statistical thinking ability of students of mathematics and mathematics education courses as well as those who have already graduated S1 at a state university in Bandung is not optimal, less than 10% who entered the Analytical level. Especially for statistical thinking ability related to analyzing and interpreting the data, none of students have the Analytical level. Furthermore Martadiputra, B.A.P. (2012) defines statistical dispositions as a tendency of people to think and act in a positive and constructive manner that takes place in statistical activities. Dasari, D. (2009) showed that: a statistical disposition influenced student prior knowledge and statistical learning model factors obtained. Not optimal students’ statistical thinking ability and statistical disposition allegedly caused by a models, approaches, or learning strategies (conventional learning) lack of precise which is used in statistical learning. Therefore, researchers are trying to develop a basic statistical learning model called the modified learning of MEAs that can optimize the enhance students’ statistical thinking ability and statistical dispositions.

The modified learning of MEAs is one of the statistical learning developed by Martadiputra, B.A.P. (2012) with: 1) modify the teaching material of previous MEAs using a Didactical Design Research (DDR), and 2) completion of learning steps MEAs prior to adding one extra step at the beginning and an additional step at the end of the lesson that will enhance students’ independent learning.

Based on the above, the problem arises:

1. Whether enhancement statistical thinking ability that students acquire learning that modified learning of MEAs higher than students who receive conventional learning?
2. Are there differences student’s statistical thinking ability between students who have initial statistical capable of low, medium, and high?
3. Whether enhancement statistical disposition who obtain a modified of MEAs learning higher than students who receive conventional learning?
4. Are there differences statistical disposition between students who have initial statistical capable of low, medium, and high?
5. Is there a association between enhancement of statistical dispositions and enhancement of statistical thinking ability?

To solve these problems, the authors conducted a study that aims to:
1. Knowing whether enhancement statistical thinking ability that students acquire learning that modified learning of MEAs higher than students who receive conventional learning.
2. Knowing and analyzing whether are there differences student’s statistical thinking ability between students who have initial statistical capable of low, medium, and high.
3. Knowing whether enhancement statistical disposition who obtain a modified of MEAs learning higher than students who receive conventional learning.
4. Knowing and analyzing whether are there differences statistical disposition between students who have initial statistical capable of low, medium, and high.

Determine association between enhancement student’s statistical dispositions and enhancement student’ statistical thinking.

2. REVIEW OF LITERATURE

Statistical Thinking Ability
Referring to Snee (1990), Jones, et al. (2000), Delmas (2002), Chance (2002), and Ben-Zvi & Friedlander (2010), Martadiputra, B.A.P. (2012) defines students’ statistical thinking ability as the ability to understand and comprehend how to: describe data, organize data, representing data, and analyzing and interpreting the data, as well as the ability to apply statistical understanding of the real issues by providing critique, evaluate, and make generalizations. Further statistical thinking ability are divided into four levels: (1) Idiosyncratic, (2) Transitional, (3) Quantitative and (4) Analytical.

Statistical Dispositions
Martadiputra, B.A.P (2012: 55) defines a statistical disposition or productive disposition toward statistics as propensity of students to think and act in a positive and constructive manner that takes place in statistical activities. Dimensions and indicators of statistical disposition are presented in Table 1 below.

<table>
<thead>
<tr>
<th>NO.</th>
<th>DIMENSIONS</th>
<th>INDICATORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Passion and a serious concern in the study</td>
<td>1. The interest in basic statistics course.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Punctual attendance in lectures.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Desire to work and collect assignments.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Seriousness in following the lecture.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5. Active in following the lecture.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Confident in answering questions.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Confident in communicating the idea / ideas.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Confidence in solving problems.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5. Confidence in using statistics.</td>
</tr>
<tr>
<td>3.</td>
<td>Flexibility in exploring ideas and alternative solutions to problems</td>
<td>1. Flexibility in exploring statistical ideas.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Flexibility in finding alternative statistical problem solving.</td>
</tr>
<tr>
<td>4.</td>
<td>Persistence in the face and resolve the issue</td>
<td>1. Persistence in the face of statistical problems.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Persistence in solving statistical problems.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Persistence in understanding the issues, procedures, concepts, or some other important aspects of basic statistics.</td>
</tr>
<tr>
<td>5.</td>
<td>Monitor and reflect on ideas</td>
<td>1. Examine the ability of the ideas and thoughts expressed statistical faculty and or students.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Desire to respond to the ideas and thoughts of others based statistical own thoughts.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Desire to learn the course material.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Desire has a source other than the required reading teacher.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Desire to learn and ask things nonroutine.</td>
</tr>
<tr>
<td>7.</td>
<td>Share your opinion with other people</td>
<td>1. Desire to share opinions, ideas, and ideas with a lecturer.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. The desire to share opinions, ideas, and ideas of students.</td>
</tr>
</tbody>
</table>
Modified Learning of MEAs


1) Modify the teaching material of previous MEAs using a Didactical Design Research (DDR). According Suryadi (2010), DDR consists of three phase: (1) analysis didactic situation; (2) analysis metapedadidaktik; and (3) analysis retrospektif. So the teaching materials of modified learning of MEAs have been tested and refined so that the learning barriers that may arise already anticipated by the lecturer. Modified learning of MEAs still maintain six principles of MEAs: 1) principle of construction; 2) principle of reality; 3) principle of self-assessment; 4) principle of documentation; 5) principle of reusability, and 6) principle of effective prototype.

2) Completion learning steps of earlier MEAs by adding one extra step at the beginning and an additional step at the end of the lesson. At the beginning of the modified learning of MEAs lecturer asks a series of questions to determine the extent to which students have mastered the basic concepts of the material to be taught. While at the end of the modified learning of MEAs, lecturer assign students to learn their own and make a concept map material will be taught at the next meeting. It is considered necessary to improve the independence of learning (statistical disposition) students.

Characteristics of conventional teaching, MEAs learning, and modified learning of MEAs presented in Table 2.

<table>
<thead>
<tr>
<th>No.</th>
<th>Characteristic</th>
<th>Conventional Learning</th>
<th>MEAs Learning</th>
<th>Modified of MEAs Learning</th>
</tr>
</thead>
</table>
| 1.  | Teaching materials | - Without DDR.  
- Teaching materials are presented in the form of textbooks.  
- The concept is explained directly by the lecturer.  
- Lecturer gives example problems and its solution.  
- Lecturers provide practice questions. | - Without DDR.  
- Teaching materials are lecturers who presented creations in the form of real statistical problems that are open-ended  
- Preparation of teaching materials should contain six principles of MEAs: construction, reality, self-assessment, documentation, reusability, and effective prototype. | - Using DDR  
- Teaching materials are lecturers who presented creations in the form of real statistical problems that are open-ended learning that meet the six principles of MEAs.  
- Before use in learning, teaching materials tested.  
- Do analysis didactic situation, analysis metapedadidaktik, and analysis for the improvement of teaching materials retrospektif. |
| 2.  | Lecturer | - Lecturers act as a source of learning, giving examples of questions and answers, provide practice questions and give an evaluation. | - Lecturers act as a facilitator and motivator.  
- Lecturer directs students to be actively involved on an individual basis to understand the problem, work in teams to solve | - Lecturers act as a facilitator and motivator.  
- Lecturer directs students to be actively involved on an individual basis to understand the problem, work in teams to solve |
<table>
<thead>
<tr>
<th>N.</th>
<th>Characteristic</th>
<th>Conventional Learning</th>
<th>MEAs Learning</th>
<th>Modified of MEAs Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>problems, presenting problem resolution, and facilitate classroom discussions to make inferences.</td>
<td>problems, presenting problem resolution, and facilitate classroom discussions to make inferences.</td>
<td>At the time of the lesson, lecturer, identified, each potential student, learning barriers (learning obstacles) that appears, observe the correspondence between the implementation of learning plans.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>• Finish learning for teachers to reflect on improvement of teaching materials.</td>
</tr>
<tr>
<td>3</td>
<td>Student</td>
<td>• As a recipient of knowledge given faculty and completing practice questions,</td>
<td>• Students as problem solvers of problems given by.</td>
<td>• Before learning students are required to learn the material his own by making a summary of the material to be taught.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• At the time of the lesson, students as problem solvers who dikreasi lecturer. Students actively learn individually, team, or class.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• At the end of pembelajaranan students to reflect on the learning materials</td>
</tr>
<tr>
<td>4</td>
<td>Interaction</td>
<td>• Interaction between lecturers and students is one-way or two-way.</td>
<td>• Interaction the multidirectional nature, lecturers and students, students with students.</td>
<td>• Interaction the multidirectional nature, lecturers and students, students with students.</td>
</tr>
</tbody>
</table>

Sources: Martadiputra, B.A.P (2012: 119)

3. RESEARCH METHOD

In this study the researcher using the Research & Development Method (R & D). The research is divided into two phase. In the first phase (preparation phase) conducted research development (development research) design study with no control group: X O to determine the effectiveness of teaching material and modified learning of MEAs, as well as validity, reliability, discrimination, and difficulty index items of initial statistic ability test, and statistical thinking ability test conducted on the class mathematics education student at a state university in Bandung for one semester.

In the second phase, the researcher tested the modified learning of MEAs for all mathematics education students who take the course Basic Statistics at a state university in Bandung, which consists of three classes: control, experimental 1, and experimental 2 by using a quasi-experimental control group design with pretest and posttest forms: O X O; O X O; O X O.

The design of the experiments using a Split-Plot Nested Design (Gaspersz, V. 2006), because this study is experiments dealing with the plot size problem, the factors considered by the researcher is effect learning
models on enhancement statistical thinking ability greater than grouping students factors based on its initial statistical capabilities. Initial statistical capability factors (low, medium, high) was placed as the main plot, learning factors (conventional and modified of MEAs) are placed as a subplot, while the factors of students (regular and repeat) placed as children plot.

Experimental class consists of two classes, regular students (41 students) and the students repeat classes (12 students) who obtained a modified learning MEAs. While the control class is the class of regular students (39 students) who acquire conventional learning. Before all classes treatment, researcher do the measurement of students’ initial statistical capability. Based on the results, students in each class to be grouped into three categories: high, medium, and low. Initial and end of students’ statistical thinking ability measured using statistical thinking ability test. Enhancement students’ statistical thinking ability obtained by using the normalized gain (N-gain). To answer the research problem of processing and analyzing data using parametric and nonparametric statistical rules.

4. RESULT AND ANALYSIS

a. Comparation Enhancement Statistical Thinking Ability Between Students Who Obtained Modified Learning of MEAs with Student Who Obtained Conventional Learning

Table 3
Table Winner of Students’ Statistical Thinking Ability

<table>
<thead>
<tr>
<th>Class</th>
<th>Group</th>
<th>N-Gain</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. Dev</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>Low</td>
<td>0.3590</td>
<td>0.1595</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.4585</td>
<td>0.1108</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.4728</td>
<td>0.1745</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>0.4436</td>
<td>0.1367</td>
<td></td>
</tr>
<tr>
<td>Experiment 1</td>
<td>Low</td>
<td>0.5611</td>
<td>0.2397</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.7140</td>
<td>0.1575</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.6949</td>
<td>0.0884</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>0.6828</td>
<td>0.1793</td>
<td></td>
</tr>
<tr>
<td>Experiment 2</td>
<td>Low</td>
<td>0.7013</td>
<td>0.1376</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.7269</td>
<td>0.2171</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.7451</td>
<td>0.1450</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>0.7208</td>
<td>0.1546</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>Low</td>
<td>0.5254</td>
<td>0.2282</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.6092</td>
<td>0.1907</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.5788</td>
<td>0.1930</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>0.5863</td>
<td>0.2003</td>
<td></td>
</tr>
</tbody>
</table>

Sources: Martadiputra, B.A.P. (2012)
From Table 3 showed that:
1. The highest enhancement of students’ statistical thinking ability owned by class experiment 2, class experiment 1, and the lowest owned class control;
2. The highest enhancement of students’ statistical thinking ability owned by the medium group, the high group, and the low lowest owned low group.

Table 4
Difference Enhancement of Students’ Statistical Thinking for Control, Experiment 1, and Experiment 2 Class (The level of significance at α = 0.05)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Class</th>
<th>Normality Test (K-S)</th>
<th>Homogeneity Test</th>
<th>Difference Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-gain Students' Statistical Thinking</td>
<td>Control</td>
<td>0.200* Normal</td>
<td>0.663</td>
<td>One Way Anova</td>
</tr>
<tr>
<td></td>
<td>Experiment 1</td>
<td>0.200* Normal</td>
<td></td>
<td>(Scheffe) 0.000</td>
</tr>
<tr>
<td></td>
<td>Experiment 2</td>
<td>0.200* Normal</td>
<td></td>
<td>(Scheffe) 0.768</td>
</tr>
</tbody>
</table>

Sources: Martadiputra, B.A.P. (2012)

From the One Way ANOVA test in Table 4 show that: enhancement of students’s statistical thinking ability between control, experiment 1, and experiment 2 is significance difference. Further posthoc test results using Schaffe test shows that:
(1) Enhancement students’ statistical thinking ability of regular students (E1) who obtain modified learning of MEAs higher significantly than regular students (K) were obtained conventional learning;
(2) Enhancement students’ statistical thinking ability of repeat students (E2) who obtain modified learning of MEAs higher significantly than regular students (K) were obtained conventional learning;
(3) No difference significant in enhancement students’ statistical thinking ability between regular class (E1) and repeat class (E2) who obtained a modified learning of MEAs.

b. Comparison Enhancement Statistical Disposition Between Students Who Obtained Modified Learning of MEAs with Student Who Obtained Conventional Learning

Table 5
Difference Enhancement of Students’ Statistical Thinking Ability for Low, Medium, and High Group (The level of significance at α = 0.05)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Class</th>
<th>Normality Test (K-S)</th>
<th>Homogeneity Test</th>
<th>Difference Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-gain Students' Statistical Thinking Ability</td>
<td>Low</td>
<td>0.200* Normal</td>
<td>0.663</td>
<td>One Way Anova</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.172 Normal</td>
<td></td>
<td>0.270</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.200* Normal</td>
<td></td>
<td>0.270</td>
</tr>
</tbody>
</table>

Sources: Martadiputra, B.A.P. (2012)
c. Comparison Students’ Statistical Disposition Between Students who Obtained Modified MEAs Learning and Students with Obtained Conventional Learning

Table 6
Difference Students’ Statistical Disposition Test
for Control, Experiment 1, and Experiment 2 Class (Level of significance at $\alpha = 0.05$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Class</th>
<th>Normality Test (K-S)</th>
<th>Homogeneity Test</th>
<th>Difference Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-gain Statistical Disposition</td>
<td>Control</td>
<td>0.000</td>
<td>Not Normal</td>
<td>Kruskal Wallis</td>
</tr>
<tr>
<td></td>
<td>Experiment 1</td>
<td>0.039</td>
<td>Not Normal</td>
<td>Mann Whitney</td>
</tr>
<tr>
<td></td>
<td>Experiment 2</td>
<td>,200*</td>
<td>Normal</td>
<td>Mann Whitney</td>
</tr>
</tbody>
</table>

Sources: Martadiputra, B.A.P (2012)

From the results of Kruskal Wallis test in Table 6, showed that there were differences significant enhance in statistical disposition between students who receive a modified MEAs learning with students who receive conventional learning. Further follow-up of test results by using the Mann-Whitney test found that: (1) an enhance students’s disposition statistical regular and repeat students who obtain modified MEAs learning significantly higher than regular students who acquire conventional learning, (2) there was no difference in enhance students’ statistical disposition between regular and repeat students who obtain repeat learning that MEAs modified.

d. Comparison Enhancement Students’ Statistical Disposition Between High, Medium, and Low Group

Table 7
Difference Students’ Statistical Disposition Test
for Low, Medium, and High Group (Level of significance $\alpha = 0.05$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Class</th>
<th>Normality Test (K-S)</th>
<th>Homogeneity Test</th>
<th>Difference Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-gain Statistical Disposition</td>
<td>Low</td>
<td>0.081</td>
<td>Normal</td>
<td>Kruskal Wallis</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.000</td>
<td>Not Normal</td>
<td>Mann Whitney</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.148</td>
<td>Normal</td>
<td>Mann Whitney</td>
</tr>
</tbody>
</table>

Sources: Martadiputra, B.A.P. (2012)

From the results of One Way ANOVA test in Table 7 show that no differences significance in statistical disposition between the groups of high, medium, and low.

e. Association Between Statistical Disposition with Students’ Statistical Thinking Ability

From Table 8, the results of the test and Chi-Square and Contingency Coefficient was no statistical association between enhance students’ statistical disposition with students’ statistical thinking ability for control class students who acquire conventional learning students and experiment class students who obtain a modified MEAs learning.
Table 8
Association Between Enhance Statistical Disposition with Students’ Statistical Thinking Ability for Control, Experiment 1 and Experiment 2 Class

<table>
<thead>
<tr>
<th>No.</th>
<th>Association</th>
<th>Statistic Test</th>
<th>Sig</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Association between enhance statistical disposition with statistical thinking ability of control class students</td>
<td>Chi Square and Contingency Coefficient</td>
<td>0.887</td>
<td>• No significant association</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Association for only 0.61%</td>
</tr>
<tr>
<td>2.</td>
<td>Association between enhance statistical disposition with statistical thinking ability of experiment 1 class students</td>
<td>Chi Square and Contingency Coefficient</td>
<td>0.571</td>
<td>• No significant association</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Association for only 2.79%</td>
</tr>
<tr>
<td>3.</td>
<td>Association between enhance statistical disposition with statistical thinking ability of experiment 2 class students</td>
<td>Chi Square and Contingency Coefficient</td>
<td>0.820</td>
<td>• No significant association</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>• Association for only 3.61%</td>
</tr>
</tbody>
</table>

Sources: Martadiputra, B.A.P. (2012)

5. CONCLUSIONS
Based on the results of this study concluded:

a. Enhance students’ statistical thinking ability who receive modified MEAs learning significantly higher than students who receive conventional learning.

b. There was no difference enhance students’ statistical thinking ability between students who have a initial statistical capabilities high, medium, and low.

c. Enhance students’ statistical dispositions who receive modified MEAs learning significantly higher than students who receive conventional learning.

d. There was no difference enhance students’ statistical disposition between students who have a initial statistical capabilities high, medium, and low.

e. There is no statistical association between an enhance in statistical disposition with statistical thinking ability for control class students who acquire conventional learning students and experiment class students who obtain a modified MEAs learning.

REFERENCES


Development Learning Material for Student Mathematical Thinking Ability

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ABSTRACT

Learning material is an important factor that can influence the quality of student mathematical thinking ability. A good learning material should be able to facilitate students in finding and constructing the correct concepts independently. Thus development of mathematics learning material that can train and increase the students capabilities and understanding of subject while giving them chance to construct their knowledge independently is sorely needed. The aims of this research are to produce a valid and practical learning material for develop student mathematical thinking. It is presented in the form of a lesson plan, and a student worksheet. This development was done by using the 4-D models: define, design, develop and disseminate. The subject were the eleventh-year students. The result showed that the development of learning material generated is valid in term of content and construct. The learning material is practical as well since it was clear and easy to used in mathematics learning. Based on those result, it can be concluded that a valid and practical learning material for develop student mathematical thinking has indeed been successfully produced.

Keywords: Learning material, Mathematical thinking Ability.

1. INTRODUCTION

The rapid development of science and technology today, have implications for rapid flow of information that is accessible by everyone. However, not all information has a positive impact. Therefore any good thinking skills in order to obtain information that can be useful. Thinking skills that must be possessed by each of them is the ability to think critically, creatively, analytically and logically.

One way to equip each person with the ability to think can be done through education. Developed educational programs need to emphasize on the development of thinking skills students need to have. Developing the ability to think this can be done through learning, one of which is the learning of mathematics, because mathematics has a structure and a strong and clear linkages between concepts.

Development of thinking skills in mathematics is also supported by the Government as contained in the Competency Standards Curriculum 2006. Competency Standards Curriculum 2006 [1] mentions that mathematics needs to be given to all students ranging from elementary school to equip students with the ability to think logically, analytical, systematic, critical, and creative as well as the ability to cooperate. The competencies required for students to have the ability to acquire, manage, and use information to survive the ever-changing circumstances, uncertain, and competitive. Besides the curriculum also mentions that one of the goals of learning mathematics is to develop a creative activity involving imagination, intuition and discovery, to develop divergent thinking, original, curiosity, make predictions and conjecture and try.

Critical thinking skills in mathematics education such as the ability to think critically and creatively mathematically. By the time a person is given a problem, he is expected to deal critically and try to find a creative solution with the best in order to obtain completion. Thus developing critical thinking skills can be done in the same environment as develop the ability to think creatively. So to develop critical and creative thinking skills can be performed simultaneously.

Developing and implementing instructional materials that contain mathematical tasks appropriate to enable students to use critical thinking skills and actively creative is something that is very difficult for teachers and researchers of mathematics education in general. Therefore we need an example of teaching materials and learning the proper trajectory in developing critical thinking skills and creative mathematical students, so that teachers can use in the classroom and make it an example to create and modify teaching materials and the learning trajectory.

In order to develop students' thinking skills teachers have to manage the learning and use of teaching materials that can facilitate the emergence of thinking skills in students. But the reality in schools teachers still
lack of teaching materials that can menumbuhan the ability to think. Lack of appropriate sources of teaching materials is one of the factors that lead teachers have difficulty in menumbuhan students' thinking skills. Accordingly, the need for teaching materials that can foster students' ability to think mathematically.

In addition, the results study from [3] showed that high school students in the city of Bengkulu basically have the potential to think creatively, but based on the results of interviews with some of the teachers obtained the result that the teachers in obtaining suitable materials to train students to develop critical thinking skills and creative mathematical students. Therefore this study aimed to develop a valid teaching materials and practical to develop the ability to think critically and creatively mathematically.

2. RESEARCH METHOD
The research was conducted in the academic year 2012-2013. Subjects were students of class XI SMA N 7 Bengkulu city, which numbered 32 people. Model of the development of this research is the development model of 4-D (four D), which consists of 4 stages. According to Thiagarajan cited by Trianto (2012) The stages are: Define, Design, Develop, and Desseminate. Further products will be validated by experts and then will be tested on students. The next stage is carried out analysis of the validity of the teaching materials developed.

1. Development Procedure

The procedure used in this study refer to product development in the form of teaching materials for students of SMA Negeri 7 Bengkulu city. Development of teaching materials based on a step-by-step development of model 4-D (four D) as follows:

![Learning Material Development Diagram]

1. Defining Phase
Implementation of this phase include:
  a. Students analysis
Students analyzes conducted in this study is looking at the background knowledge of students regarding the content.
b. Curriculum analysis

Curriculum analysis aimed at identifying and detailing the learning material in high school. Analysis was performed on a standard of competence, basic competence as well as existing indicators relating to the subject matter.

2. Design Phase

Design phase aims to design instructional materials that will be developed lesson plans and worksheets that can foster critical thinking skills and creative mathematical students. At this stage also prepared about the tests and observation sheet which will be used to observe the activities of the students during the learning process. The results of this initial design called the prototype 1. Cultivated instructional materials focused on three main characteristics, namely content, konstrukt and language. The third characteristic is a guideline for validators to assess the teaching materials developed. Table 1 presents the description of these three things.

Table 1 Main Characteristics for Development of Learning Material

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isi</td>
<td>1. Komponen Isi pada Rencana Pelaksanaan Pembelajaran</td>
</tr>
<tr>
<td></td>
<td>c. Kesesuaian dengan indikator pencapaian KD dalam silabus.</td>
</tr>
<tr>
<td></td>
<td>d. Kesesuaian tujuan pembelajaran dengan SK, KD, dan indikator pencapaian KD.</td>
</tr>
<tr>
<td></td>
<td>e. Kesesuaian materi dengan SK, KD, dan indikator pencapaian KD.</td>
</tr>
<tr>
<td>Konstruk</td>
<td>1. Langkah-langkah pembelajaran sudah sesuai dengan pembelajaran</td>
</tr>
<tr>
<td></td>
<td>dengan pendekatan open ended.</td>
</tr>
<tr>
<td></td>
<td>2. Sesuai dengan karakteristik siswa SMA kelas X.</td>
</tr>
<tr>
<td></td>
<td>3. Sesuai dengan Indikator Kemampuan Berpikir Matematis</td>
</tr>
<tr>
<td>Bahasa</td>
<td>1. Bahasa mudah dimengerti.</td>
</tr>
<tr>
<td></td>
<td>2. Penggunaan kalimat efektif.</td>
</tr>
<tr>
<td></td>
<td>3. Bahasa tidak menimbulkan penafsiran ganda</td>
</tr>
</tbody>
</table>

The design of student activity book made of the considerable consistency between learning goals and indicators of material, kelogisan and systematic scheme of relationships between subjects, the selection of material compatibility with the characteristics of the students, and the fit between the material it wants to develop thinking skills.

Draft RPP made-oriented learning approach that enables students to construct their own knowledge. Selected learning approach is open-ended approach.

Student Worksheet draft is done in accordance with the number of contact hours. The problems presented are problems in everyday life and often encountered students. LKS are developed in accordance with the indicators ability to think critically and creatively mathematically.

3. Development phase

Prototyping process is performed to evaluate whether the teaching materials developed are in accordance with the criteria. This prototyping activities carried out by three approaches: expert review and one-to-one, the evaluation group (small group) and test (field test). After checking and comparison, as a basis for revising the teaching materials developed. Test the validity of teaching materials on each prototype is seen is the content and accuracy of learning materials, conformity with the purpose of learning, the language used, and the indicator kesesuaian critical and creative thinking skills mathematically.

The following table sets forth the components are validated.

1. Expert Review and One-to-one
a. Expert Review
At this stage the first prototype validated by experts and teachers who teach in SMAN 7 Bengkulu City. This phase aims to obtain a valid teaching materials design. Instructional materials that have been made seen, assessed and evaluated. Validation test used is the content validation, validation and test validation language constructs. Comments and suggestions on the draft of the validator materials made poured on validation sheet and referenced in revising and teaching materials have been declared invalid.

b. One-to-one
Phase one-to-one carried out in line with the expert review stage. Activities undertaken at this stage is testing this prototype 1 in 5 grade students of SMAN 7 Bengkulu city. Students are required to observe, read, comment on and doing worksheets and test questions diberian gradually. Test one-to-one was carried out to determine the clarity and readability of materials and see a match between a set time to implementation in the field. This activity is done in a way so that the direct interaction difficulties that may occur can be seen clearly.

c. Small Group (Small Group)
Revised and comments from experts and one-to-one on the first prototype to be a reference to design prototype 2. The results of this second prototype evaluated through testing on small groups. Kelompom small group is a different group with research subjects. Tests carried out on small group learning activities to see keterlaksanaan-based open-ended approach. At the stage of a small group of 15 people consisting of non students study subjects. Based on observations and student feedback is revised and improved teaching material again. Results of prototype 3 is expected to produce a valid teaching materials and practical.

3. Deployment Phase (Field Test)
At this stage of testing done real research on the subject as a field test. The products have been tested on the field test must have met the quality criteria. Quality criteria of a good teaching material according to Akker is in compliance with the validity of the expert review, the use of these materials is easy and can be used (practically) and effectively used to see the students' ability to think mathematically.

3. RESULT AND DISCUSSION

RESULTS
Development of teaching materials made through the analysis, design and evaluation.

a. Analysis Students
Analysis shows students that the material is not new material opportunities for students, because the students have acquired the basic material in class IX SMP. In addition to the students of class XI SMA Negeri 7 Bengkulu city average age was between 16-17 years old and is at a transitional stage between concrete operational thinking to formal operational.

b. Analysis Curriculum
At this stage, the material analysis opportunities that will be developed based on the open-ended approach. Competency standards for subject matter for class XI SMA opportunity is to solve the problem with the concept of opportunity theory, the basic competence enumeration describing rules, permutations and combinations.

c. Teaching Material Design
The design of devices based approach to learning math that made open-ended aims to develop students' mathematical thinking skills especially the ability to think critically and creatively mathematically. Developed learning tools include: lesson plan, teaching materials and assessment instruments. The results of the preliminary design of a prototype 1.

d. Evaluation
At this stage the results of the preliminary design that was created evaluated by expert review, one-to-one and small group and trial. Evaluation by expert review, one-to-one and small group is the stage to see the validity and practicality of the teaching materials developed, while the field trials are testing to see the potential effect on students' ability to think mathematically. Based on the advice of the expert review and the results of one-to-one, then the results of the revised prototype 1 prototype 2 produces a better quality. Table 2 presents the changes before and after revision for prototype 1.
### Table 2. Before and After Revision Changes to Prototype 1

<table>
<thead>
<tr>
<th>Saran Sebelum Revisi</th>
<th>Saran Sesudah Revisi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Materi ajar harus disesuaikan dengan pendekatan/model pembelajaran yang digunakan (pendekatan open-ended).</td>
<td>1. Penyusunan materi sudah sesuai dengan pendekatan open-ended yaitu dimulai dengan memberikan masalah terbuka</td>
</tr>
<tr>
<td>2. Bahasa yang digunakan harus jelas, tidak menimbulkan makna ganda</td>
<td>2. Bahasa yang digunakan masih bermakna ganda</td>
</tr>
<tr>
<td>4. Materi ataupun soal harus dibuat sesuai dengan konteks dan berbeda dengan materi yang sudah ada di buku paket</td>
<td>4. Masih banyak materi atau soal yang sama dengan buku paket</td>
</tr>
<tr>
<td>5. Ilustrasi atau gambar yang digunakan harus sesuai dengan maksud dan tujuan materi atau soal</td>
<td>5. Ada beberapa gambar yang tidak relevan dengan materi atau soal</td>
</tr>
</tbody>
</table>

### Table 3. Changes before and after the revision on prototype 2

<table>
<thead>
<tr>
<th>Saran</th>
<th>Sebelum Revisi</th>
<th>Sesudah Revisi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Soalnya diganti dengan permasalahan yang dekat dengan siswa</td>
<td>1. Permasalahan yang ada belum tentu diketahui oleh siswa atau belum dikenal dalam keseharian siswa</td>
<td>1. Diganti dengan permasalahan yang dekat dengan keseharian siswa</td>
</tr>
<tr>
<td>2. Indikator kemampuan berpikir kritis dan kreatif belum terlihat jelas pada soal tes</td>
<td>2. Soal tes belum mencerminkan indikator kemampuan berpikir kritis dan kreatif matematis</td>
<td>2. Pada soal tes sudah dimunculkan indikator kemampuan berpikir kritis dan kreatif matematis</td>
</tr>
</tbody>
</table>

**e. Prototype 2**

Based on the evaluation results obtained on prototype 1 prototype 2. Prototype 2 is evaluated through a trial limited to a small group. Based on the results of tests on a small group of students as well as the opinion of the revised prototype 2. Revision 2 prototype aims to correct existing deficiencies. Prototype 3 is an instructional materials that have met the criteria for a valid and practical quality. Changes before and after the revision can be seen in Table 3.

**f. Field Test (Field Test)**

Prototype 3 is already valid and practical, subsequently conducted a field trial to see the potential effects on the ability to think critically and creatively mathematical students. This stage is the effectiveness of the prototype testing stage 3. Teaching materials based opportunities open ended approach, observation sheets...
, and test instruments the ability to think critically and creatively mathematical subjects tested on the students of class XI IPA 2 SMA Negeri 7 Bengkulu city, amounting to 32 students. The third prototype testing done as much as 6 meeting. At trial implementation in the classroom, observed by 2 observers in charge of observing the activity of the students during the learning process. In addition students are given tests of thinking skills critically and creatively mathematically after learning. Giving the test was conducted in order to see the ability to think critically and creatively mathematical learning of students after being given the use of teaching materials based on open-ended approach.

g. Description and Analysis of Data

Instructional materials that have been given to students in learning contains open-ended problems that must be solved students critically and creatively. In solving the problems students are divided into several groups. Results of analysis for solving problems in the practice of teaching materials are presented in Table 4.

Table 4. Analysis of the results of Exercise Settlement Issues in Instructional Materials

<table>
<thead>
<tr>
<th>No</th>
<th>Kelompok</th>
<th>Nilai LKS 1</th>
<th>Nilai LKS 2</th>
<th>Nilai LKS 3</th>
<th>Nilai LKS 4</th>
<th>Nilai LKS 5</th>
<th>Nilai LKS 6</th>
<th>Rerata</th>
<th>Kriteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>80</td>
<td>90</td>
<td>80</td>
<td>85</td>
<td>75</td>
<td>82,5</td>
<td>Sangat Baik</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>85</td>
<td>80</td>
<td>75</td>
<td>85</td>
<td>90</td>
<td>81,6</td>
<td>Sangat Baik</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>III</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>75</td>
<td>80</td>
<td>76,6</td>
<td>Baik</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>IV</td>
<td>80</td>
<td>80</td>
<td>75</td>
<td>70</td>
<td>80</td>
<td>76,6</td>
<td>Baik</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>V</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>80</td>
<td>75</td>
<td>76,6</td>
<td>Baik</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>VI</td>
<td>75</td>
<td>75</td>
<td>80</td>
<td>80</td>
<td>75</td>
<td>77,5</td>
<td>Baik</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>VII</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80,8</td>
<td>Sangat Baik</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>VIII</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>75</td>
<td>79,2</td>
<td>Baik</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 shows that the three groups have achieved excellent criteria and five other groups received either criterion. This proves that the teaching materials developed have reached the criteria of practicality. Based on the average obtained it can be concluded that the developed prototype 3 has achieved both categories.

h. Description of Test Results Critical and Creative Thinking Mathematically

After the lesson is completed, students were given a test of critical thinking skills and creative mathematical. The results of tests of thinking skills critically and creatively mathematically can be seen in Table 5 and 6.

Table 5. Critical Thinking Mathematical Students Skills Test

<table>
<thead>
<tr>
<th>Indikator</th>
<th>Rerata Nilai</th>
<th>Kategori</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mengevaluasi</td>
<td>3,2</td>
<td>Sangat Baik</td>
</tr>
<tr>
<td>Mengidentifikasi</td>
<td>3,5</td>
<td>Sangat Baik</td>
</tr>
<tr>
<td>Menghubungkan</td>
<td>3,2</td>
<td>Sangat Baik</td>
</tr>
<tr>
<td>Menganalisis</td>
<td>2,8</td>
<td>Baik</td>
</tr>
<tr>
<td>Memecahkan Masalah</td>
<td>2,9</td>
<td>Baik</td>
</tr>
</tbody>
</table>

Tabel 6 Creative Thinking Mathematical Students Skills Test

<table>
<thead>
<tr>
<th>Indikator</th>
<th>Rerata Nilai</th>
<th>Kategori</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kebaruan</td>
<td>2,7</td>
<td>Baik</td>
</tr>
<tr>
<td>Kefasihan</td>
<td>3,5</td>
<td>Sangat Baik</td>
</tr>
<tr>
<td>Kelancaran</td>
<td>3,2</td>
<td>Sangat Baik</td>
</tr>
<tr>
<td>Kepekaan</td>
<td>3,3</td>
<td>Baik</td>
</tr>
<tr>
<td>Keterperincian</td>
<td>2,9</td>
<td>Baik</td>
</tr>
</tbody>
</table>

The third prototypetest results show that the learning materials can develop the ability to think critically and creatively with students' mathematical average values are in the category of good and very good.
DISCUSSION

After going through the process of development ranging from process validation to obtain the revision -based learning approaches are categorized open-ended valid and practical. At the beginning of the learning process, the researcher explained the math learning based on open-ended approach that has the characteristics of a dialogue or interaction with other students in the group to understand the teaching materials, which were prepared using four comprehension strategies, namely to conclude or summarize, formulate questions, explain and make predictions. This allows students to undertake appropriate steps in understanding and teaching materials to help students construct meaning from a text so that students can increase their understanding of the material.

At the beginning of learning by using teaching materials based on open-ended approach is not expected to show how learning in accordance with the steps or mechanisms in the open-ended approach, although earlier the teacher has explained and an example to the students of the learning activity. This happens because the students have never received this model of learning so that they are not familiar. After they were led learning how to do this activity using teaching materials that the teacher, they can gradually understand.

The interaction between the group began to look, but most groups working on assigned tasks and discuss together and only partially working group with the task of dividing the tasks to each member of the group in accordance with the strategy of open-ended approach.

After the group discussion is completed, the teacher appointed representatives from each group to lead the dialogue or discussion groups presented their results to other groups in front of the class and other groups can provide feedback or corrections. Not all sets can be performed due to time constraints. At the next meeting, students in general have shown how learning-based approach to open-ended to be expected, although there are groups who still need an explanation from the teacher to debrief the activity of learning. At the moment summarize or conclude, students identify the important things and the main idea of the student worksheets.

When students formulate questions, they first identify the type of information that is clear enough to establish the main content for a question and ask themselves to ensure that they can actually answer their own questions. Meanwhile, when students were asked to explain, they took steps to explain or clarify parts of the text parts are difficult to understand, with a re-read or ask for help. Predicting occurs when students anticipate what they might read next based on cues in the text and the ideas that have been completed are presented. Based learning approach is applied to an open-ended using teaching materials that have been developed to create process information on student activities, which means students will examine, process and create information presented in the teaching and learning materials by using comprehension strategies in four open-ended approach.

Students read teaching materials and are conditioned to observe before making conclusions, questions, explain and predict the situation through understanding the structure linking the learning they already have with the new information presented in teaching materials.

Thus, students will gain significant mathematics learning. For example, one of the concepts in the teaching material presented in class is when students are given the concept of permutations, the students have learned about counting rules and definitions and factorial notation in the previous discussion. To further stimulate the curiosity of students, they are given a situation and then the students were questioned about the number of ways to arrange r elements taken from the n elements in a way that previously tried they have learned and then use the formula. Furthermore students were instructed to create a new question of the problems that have been given previously and then explain it.

4. CONCLUSIONS

Results of this study indicate that teaching materials have been developed to meet the quality as valid teaching materials, practical and effective. Instructional materials are developed instructional materials for material-based learning opportunities with the open-ended approach. Teaching materials obtained can serve as an example for the development of teaching materials in different materials or learning through the different models.

REFERENCES

MATHEMATIC’S TEACHER WITH GOOD CHARACTER: A KEY FOR SUCCESSFUL IMPLEMENTATION OF CHARACTER EDUCATION THROUGH MATHEMATICS INSTRUCTION

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ABSTRACT

When cultural derailments are getting worst and weakening characters has put Indonesia in the lowest point position of morality, it is very easy to point our finger to education institutions as the most responsible institutions to develop and nurture good characters of the nation future generations. As the response to this condition, the Ministry of Education and Culture determines that character education must be implemented in all education levels and through all subjects in curriculum, including mathematics. As the frontline in an education system, teachers play significant role in implementing character education. This paper highlights the profile of teachers who possess the capacity to develop good characters through mathematics instruction. Through a research and development process, this study is not only able to produce teaching materials for the Introductory Courses of Basic Mathematics that have been validated, but also discover that teacher’s profiles who will be able to develop good characters are those who teach using contextual teaching and learning approach and always take every opportunity available to develop the characters. The teachers also act as the role model for the character they need to develop.

Keywords: Character education, Teachers, Mathematics learning

1. INTRODUCTION

Indonesia is now in a massive derailed condition due to the condition of poverty, ignorance, and unemployment which is getting increased. Moreover, those miserable conditions are getting worse when intolerable juvenile delinquency, drugs, crimes, terrorism, anarchy demonstration, corruption, and others are happened everywhere in this country. In other words, the morality of the nation is in the lowest point.

This miserable condition has encouraged government to take an initiative to prioritize moral development efforts in the national development plan. As the consequence, every efforts intended to develop this nation must be directed to provide positive impacts of character development. The Ministry of Education and Culture (Kementrian Pendidikan Nasional), as one of the institutions to take responsibility to develop nation character, develop a planning for implementing character education in all education levels in Indonesia. Character education must have focus on developing foundation of intellectual curiosity through forming habits and conducting intervention that are expected to produce and nurture good characters in school culture.

“Character is traits, attitude, behavior, or personality of people that is formed through internalization process of all virtues believed and used as the basis of views, attitudes, thoughts, and behaviors” (Kementrian Pendidikan Nasional, 2010: 4). Meanwhile, Berkowitz and Bier (2005:8) say, “Character education is a national movement creating schools that foster ethical, responsible, and caring young people by modeling and teaching good character through emphasis on universal values that we all share. It is the intentional, proactive effort by schools, districts, and states to instill in their students important core, ethical values such as caring, honesty, fairness, responsibility, and respect for self and others.”

In relation to the development of character education and nation culture, Kementrian Pendidikan Nasional (2010) express, that the process of value development that become the basis for character needs to be implemented sustainably, and implemented through many subjects in curriculum (Civics education, History, Geography, Economics, Sociology, Anthropology, Indonesian Language, Social Studies, Science, Mathematics, Religion Studies, Physics, and Arts). It is clear that developing great characters is not only a responsibility of
Civics education and Religion studies teachers, but also a responsibility of all teachers in all subjects including mathematics. Based on religion, Pancasila as basic state, culture, and national education goals; Kemendiknas identify 18 values developed in character education. They are: religious, honest, tolerant, discipline, persistence, creative, independent, democratic, curiosity, the spirit of nationality, love of country, appreciate the achievement, friendly, love peace, love to read, care for the environment, care for the social, and responsibility.

Soemarmo (2012) prefer to review the quality between the values developed in character education with mathematical disposition and habit of mind characteristics. In other words,sheds the relationship between values developed in character education can be integrated in mathematics instruction. In a different way Suyitno (2012) express that values contained in mathematics materials such as consistency, discipline, honesty, and creativity have potential to develop other values such as tolerance, democratic, independency, unity, responsibility, cares, empathy, sympathetic, faiths, and so forth. Different with Sumarmo and Suyitno, Sutama (2011) raise the following 8 values can be developed through mathematics instruction: responsibility, honesty, discipline, modesty, hardworking, independency, fairness, courage, and cares. Based on teaching experience, researches sure, especially in introductory courses basic mathematics, values can be developed through mathematics instruction are: honest, discipline, and persistence.

As the frontliner of an education system, teachers play a very significant role. In a model of character education classroom, Lickona (1997) described the role of teachers as a caregiver, model, and mentor. Surely, before playing the role as a model of the character to develop, a teacher should possesses and nurtures the character him/herself. It does not mean he/she should get a character advance learning about character. As an adult, teachers are supposed to have those great characters that will be developed and taught to their students.

Those conditions encourage the researcher to review the profile of teaching process that can develop and nurture students’ great characters. How is the profile of a lecturer/teacher who can develop and nurture great characters of students? This question is one of the four problems formulated in the research about the Development of Teaching Materials of Contextual-based Introductory Courses of Basic Mathematics for Developing Students’ Good Characters. Consequently, the answer and its analysis will not be separated from the overall study.

2. METHOD

This study is a development of Introduction to Basic Mathematics course with focuses on Elementary Logical Mathematics. The main objective of this study is to design a contextual-based Basic Mathematics course which can improve students’ good characters such as honest, discipline, and persistence. This course development follows the series of developmental research conducted through thought-experiment and instruction-experiment cycle (Herman, 2002: 2). The steps taken to conduct this developmental research were adapted from Borg and Gall R&D Model (Ghufron, 2011: 12) which consisted of the following ten steps: 1) preliminary research includes literature review and initial survey; 2) planning; 3) initial product development; 4) preliminary field-test; 5) revision on limited field-test; 6) main field-test; 7) revision on main field-test; 8) operational field-test; 9) revision on operational field-test; and 10) dissemination and socialization of final products.

In general, this study was conducted in three phases in the duration of two years. The three phases were as follows: preparation and implementation in the first year, and refinement phase in the second year. Preparation phase included preliminary design, validation, and limited field-test. The implementation phase was the main field-test; meanwhile, refinement phases included validation by experts and revision which were followed by scientific publication. The design of this study is presented in form of flowchart presented in Picture 1, 2, 3 which are consecutively representing the Phase 1, Phase 2, and Phase 3 of this research and development.

![Flowchart](Picture 1)

**Picture 1** The First Phase of the Research and Development

The first phase of this research and development was initiated by studying curriculum in order to define the objectives, determine in which competency the materials need to be developed, and identify main materials to teach. Then, a literature review was conducted in order to collect and select relevant materials, and reorganized them systematically to finally develop a material design namely the Design of Introduction to Basic
Mathematics (Disain Pengantar Dasar Matematika/DPDM) and research instruments. DPDM and the instruments were then validated, implemented in a limited field-test, analyzed and revised, and finally resulted in DPDPM Revision 1.

![Diagram showing the process from DPDM Revision 1 to Field Test 1 to Analysis of Field Test Results to DPDM Revision 2]

**Picture 2 The Second Phase of the Research and Development**

As it was in Phase 1, Phase 2 is research and development stage conducted in the first year. DPPM Revision 1 resulted in the Phase 1 was tried-out in real classroom situation. It included students of the Department of Mathematics Education Faculty of Teacher Training and Education Nusantara Islamic University (FKIP Uninus) semester 1 Academic Year 2012/2013. The results of this process were: classroom observation recording, observation sheets of lecturer and students activities, formative and summative scores; and these results were used to refine the teaching materials design which resulted in DPDM Revision 2.

![Diagram showing the process from DPDM Revision 2 to Workshop (DPDM Validation and Instrument) to Scientific Publications and Analysis to DPDM Revision 3 to Field Test 2]

**Picture 3 The Third Phase: Research and Development**

Phase 3 in this research and development process was conducted in the second year. First activity conducted in this phase was workshop intended to discuss DPDM Revision 2 which was concluded in validation from experts and users. Field test for DPDM Revision 3 was conducted on students of the Department of Mathematics Education FKIP Uninus semester 1 Academic Year 2013/2014. The result of this field test was being consulted to experts, and editorial team worked together to finally design learning materials for Introduction to Basic Mathematics with Character. The process was then finally wrapped up by scientific publication and ISBN application.

### 3. RESULT AND DISCUSSION

The steps taken thorough this research and development study has resulted in the Design of Introductory Course of Basic Mathematics (DPDM) Revision 2 which has been field-tested. The followings is description of how character education processes were implemented in teaching materials: Study on Statement and Operation in Statement, also Argument and Deduction Method were implanted by character of honest and discipline. Meanwhile, Argument Validity Proofs were implanted by character of persistence. In order to provide setting and context for discussions, steps taken by the researcher in the first year will be described in the next parts.

Literature study was begun by collecting books about Introductory Course of Basic Mathematics. In addition, this study also collected references downloaded from internet sources. In the period of one month, there were no less than ten sources and fifteen references could be successfully collected by the researcher. In accordance with the needs of this study, the collection and analysis process of the references were continued simultaneously in the planning phase of the Design of Introductory Courses of Basic Mathematics (DPDM).

Three months period allotted to design DPDM is fairly insufficient. However, it does not mean that the researcher failed to design DPDM in that time. Rather, the design resulted from this period of time did not contain characters that were elegantly and fairly implanted to the course design. The integration of characters into course materials is not easy, especially when it has to be proven by providing the terms of character explicitly as the nurturant effect of mathematics concepts.
In order to reach the objectives of this study, the designing process of DPDM was aligned to the development of instruments. Consultation with experts and practitioners was really helpful for the researcher in designing the DPDM that was in accordance with syllabus and contain of characters needed to develop. Based on the consultation results, the researcher then conducted revision on DPDM which was later followed up by limited field-test to ten students who have taken the introductory courses. Inputs from students and observers were then analyzed in order to revise the DPDM.

Before conducting field-test to students in the first semester, revised DPDM was validated by two experts: a lecturer of the Indonesia University of Education who holds a Doctorate degree and a Professor of Civics education in FKIP Uinus. The validated teaching materials were then enacted as DPDM Revision 1, and it was ready to implement in the Introductory Courses of Basic Mathematics in the first semester of the Academic Year 2012/2013.

Although the DPDM had gone through many revisions, it was still need many improvements. Scenarios intended to guide students to develop their own knowledge, as one of the features of contextual teaching and learning, could not be able to facilitate in the DPDM. Therefore, in the teaching and learning processes, the researcher decided to equip the DPDM with Students Activity Worksheets (Lembar Kerja Mahasiswa= LKM) which were designed to facilitate contextual teaching and learning process. In addition to fulfill constructivism ideals, the development of LKM was also intended to get learning processes take place cooperatively. In order to develop the characters of honest, discipline, and persistence, LKM was discussed/learned cooperatively in which all students involved in this cooperative work must be able to take individual responsibility for the results.

In every learning process, there is one faculty member who acts as a lecturer and another one acts as an observer, with some times videotaped the unfolding events in the classroom. Data was also collected from several other sources including: field notes, student journals, and questionnaire completed by students. The observation results were then discussed to get reflection on what was happening and provide basis for the next sessions. Considering that the revised DPDM would be reviewed at the end of the semester, the reflection results would be implemented for revising the LKM only. This is due to the fact that LKM was provided in every session.

By implementing this learning system, at the end of the semester, the researchers were able to design teaching materials that were in accordance with the DPDM Revision 2. While continuing the Phase 3 of the research, the researcher published five articles as follows: first article highlights the whole process of the research which conducted by the three researchers; articles two to five provide highlight research questions one by one for every researcher.

In addition to resulting in teaching materials, the research team has also successfully arrived to a conclusion that teaching processes which can develop the character of honest, discipline, and persistence are those which involved contextual teaching learning approach and took every opportunities available to develop those characters, and are conducted by lecturers who posses good characters. In other words, lecturers (teachers) of mathematics who takes opportunities available to develop those characters and acts as the role model of them are the keys for implementing character education through mathematics learning.

Furthermore, researcher will propose an explanation of the conclusions reached: about contextual teaching and learning approach, and also teacher with good character. The seventh characteristics of contextual teaching and learning approach can be reliably as implementation tool of character education. Constructivism and inquiry guiding students to become independent and perisstance; questioning foster curiosity; learning community realize democratic values, respect, mutual cooperation, responsibility, and orientation on excellence; modeling indicates no bias model emulated, especially teachers; reflection bring forth honesty and awareness to constantly introspect ourselves each time have to do something; authentic assessment familiarize students to be honest, discipline, and be able to measure themselves.

Undoubtedly, character education can only be implemented by teachers who possesses good character. Requires students work hard will be effective the teacher becomerole model of persistence character for their students. Character of discipline can only be developed by disciplined teachers who act as the role model. Conversely, Schools cannot model punctuality when student tardiness is not sanctioned, and moreover when teachers arrive late. Also, a teacher who intends to develop the character of honest could never be able to reach it if the teacher does not possess the character and show it to his students. Male teacher who have affair with his female student cannot teach law-abiding to his students, since he is breaking the law and the school allows it. According to Prabowo and Sidi (2010: 171), “teachers who do not possess the character they need to develop could never success to develop students’ character since they will not be able to develop good habits which caused by the lacks of role model for students.”

Characters are caught, not taught. All education stakeholder want the teacher have good character, because he or she inescapably influence the moral development of the student in their charge. In other words; the character of the teacher believed to have a direct effect on the moral development of students, and as the consequent all parents want teachers of good character in the classroom, and he or she act as moral exemplars and model.
In addition to become a role model, teachers must also be able to take any opportunities available to develop characters. Praises and reprimand are effective tools for nurturing characters that need to develop. Moreover, rewards and punishments are also still needed in the learning process. Rewards to high achievers can become a very effective motivator and booster. Meanwhile, sanction given to misbehave students can become an effective tool for preventing the behaviors happen again. Meanwhile, teachers should not only able to take every opportunity available, but also creating those opportunities in order to implement the character education through mathematics learning. Sabandar (2011: 7) says, “Since the teachers must intentionally create a didactical situation to trigger right momentum to initiate character building and values in mathematics classroom, they must make a design and plan an appropriate lesson. After deciding what mathematics topic/problems will be presented in class, the teachers might list the kind of characters (perseverance, appreciation, comply with the rule, appreciation, respect, hard work) they want the learners to learn about and also contextual problems (mathematics topic) relevant to the character and value.”

4. CONCLUSIONS

The integration of character education in mathematics instruction is not something new in education. Mathematics teachers who care about the future of their students will equip them with good characters and act as the role model for the characters. Opportunities to implement character education in mathematics instruction can be created through modeling and implementing appropriate approach to teaching. In addition, those opportunities can also be created through designing teaching materials which are in accordance to didactic principles of learning process. It is in the hand of teachers those opportunities can be created in order to develop great characters of our students.

REFERENCES


BABY WATER MELON FOR CREATING THE FORMULA OF CYLINDER VOLUME IN JUNIOR SECONDARY CLASSROOM: AN EXPERIENCE IN LESSON STUDY

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ABSTRACT
Mathematics lesson using manipulative instruments was still an important endeavor for junior secondary students, as the students are still in the transition between the concrete and abstract thinking level. This article is written based on a reflection of authors’ experience in teaching mathematics using an unusual manipulative materials. Mathematics teaching using baby watermelon was inspired by a teaching the area of circle using ‘triangle-like’ approach (Turmudi, 2006), and the sphere volume formula using water melon (Turmudi et.al, inpress).

A lesson study program of mathematics was carried out in junior secondary school in Garut, West Java. Prior to this program, a number of activities were taken place, so that teachers have a new sight in mathematics teaching innovation. One of the activities was preparing mathematics lesson using baby water melon as media. Students were challenged to create a “cuboid-model”, observing part by part, constructing the “wedges” to make a “cuboid-model”, investigating the length, the width, and the height of “cuboid”. Later on, the students discussed to find its volume. The students were able to re-invent the volume of a cylinder as $\pi r^2 h$. This was our findings of the small research that students could formulate it by using baby watermelon.

Keywords : lesson study, water melon, baby water melon, volume of cylinder

A. Background
Mathematics is a subject that usually most students do not like and avoid it. However, the effective teacher has tasks to enable students to have well mathematical understanding and teacher was responsible to make mathematics more enjoyable and fun. The low achievement of Indonesian students nationally or internationally was still main issues in education as expressed by PISA-survey 2006 (Balitbang, 2011a). The rank of Indonesian students was 61th out of 65 participating countries with a mean score of 371, under the international average score of 500. Similarly, the TIMSS survey results indicated that Indonesia students achievement was also at the low level (Balitbang, 2011b). This condition motivated the teachers to always make an effort to improve their quality of teaching by providing lesson to the students more simple, challenging, attractive, and easier,

Simple does not mean that the lesson just delivered to the students superficially, instead the lesson were prepared for the students in order they were able to construct their mathematical knowledge based on their previous experiences and knowledge, so that they could achieve their factual, conceptual, procedural, as well as metacognitive knowledge of mathematics.

Some situations in mathematics teaching in Indonesian classroom consistent with studies of Silver (1999), Romberg & Kaput (1999), Senk & Kaput (2003), and Ernest (2004). Romberg & Kaput (1999) stated that mathematics classes mostly consisted of three segments: an initial segment where the previous day’s work is corrected. Next, the teacher presents new material, often working one or two new problems followed by a few students working similar problems at the chalkboard. The final segment involves students working on an assignment for the following day (p.16).
Moreover the previous Minister of Education and Culture in the Republic of Indonesia, Wardiman Djojonegoro stated at the opening ceremony of the International Seminar in Mathematics and Science (Djojonegoro, 1995):

Most schools and teachers treat students as a ‘vessel’ something to be filled with knowledge. Another well known example is the tendency towards right-answer/fact-based learning. School and teachers focus on getting the right answer from the students at the cost of developing the processes that generate the answer. As a result, students resort frequently to superficial accomplishments. Rote learning falls into this category. (p. 36)

Willingness to reform mathematics teaching was favoured by Djojonegoro, (1995) who argued as follows: I would like to challenge you to create greater understanding on how students learn as prerequisite for improving our teaching methods in mathematics and science, and improving the education of teachers for these subjects (p.36).

The above quotation suggests that, according to the former Minister, students have rarely been given the opportunity to experience the intellectual excitement of generative mathematics inquiry. A small survey for mathematics teachers in Junior secondary School in Bandung area (Turmudi & Eri Erlina, 2012) indicated that volume formula of 3-dimensional space is given to the students as a ready well-prepared formula. Very seldom the students are given opportunity to construct their own potential knowledge and to re-invent their understanding of formula for the 3-dimensional shapes.

Mathematics teaching innovation tends to deal with three things: how to perceive mathematics, how to teach mathematics and how to assess mathematical understanding. There has been persistent criticism of previous views of mathematics in which mathematics was perceived as a fixed and static body of knowledge (Romberg & Kaput, 1999), as formal systems, rules, and procedures (Clarke, Clarke, & Sullivan, 1996), as a set of rules and correct procedures (Ernest, 2004), or as a large collections of concepts and skills to be mastered (Verschaffel & De Corte, 1996). Advocated instead is a view of mathematics as a dynamic subject, as a human activity (Freudenthal, 1991; Romberg & Kaput, 1999), as a human-sense and problem solving activity (Verschaffel & De Corte,1996), or as humanized and anti-absolutist (Cockcroft, 1982; NCTM, 1980, 1989).

These innovative views, in one hand, the teacher’s task in this notion was to articulate mathematical structure and content in a certain time frame, with an expectation that the students could articulate what their thinking even sometimes they do not understand the meaning behind the mathematical expressions. On the other hand, in the constructivist perspective, mathematics should be presented to the learner using inductive strategies in which the learners have to construct their mathematical understanding of concepts, properties, procedures, with high order thinking skills so that they have an awareness that they know or they do not know or it be called as metacognitive knowledge.

Regarding how to build mathematics lesson according to constructivist perpective, the authors of this article, as part of learning community of mathematics teachers in an In-House-Training program sponsored Directorate of Junior High School, Ministry of Education and Culture Republic of Indonesia, attempted to design mathematics teaching materials which involving students in the processes of enactive, iconic, dan symbolic (Bruner, 1961). The students can imagine how was a cylinder shape look like. Students were asked to change (transform) cylinder to be a rectangular block (cuboid) with the same volume. Modifying to other shape without change it volume. The volume of cylinder is still constant using conserve principle, so that the students feel easy to understand. Transformation and modification processes of how a cylinder changed to be a 3-D shape of a “rectangular block –like” (or cuboid-like) was not easy, because we have to manipulate a solid geometrical shape.

Initially we as a team tought that using a log-wood (Figure 1) would be easy to use and students were also easy to manipulate, but infact was not.

**Figure 1: Log-woods (from the tree)**

**Figure 2: Baby watermelon**
Practically, a carpenter has a difficulty to saw the log-wood, because he does not have any special saw to slice a log-wood without lose it volume. The second author of this article has come for asking the carpenter to make a cylinder model with it slice, but the carpenter fails to do. He fails to make an expected model of cylinder with slice. Therefore we were worry that the conservation law could not be applied in this situation.

An idea to use baby water melon (see Figure 2) was found unintentionally after going around by using motor cycle. In the fruit store of super mall, we found baby water melons, then we thought that if we cut off the both ends, we would have solid cylinder (Figure 12a).

This cylinder model would be easier to be sliced by the students in order to be “wedges”. Baby water melon becomes a model to explain how is a cylinder modified to be a “cuboid-model”, therefore its volume could be approached by using “cuboid-model” volume.

B. Theoretical Framework

Excellent teachers of mathematics are purposeful in making a positive difference to the learning outcomes, both cognitive and affective, of the students they teach. They are sensitive and responsive to all aspects of the context in which they teach. This is reflected in the learning environments they establish, the lessons they plan,..., their teaching practices, and the ways in which they assess and report on student learning (AAMT, cited in Goos, Stillman, & Vale, 2007).

The teacher who can present his/her mathematics lesson differently will give opportunity and chance for learners to think creatively. One of ideas to help teacher was by using LS as kind of professional development. Lesson Study is a kind professional development model, where the teacher sit and discuss together to think the way how to teach mathematics for students. They mutually learn each other, as the standard tenet of LS process which is lead by an expert in mathematics instruction and a number of head of schools, or a number of school neighbours and supervisor teachers. According to Fernandez & Yoshida (2004), “Lesson study is a direct translation for the Japanese term jugyokenkyu, which is composed of two words: jugyo, which means lesson, and kenkyu, which means study or research” (p.7). As denoted by this term, lesson study consists of the study or examination of teaching practice. In the Confucian tradition, lesson study is said to be premised as “Seeing something once is better than hearing about it one hundred times” (Yoshida, 2005). Its ultimate purposes to gain new ideas about the process of teaching and learning based on a better conceptual understanding of children’s thinking so the observation of actual research lessons is at the core component of the LS process. Yet, the LS cycle encompasses much more than studying children’s responses while observing a lesson. It requires time to intensively and collaboratively investigate all aspects of the content to be taught and instructional materials available, and reviewing the reflection in the post-lesson review session (Takahashi, Watanabe, Yoshida & Wang-Iverson, 2005).

In relation to the collaboration among elements in the LS, Supriatna, et.al., (2010) explained that LS in Indonesia can be meant as a professional development model to enhance the quality of teaching through study of their lesson collaboratively and continuously based on the collegiality principles and mutual learning to building learning community (p. 14).

Based on Regulation for the Teacher & Lecturer No 14, Year 2005, for professional development (PD) of the teacher, that LS is to be one of the important kind of PD program for the teacher. The steps in the LS follow the formula was the step of plan-do-see. Plan means that the teacher follows how to make plan of the lesson the teacher need to identify current problem in mathematics instruction and what should be considered to be changed whether the strategies and media for teaching, or students worksheet. The following diagram shows the procedure in doing the LS.

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![Plan-Do-See Diagram](image-url)

Figure 3: Plan-Do-See Diagram
Do, means that the teacher (model teacher) have to implement the plan that has been made by the teacher collaboratively. Other member of LS should role play as observer (purely observer) of the lesson which focused to observe on the students learning (student interaction among them, students interaction with the mathematics materials, and students interaction with the teacher). The results of their observation should be presented or shared in the see session (reflection session, a reflection toward teaching process).

In the plan step, the teachers in a group discussed and shared the problem of instruction that usually happened in the classroom. The teaching patterns, according to their experiences that the teachers just informed the concepts of mathematics for students, and the teachers asked students to follow what has been done. The position of the students in this habitual action of the teachers as receptors, rarely the students raised their hands and asked for certain concepts as well as certain procedures to solve mathematical problems. Ironically, Silver (1989) argued that “daily activity for most students in mathematics classes consists of watching a teacher work problems at the board and then working alone on traditional problems provided by the textbooks or by a worksheet” (p.280). In the plan step, the facilitator asked teachers as member of LS to re-evaluate their daily teaching, whether their teaching approaches influence students learning. Instead, the teachers were asked to judge the teaching which involve students to actively construct their understanding of mathematics, by giving them a stimulus as situation in which students have opportunity in making a conjecture, then collecting data to support their conjecture, investigating, and exploring the properties of mathematics in the contextual problems.

In order the teachers have a various capabilities in the teaching strategies, then the teachers involve in discussion to plan for open lesson in LS.

An experience in applying contextual learning-in an LS- most teachers felt confidence and they propose to be model teachers for the next session, most students felt happy to learn mathematics (Turmudi & Haryanto, 2011; Turmudi & Erlina, 2012; Turmudi, Julia, & Mahirudin, inpress), this situation would bring students to have good experiences to learn mathematics.

Figure 4: Scale for presenting algebra model of $x+2=8$

For example in the learning process of linear equation, the teacher applied the plan as the result of collaborative work in Laboratory Junior High School (Turmudi & Dwi Haryanto, 2011). In the open lesson the teacher use the scale (ballance) to explain the equation. The results showed that the model teacher was more confidence, because he could apply for teaching using scale as media for learning linear equation (Turmudi & Dwi Haryanto, 2011). The linear equation was easier for students to understand. For example when the algebraic form of $x+2=8$ explained to the students they have bariers to understand this meaning and could not find the value of $x$, but when they use the scale (ballance) they were easy to show how many marble in the bag (see Figure 4)

These facilities enable students to relate mathematical expression and the reality so that they were easy to think mathematically. The difficult and more complicated equations were easy for students to understand and to solve them. The students recognize the equation though scale (balance) as an initial understanding of algebraic procedures and principles. Furthermore, Turmudi & Erlina (2012) introduced the way how to construct the volume of pyramid by using beach sand. Though this formula could not be found directly by the students, they could manipulate algebraic forms of both shapes pyramid and prism. By using pyramid and prism containers and the beach sand, they manipulate, play, measure out, and compare their capacities. The results indicated that they could show the volume of prism as the area of base time it height ($V=A\times h$), and the volume of pyramid is 1/3 time the area of base time height ($1/3\times A\times h$). This finding give a valuable experience for students, model teacher, and LS participants. Students’ difficulties in understanding of algebraics relationship among two relationships ($V=A\times h$), and ($V=1/3\times A\times h$) suggested for teacher to enable students to relate both relationships, so that they could use transitive principle in mathematics.

The third experience in applying LS was taking place when a model teacher implementing mathematics lesson using a round melon to introduce and find the volume of a sphere in a Junior High School classroom in Bandung. A melon, as a model of a solid ball, was sliced and cut off to be a number of “pyramid-like” shapes. Junior high school students in an open lesson of LS could construct how was the volume of sphere using an approach of pyramid volume by considering that the sum of pyramid bases as the area of the melon skin.
Of the three open lessons in Bandung, we learnt that mathematics teachers glad and happy in joining a professional development as such a LS. Initially, they were not eager to be a model teacher in the open lesson session, but recently the teachers tend to snatch away to be a model teacher in the LS forum (Turmudi, Harningsih, Juandi & Whatma, 2010). Moreover, the findings of the LS process that the LS participants always mutual learn each other, so that collegiality principles and mutual learning (Supriatna et.al., 2010) took place for building learning community to be more creative. Each session in LS included some elements of exploring and investigating mathematical ideas by the students as participants of the LS.

The following illustration is an example of mathematical tasks presented in the first session which were intended as an introduction to thinking about primary school mathematics other than the traditional algorithmic terms.

Jesse’s work and a student’s work on factor pairs of 950. In board, the student vertically lists the factors pairs as “1 × 950 = 950, 5 × 190 = 950, 50 × 19 = 950, 10 × 95 = 950” and justifies reasoning in writing: “This is how I got my answers. 5 × 190 = 950 because 5 goes into 950, 190 times so 5 × 190 = 950. 50 × 19 = 950 because if I was to skip count by 50’s 19 times I will get 950. 10 × 95 = 950 I know that cause if I was to have 10 × 90 will equal 900 so I will 50 more so change 0 into a 50 and I will get 950. Also 1 × 950 = 950 because I can’t equal itself. That is how I got my answers.”

Jesse not only used the investigation materials as provided by its developers, but he also demonstrated specialized content knowledge, knowledge of content and students, knowledge of content and teaching (Ball et al. 2008) as well as a positive pedagogical disposition towards investigations.

Teachers’ recognition to the students’ previous knowledge will enable students learn new mathematics that they encounter. This supported by Ladson-Billings (1997) which stated about dealing with the social context of the mathematics classroom. In particular, she identified successful teachers of African-American students as practitioners of a culturally relevant pedagogy. She suggested that students learn best in an environment that acknowledges the prior knowledge the students bring to class, rather than one that focuses on what students do not know. For example in introducing the ball and its volume, the students were rewarded that they have already known the formula of pyramid volume and the area of ball skin (Turmudi, Julia, & Mahirudin, in press) in introducing the volume of pyramid, the students have already known the volume of prism and they have skills to compare (measure out) the volume of prism and volume of prism using the beach sand (Turmudi & Erlina, 2012), and in introducing linear equation, students were given an experience how to weigh things, i.e marbles or candies (Turmudi & Haryanto, 2011). Shortly, the teacher uses students’ previous knowledge to reach mathematical concepts that they learn.

Of course in giving mathematical concepts for students, it does not mean that students have to find out all mathematical concepts, procedures, and facts, instead the role of teacher in this teaching was to facilitate the learners in constructing factual, conceptual, and procedural mathematical knowledge. As supported by an idea that “children should not be expected to create or invent the entire history of mathematics on their own regardless of all the rich experiences we could dream up. Rather, they should be guided through this process by involving them in cleverly designed activities that tie together in important conceptual ways”. For instance, in the activity that previously described, the students developed a beginning concept of area and volume. However, the formulas they created are appropriate only for rectangular solids. What about spherical shapes? What about really odd containers like ponds? Continuation of the original problem to more complex problem situations allows students to expand their understanding of volume and connect it with other geometric concepts dealing with shape.
With regard to the LS, our knowledge of how students learn has deepened. The field of mathematics itself is undergoing change dramatically. Traditional shopkeeper calculation, appropriate for the lowtech past, has become less and less important in an information age as currently happened. These and other reasons for rethinking the nature of school mathematics have spurred the reform movement.

The most crucial and important thing was that the teachers should have knowledge and experiences how to develop lesson based on the innovative perspective and how to support student learning by using local materials as media, or as a teaching material, or as a “bridge” for student to understand the concepts of mathematics.

In this article, we were going to know whether the students at school, the place for implementing the LS program, learn the volume of cylinder using baby water melon, how was the students understanding to this concepts, and how was the teachers react to the open lesson of LS which use baby water melon as media for learning cylinder volume.

C. METHODS AND DATA COLLECTION

Data on how the teachers prepared lesson collaboratively among the member of LS were collected. Initially the teachers (LS participants) discussed the current teaching that they usually used in the classroom which a very typical pattern: present-example-copy-test-check-test. This instructional pattern looks like effective from the time perspective, because the curriculum target can be covered by the teacher, whereas the students’ proficiency need to be questioned. This situation motivated the teachers to alternatively design the lesson that provides the students opportunity to think and construct their mathematical understanding. Manipulative teaching materials was used as suggested by Bruner (1961) as enactive learning. Through this manipulative materials students actively attempted to observe, to explore, and to investigate the properties of mathematical structures, to predict, to make a conjecture, and then finally to build the formula of mathematical concepts and procedures.

In the LS context students identified the cylinder form that has been modified to be prisms. The prisms look like triangle prism as could be seen in the Figure 6.

Next, students treated the “prism-like” as the slices of baby water melon which originally from melon as a solid cylinder. They work collaboratively to stuck the “prism-like” to be other type of geometrical shapes. One type of shape was a cuboid as stuck vertically (Figure 7a) or horizontally (Figure 7b).

Both figures indicate a rectangular-block or we could say as “cuboid-model”. Previously they have known the formula for the volume of cuboid. In the beginning of the lesson the teacher mentioned the formula of the volume of cuboid which has the length of \(l\), width of \(w\) and the height of \(h\). Easily, the Year-9 graders of junior high school in Garut, West Java recognized as cuboid and could find its volume. However, they really got difficulty to determine the “cuboid model” made of baby water melon.

Intensively the students tried to link between the radius of circle as the base of cylinder, and the height of cylinder with the length, width, and height of “cuboid model” as the result of modifying cylinder. When two models were put side by side we could see as in the Figure 8(a) and Figure 8(b). The Figure 8(a) was the modified form of Figure 8(b).
Then the Figure 9(a) as a cuboid model, look like a rectangular block or cuboid with the length of \( l \), the width of \( w \) and the height of \( h \).

In here, students little bit stuck could not determine which one the length, which one the width, and which one the height with respect to the radius and the height of the baby water melon cylinder. In group, students were asked to trace where were from the length, the width and the height of cuboid.

They were able to response that a “cuboid-model” made of “solid cylinder” of baby water melon.

Students try to investigate which part the length of “cuboid model”? By modifying cylinder to be “coboid-model” students were able to recognize the length of “cuboid-model” which from the circumstance of the circle (base of cylinder). Then how to find the width of “cuboid-model”, students explain to the teacher that the width of “cuboid-model” was the height of cylinder. Moreover, the height of “cuboid-model” was the radius of the cylinder. Therefore students have found that the measure of “cuboid-model” were as in the Figure 10a & 10b.

When the students recognize the length, width, and the height of a cuboid, then they will be easy to know the volume of the cuboid. Similarly, the “cuboid model” was recognized the length as \( \pi r \), the width as \( h \), and the height as \( r \), then the volume of coboid model would be \( \pi r \times h \times r \) or \( \pi r^2h \).

How can this conclusion be made by the students, the following dialogue will clearly explain the situation.

Conversation between teacher and one of groups in the lesson.

Teacher : Try to pay attention to the shape of cylinder which was modified to be “cuboid model”
Student : Yes Sir
Teacher : What is that? [the teacher pointed to the “cuboid model”]
Student : Look like a cuboid
Teacher : Do you remember how to find the volume of cuboid?
Student : Yes sir
Teacher : How do you find the volume of cuboid?
Student : Volume of cuboid was length \times width \times height
Teacher : Well done, now you have to notice this cuboid model [the teacher pointed to the model of cuboid]
Which part to be considered as the length, as the width, and as the height of cuboid?
Student : In our result of discussion, the length of “cuboid” was the half of the length of circle perimeter so we consider as \( \frac{1}{2} \times 2\pi \times r = \pi r \)
Teacher : OK, what about the width and the height?
Student : In our group, the width of “cuboid model” is the height of cylinder \( h \), and the height of “cuboid model” is radius of the circle \( r \).
Teacher : If so, how to find the volume of “cuboid model”?
Student : The volume of “cuboid model” is \( length \times width \times height = \pi r \times h \times r \) or can be write as \( \pi r^2h \)
The above dialogue showed that the model teacher did not inform directly about what they know, instead the teacher promote some questions as kind of guided-re-invention to help learners construct knowledge of volume.

**Open lesson- Implementation and Observation**

The implementation of open lesson was done on Saturday, November 10th, 2012, started from 09:30 by model teacher (the second author of this article). We open the lesson by greeting the students “Assalamu ‘alaikum wr. wb.”, and the students response “Waalaikum salam wr wb”

While distributing the materials, the teacher remaining the students about how to find the volume of cuboid. Most students were still remember how to find the volume of cuboid (rectangular block). Now we were going to learn the cylinder and it properties. One of the properties of cylinder was the volume of cylinder. But I did not want to directly tell you how to find the volume of cylinder. Instead, I provided the baby water melon as cylinder for helping you all to construct and build the volume formula of cylinder.

The mathematics materials (worksheet) and cylinder model (baby water melon, see Figure 13a) have been distributed to the learners in the six groups of students.

In general, it was quite difficult to imagine that the shape of the melon fruit was a cylinder. However, in fact the baby water melon when the ends of fruit was cut, then we would get the baby water melon (as can be seen in the Figure 13a) as cylinder. Beside the baby water melon we also provide the knife and the rulers for students to cut and modify the baby water melon to be a cuboid-model.

After giving instruction for the students that today’s lesson was different from the usual one, because some visitors would see what we were doing in our lesson today, the teacher mentioned that we were going to generate the volume of cylinder through our experiences in finding the volume of prism or the volume of cuboid (rectangular-block). Later you would slice the baby water melon and stuck these parts to be “model of cuboid”. Then you would recognize the length, the width, and the height of “cuboid model”. One of the shape students made was the Figure 10b.

By slicing the “cylinder melon” and constructing part of melon to be other shape of geometry, students explorated to various types of geometrical shapes. They identify the length, the width, and the height of the cuboid-model, as modified of cylinder. Then by using their knowledge about the volume of cuboid, students find the volume of cuboid model as modified shape from cylinder of baby water melon. Among 6 groups, there were at least two different methods to organize part of slicing melons as could be seen in the figure 12, 13(a) and 13(b) in the following:
Students in each group designed the shape of cuboid-model with different positions. Using the teacher’s scaffolding, students are able to recognize the length, the width, and the height of the cuboid-model.

They are able to modify a cylinder to be a cuboid-model, and they were also able to create the formula of the cuboid model. Since a cuboid model was made of a cylinder watermelon, therefore they were able to formulate the volume of cylinder as $\pi r^2 h$, as previously mentioned.

The teacher has a deep impression towards the way how students create and modify a cylinder to be a cuboid-model using a baby watermelon. Before instruction taking place, there have not happened in someone teaching to teach volume of cylinder by using melon, except a lesson by using round melon for constructing the volume of a sphere (ball) (Turmundi, Julia, & Mahirudin, in press) who have applied the lesson in Year 9 of Junior Secondary School in Bandung. Therefore this kind of teaching can be considered as an original and creative finding, the teacher and team are able to teach by using water melon as media, as a local material, and as a product of agriculture. The teacher lives in the remote area whereas the fruits can be used to help learners learn mathematics contextually.

D. REFLECTION

The third part of the lesson study was reflection session. During the reflection session, the model teacher spoke first, reflecting on the lesson implementation, noting what went well, and on any difficulties in the lesson before others shared their reflections. The teacher who taught in the open lesson told his critical experience in preparing the lesson. Even he told to facilitator for delaying the lesson due to difficulties to prepare media for learning.

After being fail to persuade the carpenter to slice the wood-cylinder to be a number of prism-like shapes, the model teacher wished to postpone the open lesson until everything ready to teach (see Figure 1). The failure of the carpenter, because he did not have any sharp-saw. The closer time to open lesson session, the less time for the model teacher to prepare media for learning, and the more stressful for him, therefore he proposed to postpone the open lesson of the LS. In the very difficult time, model teacher sought to change the wood-cylinder with the fruit. He has an idea to switch his plan from making the wood-cylinder model to water melon-cylinder model which ultimately successful in his teaching. In his short journey from the remote area to the down town he found the baby water melon which resemble to a cylinder. He felt sure that baby water melon can be considered as cylinder after cutting both ends (see Fig. 2 and Fig. 13a).

The member who assisted in planning the lesson shared their observation notes reflecting on the goals for the students and the design of the research lesson, comparing and contrasting what was planned and what was observed. The discussion focused on the specific notes collected by each observer. Observation notes consisted of comments and questions from the students and teachers during the lesson as well as observation notes on the use of manipulatives of baby water melon and students’ worksheets. The facilitator of the LS then provided feedback and shared in the discussion.

As we know that the Lesson Study is a reflective process. It means that Lesson study provides plenty of time and opportunities for teachers to reflect on their teaching practice and student learning, and the knowledge gained from and for the reflective practice should be shared in some format with the larger teaching and educational communities.
After cutting the ends of “baby watermelon”, we have obtained three dimensional shape of solid cylinder. While students have not known yet the formula of the cylinder volume, but they have already known the formula of the cuboid volume, namely “length × width × height”. By learning process in the LS students have sliced the cylinder-melon, and modified the cylinder-melon to be a cuboid as can be seen in the Figure 13b and 13c. In this part, students little bit confused how to know the length, the width, and the height of cuboid. However, teacher gave a box to each group to identify and to name the dimension of the cuboid as the result of modification process.

The following comment come from the participant of the LS:

“Saya pikir ini merupakan gagasan yang sangat bagus dapat mengubah sebuah tabung menjadi sebuah balok, yang saya kira sebelumnya ini belum pernah ada pembelajaran yang menggunakan semangka seperti ini” (Tony, participants of LS).

“I think it was a very smart ideas to change and modify a cylinder-model to be a cuboid-model by using water melon that have never been done before by anyone in the world (Tony, a participant of the LS)).

“Justru inilah yang menjadi gagasan asli dari tim LS dalam rangka IHT ini dapat menemukan alat peraga menggunakan semangka. Bahkan semangka sendiri umumnya berbentuk bulat seperti bola, sedangkan tim dalam lesson study-IHT ini menemukan semangka yang bentuknya sendiri menyerupai tabung (lihat Figure 15b), yang apabila ujung-ujungnya diiris maka akan berbentuk ban buang ruang yang menyerupai tabung solid” (salah seorang peserta Open Lesson, 2012; see Figure 12a).

“I think this was the original idea from the LS team in the In-House Training program that could invent the lesson using manipulative materials like baby water melon. A melon usually looks like a sphere (ball), whereas the model teacher in a team found special melon which has cylinder shape (see Figure 15b). If we cut the two ends of melon, then the result would be a solid cylinder model (participant of LS, see Figure 13a).

Eventhough a number of teachers usually used traditional teaching approach, when they were introduced with new creative, and inovative teaching approaches, they could make smart and unusual innovation as they can invent the teaching approach by using baby water melon to show the volume of cylinder.

We as a team LS in the IHT program felt optimistic that mathematics teachers at the rural area would have inovative and creative ideas when they were given opportunities to prepare and follow a professional development regarding a cerative and inovative teaching approach that could challenge students to learn mathematics and challenge the teacher to present their lesson attractively.

E. DISCUSSION AND FINDINGS

Though the model teacher has limited knowledge and experience in teaching mathematics, when they were introduced with new creative, and inovative teaching approaches, they could make smart and unusual innovation as they can invent the teaching approach by using baby water melon to show the volume of cylinder.

For example, mathematics learning by using cooperative learning used cards to learn algebra (Juariah, Syafari, & Suhendar, 2012), teaching geometry using cylinder-water melon (Suherdi & Kusmana, 2012), and teaching equation using constructivist’s perspective (Ruswanda, Latif, & Sumpena, 2012) all suggested that LS in the IHT context have positive impact for the students as well as teachers in the rural area of West Java.
The participants of the lesson study admitted honestly that they really eager to apply these experiences in their teaching in the classroom. As the result of study by Turmudi, Harningsih, Juandi dan Wihatma (2010) indicated that the teachers’ awareness in implementing mathematics teaching innovation in the classroom has significantly influence behavior in teaching. Similarly, the LS in the IHT context also has a positive impact for the participating teachers. The participants of the IHT expected that the similar professional development should be followed up and disseminated for other teachers. It was consistent with the findings of previous research (Turmudi, Harningsih, Juandi, & Wihatma, 2010) that the teacher have strong and high commitment to make an innovation in teaching and learning mathematics, and start to adapt mathematics teaching innovation (Turmudi, 2006).

An isolated and rural area does not mean that they were illiterate and left behind in absorbing knowledge about innovation in mathematics teaching, in research, and in professional development. Through LS model and open lesson, the participants have opened their perspective and horizon that learning to teach was not only happened in an isolated society, in which the teacher was only well-prepare when the supervisor would come, whether by the supervisor or by school master. In contrast in the open lesson process, the teacher would always well-prepare whenever and whoever would come to their classroom. In this context, the IHT participants showed their readiness to be a model teacher. This situation was relevant with Turmudi, Harningsih, Juandi & Wihatma (2010), Turmudi & Haryanto (2011) and Turmudi & Elfina (2012) that their interest and enthusiasm to be a model teacher in an open lesson to be more and more. The similar phenomena happened in the LS context of IHT, among 8 participants, almost all of them interested to be model teacher. However, due to the time constraints, only three of them were presenting their open lesson, and this result was part of the LS.

Among three open lessons, in this IHT of Garut regency appeared that baby water-melon lesson was the most creative and innovative lesson, because either the students or the teachers positively commented that the lesson was creative and fun. Why this was creative, because this kind of lesson was truly original and new (Plucker & Beghetto, cited in Yuan & Sriraman, 2011). There are two key elements of creativity, specifically novelty (original, unique, new fresh, different creations) and usefulness (i.e., specified, valuable, meaningful, relevant, appropriate, worthwhile creations). As far as we know in mathematics lesson just once the lesson using water melon, but this was round water melon to show the volume of sphere (Turmudi, Julia, & Mahirudin, impress), and in our opinion that have never been conducted teaching volume of cylinder by using baby water melon. Variety of structures have been made by students as could be seen in Figure 13(a) and Figure 13(b) which indicated that students creativity appeared, they did not only wait for the teacher request, instead they construct the volume formula of cylinder as \(\pi r^2 h\) using cuboid volume approach. Students were happy and they expected to use similar approach in teaching mathematics for the next session. It means that the teacher were challenged to use a resemble approach in other topic of mathematics teaching.

F. CONCLUSION AND SUGESTION

Eight-day professional development program of IHT gave a deep impression for the teachers in using new teaching approach in mathematics. The participant of IHT was junior secondary teacher in the rural area of Garut Regency, West Java, Indonesia. Though they were in the rural area, they have high enthusiasm to absorb knowledge and experience in applying mathematics teaching. The participants expected that there would be a follow-up session of the program for the future. Though they were happy and have a deep impression to the open lesson session, it still needs to conduct the follow-up research with a wider subject, whether the students’ mathematical proficiency, their reasoning, their problem solving competencies would improve. From the short study, we know that the students enjoyed the way how to learn mathematics. They also were able to construct the volume of cylinder as \(\pi r^2 h\), not only using the chalk and talk teaching, or lecturing, but using an inductive learning approach, they observe, explore, make a conjecture, predict, and collect data, and prove. Ultimately they can construct that the volume of cylinder. Therefore we recommended this kind of teaching could be disseminated (nationally or internationally) to be a wider context for the students as well as teacher, so that they learn factual, conceptual, procedural understanding of mathematics.

Participation in the studying mathematics teaching in LS by engaging in actual teaching, and then reflecting critically on it as a group of individuals who are all similarly engaged, contributes to the identity of an individual engaged with learning to teach mathematics and to the community of practitioners building knowledge of and through the enterprise of LS dedicated to mathematics teaching. By putting their experiences and practice of the LS community as a whole, it is possible to illustrate, largely from the student teachers’ experiences, how one participant re-positioned herself, through identity work as an in-service junior secondary teacher learning to teach mathematics well thereby increasing her mathematical knowledge in teaching.
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DEVELOPING STUDENTS’ 21ST CENTURY SKILLS THROUGH
INDONESIAN REALISTIC MATHEMATICS EDUCATION:
AN ALTERNATIVE APPROACH

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ABSTRACT

The world has changed. Living in the next twenty years later requires preparing our generation. They need certain proficiencies or what is called as “21st century skills” to answer challenges of their life ahead. The skills were thinking critically, solving problem, communications, collaborations, creativity, and innovations. Mathematics as a major subject in most of Indonesian schools has a chance to support children in acquiring the skills. Indonesian Realistic Mathematics Education (PMRI) contributed to the development of mathematics learning in Indonesia for more than one decade. Some researches used this approach in their instructional design for mathematics classroom. Compiling the result of the researches, this paper discussed how did the approach implemented in the classroom and its impact to the students’ skills. Those researches were not aimed to intentionally investigate the effectiveness of the approach in supporting the development of the skills. However, the investigation toward the learning process was expected to show significant contributions of the approach toward students’ 21st century skills. Based on the investigation and discussion, the approach was highly potential tool for building the skills to survive in the 21st century. By applying its principles in mathematical learning, students could be prepared to be critical thinkers, problem solvers, creative and innovative person. They might also be communicative person and work collaboratively with others. The idea discussed in this paper might give a glimpse of a future mathematics education in Indonesia, though the discourse of 21st century skills in educational sector is quite new.

Keywords:
Realistic Mathematics Education

1. INTRODUCTION

Some ancient civilizations believed that earth was flat. Living in the beginning of twenty first century has brought the same perception. Rapid change of technology made our generation believe that our world was getting “flatter”. People were easily moved from one place to another in shorter time. We even simply connected to others by using the aid of technology that we might not imagine couple years ago. This sort of change affected the way people living and interacting. In addition, people also would work differently for some new jobs that occurred as the consequences of it. It cannot be denied that our everyday life has rapidly shifting.

The discourse of this issue led us to think about how this change might affect educational system around us. We needed to start defining new competencies to be mastered by young generation for their life ahead. These competencies could be something that we did not even imagine fifty years ago. It also required us to prepare them for solving problems that might not be defined yet today. Therefore, we needed to start thinking of identifying the particular skills and education that this flat world would demand [4], [10].

Recently, 21st century skills were introduced as new term to mark an issue of those particular skills. The skills included critical thinking, problem solving ability, communications skill, collaborations, creativity and innovations. Mastery of these skills was essential for our young generation to survive in today’s world at this century.

Friedman stated that in the future the way we educate our children might prove to be more important than how much we educate them [10]. This challenge made us aware of selecting things learned by our young students. The framework for 21st century learning suggested core academic subjects which were including mathematics [9]. It also suggested that acquiring the skills must be based on the core subjects. It implied that educators could develop students’ skills through mathematical learning activities or any subject they learned.
Furthermore, some of those skills closely related to mathematics. Therefore, it was important for educators to find a way such that students could acquire the skills within the context of learning mathematics.

Starting from Curriculum 2006, Indonesia’s government has acknowledged some of the skills as the main goals of learning mathematics. It required students to think critically, solve problems, communicate, and become creative in the context of learning mathematical concept. It implied our government concern toward the shift of educational goal and today’s world demand. This was similar to Friedman’s opinion. In our world in which routine work was largely done by machines was a world in which mathematical reasoning will be far more important than memorizing mathematics facts [10]

In addition, some mathematics educators in Indonesia adapted realistic mathematics education (RME) as learning and teaching approach. This adaptation process has become a movement for a decade now. It was aimed to promote learning mathematics in the meaningful way instead of rote learning. This approach was known as Indonesian realistic mathematics education (PMRI) recently. PMRI has brought a new alternative for mathematical learning approach in Indonesia. Using this approach mathematics was perceived as a human activity. Therefore, all mathematical activity was based on problems in which learners was expected to reinvent mathematical concept during their process on solving it. This characteristic showed that PMRI approach emphasized on building students’ thinking and reasoning.

This trend of mathematics education in Indonesia might answer the challenge of 21st century skills. The alternative approach offered by PMRI seemed different from usual mathematics classroom in Indonesia. It raised a question whether there was a chance for this approach to contribute on the development of students’ 21st century skills.

However, some research on PMRI has been done in wide range of mathematics topics. It produced research-based literature on the implementation of this approach. Unfortunately, these research-based literatures did not intentionally explore the possibility of PMRI to facilitate students in developing the skills. Therefore, an investigation toward these literatures should be done to examine this issue. In this paper, we would investigate the potential of PMRI approach in facilitating students in developing the skills through mathematics learning.

2. THEORETICAL FRAMEWORK

2.1. Twenty first century skills

There were some theories regarding what might contributes to 21st century skills. Trilling and Fadel defined 21st century skills in three categories namely learning and innovation skills, digital literacy skills, career and life skills [4]. This categorization was also adapted by Partnership for 21st century skills [9].

Learning and innovation skills were critical thinking and problem solving, communication and collaboration, creativity and innovation. It would prepare them to face their complex life in this competitive digital age.

Rapid change of technology, accessible information, and development of media environment were reflected our world today. It required what was called by digital literacy skills. Based on Trilling and Fadel [4], it was defined by three competencies namely information literacy, media literacy, and ICT (information, communications and technology) literacy. The digital literacy skills were also marked by Friedman as navigation skills in virtual world [10].

As we noted that life and work environments nowadays demanded an adequate life skills. Young learners in 21st century must pay attention to it more than mastery of content knowledge. This reasoning might bring the last skills category from Trilling and Fadel – career and life skills. This skills included flexibility and adaptability, initiative and self-direction, social and cross-cultural interaction, productivity and accountability, leadership and responsibility.

Friedman did not define specifically about the skills. Friedman rather suggested five skills sets or attitudes toward learning that would be helpful for young learnersto be prepared for their life [10]. First, young learners must decide properly which class they should take to learn how to learn well. Second, students must be able to navigate in virtual world. It was also highlighted by Trilling and Fadel as digital literacy skills [4]. Third, young learners should keep their passion and curiosity to be long life learners and become an adaptable person. Fourth, curriculum should pay attention on interdisciplinary studies. It suggested making connection among history, art, politics, and science. Fifth, educators must think how to optimize right brain potential of young learners. This ability was beyond creativity level. Pink in Friedman [10] explained it as the ability to build relationships rather than execute transactions, tackling challenges instead of solving routine problems, and synthesizing the big picture rather than analyzing a single component.

However, the discussion of 21st century skills in this paper limited to learning and innovation skills. So the term 21st skills in this paper could be defined in five skills only. These are critical thinking and problem solving, communication and collaboration, creativity and innovation.
2.2. **Indonesian Realistic Mathematics Education (PMRI)**

One of principles emphasized by Freudenthal [6] in doing mathematics was the selection of learning situation within students’ current reality which appropriate for horizontal mathematizing. It implied that a problem situation involved in a mathematical learning must be experientially real or imaginable for students. In other word, by using their common sense, students could be encouraged to explore their ideas and develop their own strategies to solve the problem. This was the core of realistic mathematics education.

There were five principles that characterized RME. These principles served as guidance in designing a mathematical learning which used this approach [1]. The first principle was phenomenological exploration in which a contextual situation was presented to students. The aim of using this kind of problem was to prompt students’ thinking. Second, a mathematical learning should use models and symbols for progressive mathematization. It characterized a progression from a concrete level to the more formal level. Third, students should learn by using their own constructions and productions. It gave them opportunities to explore their ideas and solution in producing solution. Fourth, the learning environment should give enough space for students to interact each other. Interactivity in this approach considered students’ learning process both as an individual and a social process. Fifth, learning mathematics should integrate various mathematics topics in such away learners could make relation among them.

All characteristics and principles of RME also applied in Indonesian realistic mathematics education (PMRI). It could be considered as an Indonesian version of realistic mathematics education (RME). The adaptation merely referred to the use of local context in the learning activities. It was based on the meaning of word “realistic”. It suggested a problem situation involved in a mathematical learning must be experientially real or imaginable for students. As stated by Freudenthal, mathematics is as a human activity [6]. Therefore, mathematics should be taught in such a way students could learn it with experience-based instead of memorizing a ready-made algorithms or formulae.

3. **RESEARCH METHOD**

This paper was not written based on one specific research. It was actually based on literature review of the existed research. Three research-based literatures were reviewed regarding the issue of 21st century skills. All of these literatures used realistic mathematics education as an approach in instructional design during the research. The learning and teaching were implemented in three different regions in Indonesia namely Palembang, Yogyakarta, and Surabaya [2, 3, 8]. These researches covered different topics in mathematics namely subtractions, linear measurement, and area measurement.

All of those researches used a design research as a research method [2, 3, 8]. This kind of method was basically about testing an instructional design in the series of learning activities [7]. The test was aimed to investigate conjectures of student thinking and strategies for improving the learning design. A retrospective analysis was conducted afterward to highlight the students’ learning process. This analysis was intended to find supportive argument for making recommendations regarding the improvement of the instructional design [7].

However, these researches did not intentionally investigate the relation between PMRI and 21st century skills. Therefore, the investigation toward the potential relation between the approach and development of 21st century skills need to be done. We could evaluate the implementation each of RME principles in the result of each research and then investigate whether it has potential contribution to the development of the skills. This evaluation would be based on the retrospective analysis since it contained the whole learning process of students during the researches. The process of each research and the evaluation were shown in Figure 1.

![Figure 1. The design research and the investigation of the result of each research](image-url)
4. RESULT AND ANALYSIS

The result of investigation toward three different design researches on mathematics education would be explained in this section. Each research was explored in the separate subsection so that the investigation was described clearly.

4.1. Design research on subtractions

This research was aim to develop a model to support students in learning subtractions. It was carried out in Palembang. The implementation of RME principles during the research were described as follows:

a. The use of context in phenomenological exploration

Two contexts were used to emphasize the meaning of subtraction during the lesson. The first context was taking ginger candies in order to construct the idea of taking away something. The second context was making grain bracelets to develop the idea of subtraction as determining difference between two numbers.

The students gave various responses to the contextual problem given in the lesson. They showed different strategies and were able to make mathematics notation of those contexts. Even some students were able to use more than one strategy in solving the problems. It indicated that the use of context could promote students’ ability in problem solving.

b. The use of models for progressive mathematization

This principle suggested that students should develop mathematical models to handle the given problem and build a bridge to the more formal mathematics. As the students were asked to solve subtraction problems, they were asked to visualize the solution. From various visualization occurred among students, they were introduced a string of beads to represent the situation of the context. Later on, students were facilitated to use empty number line which could represent the general situation and reflect students’ thinking. All of those models were initially intended to help the students acquire the concept of subtraction. However, it showed that giving chances to students for develop model helped them to solve subtraction problem better. The use of different models made them aware of various ways to reach the solution. It implied that these models helped them build their problem solving skills.

c. The use of students’ own constructions and productions

Since students needed to solve a contextual problem which its mathematical term was not introduced yet, they must propose their informal strategies first. It enabled them to product any kind of method freely. In fact, during the subtraction lesson they used various strategies in solving the problems. Somestudents used finger calculation, some of them utilized the ginger candies they have, other students drew pictures of candies. Some other students were even able to do thematical calculation for certain number that quite easy for them to calculate.

The application of this principle did not explicitly show its contribution to the development of students’ creativity and innovation. However, students’ freedom to construct and produce any possible solution might enable them to notice that it was possible to find the solution of mathematics problem from different approach. In turn, it highlighted the idea of being creative and innovative in learning mathematics. It would make them aware that the way to find solution for mathematical problem was not always a single way out.

d. Interactivity

Applying PMRI implied that the learning environment must be in the interactive way. In solving subtraction problems, students were conditioned to work in group. It made them possible to share their idea to others and develop their understanding while at the same time they learned to respect eachother.

The students came with several different solutions to solve subtractions problem. It could give insight for them in the class discussion. They criticized other solution proposed by their friends since it was different from theirs. They finally recognized that both indirect addition and indirect subtraction were more efficient to solve the problems which have small difference between minuend and subtrahend.

This principle showed effectively the impact of students’ interaction. They could work with the other students as partner in order to find solution. They could also learn how to communicate their ideas during class discussion. Moreover, they started to critically think about other possible solution, criticize it, and then choose the best way that made sense for them.

e. Intertwining of various mathematics strands or units

The design research on subtraction emphasized the meaning of subtraction and the strategy to solve subtraction problems. It also gave more attention to the relation between addition and subtraction.

All students did not find difficulties to understand the meaning of subtraction as “taking away
something” because the already familiar within. In the other hand, most of students could solve the subtraction problems by recognizing the relation of subtraction problem with the addition problem that was given beforehand. They did not rely on counting method. It implied that they were able to understand that subtraction was implicitly the converse of addition. icated by the students only by seeing their relation with the addition problem before.

Students’ recognition of the relation might give them a new way to look at subtraction problems. In turn, they could build another approach to solve subtraction problem. They might creatively propose an innovative solution for difficult problem that involved subtraction in the future.

4.2. Design research on linear measurement

This research was initially aimed to investigate how Indonesian traditional games could be used to develop students’ reasoning and reach the mathematics goal of linear measurement. The observation of students’ learning process during the research was conducted at Yogyakarta. The following description explored how the implementation of RME principle in the actual learning during the research.

a. The use of context in phenomenological exploration

In the beginning of the instruction, mathematical activity was based on a concrete context. It was an experientially real situation for students. This research employed Indonesian traditional games as the contextual situation since Indonesia has a lot of traditional games.

The students showed the need of using “third objects” as benchmark in comparing the distance during playing gundu. This was a start of a measurement. The analysis of the lesson indicated that students considered the precision of the measurement, unit iteration, and adjustment of the unit to the size of distance that was being measured. In addition, two students gave opinion about there was no one willing to measure the distance. This opinion was criticized by the other students. As the result, they shifted from using a person’s body part as standard unit of measurement to using independence object that could be used by everybody.

It could be concluded that this game promote students’ critical thinking in measuring a distance. It also helped students to build reasoning for solving problem in comparing distance.

b. The use of models for progressive mathematization

The second principle of RME was building abridge from a concrete level to a more formal level by using models and symbols. From playing traditional games students gained informal knowledge. It was a result of their experience during the contextual activities. This informal knowledge needed to be developed into more formal concept of linear measurement. It was done by challenging them in the “making our own ruler” activity. This activity required them to make their own instrument to measure a distance. The lesson reflected that students commenced to measure in a more formal way by using their handmade measuring instrument. Blank rulers were developed afterwards as the introduction of using normal ruler. It has become a ready-used instrument of linear measurement.

The development of measuring instrument during the series of lesson was not taught as collection of facts. Instead, it was developed by putting the students into a situation where the more standard measuring instrument was indispensable. This kind of situation led the students to solve different measurement problem in the more challenging level. Therefore, it could be concluded that during the series of lesson students were implicitly trained to be a good problem solvers.

c. The use of students’ own constructions and productions

Linear measurement seemed merely a procedural concept. However, design research on this topic had given a new point of view. The students got freedom to construct their own strategies. It promoted the emergence of various solutions that could be used to develop the next learning process. In the game, students used hand spans, foot spans or their other body part as unit measurement. Later on when they realized the need of independent standard units they could reason to use third object as measuring instrument.

Developing informal knowledge of measurement units to the more formal tools might be difficult for students in the usual mathematics lesson. However, through this research they showed that they could find many ways to measure distance. They also showed that they could come up with the idea of precision, unit iteration, or adjustment of measuring tools. It implied that they began to develop their creativity to “reinvent” an innovation of linear measurement.

d. Interactivity

Game playing forms a natural situation for social interaction among students. It encouraged students to communicate their thought and work in the class discussion. In fact, they discussed an agreement in deciding a strategy for the fairness of their games during the lesson. There was a student argued about the constant length of stick used in the game. This student used the concept of conservation of length to support his acquisition of concept identical unit and the emergence of
standard measuring unit in measurement. Conclusion of another student also became a base for the whole class in perceiving the concept of identical unit and, moreover, the need of a standard measuring unit for fair measurement.

All of those interactions in the lesson indirectly promoted their communication skills in arguing, criticizing, and explaining their reasoning. It also supported students’ acquisition of the basic concepts of linear measurement.

In addition, game was a logical reason to form group or several students in playing it. Working in group enabled them to get used to learn collaboratively with the other students.

e. Intertwining of various mathematics strands or units

Intertwining suggests integrating various mathematics topics in one activity or lesson. The Indonesian traditional games used in this research supported learning for linear measurement as well as the development of students’ number sense. However, the intertwining in this research did not give different ways for students to think critically about linear measurement from the perspective of number sense

4.3. Design research on area measurement

a. The use of context in phenomenological exploration

This research brought the context of drying kerupuk in the classroom. The task was comparing two different bamboo trays in rectangular shape. The students must determine which tray was the largest one. During the discussion, the students could reach conclusion that the largest tray contained kerupuk (crackers) more than another. Even they all agree to this conclusion, the students showed different strategies in determine the number of crackers in a tray. Some of them covered the tray by putting the rectangular units in column and repeated until fully covering the whole surface. Some other students put the crackers by repeating the row or even put the units randomly. It implied that the context could encourage the students to solve the problem in their own way.

b. The use of models for progressive mathematization

This principle was originally aimed to facilitate the students to reach the more formal mathematics concept (or mathematical model). It happened when the students struggled to find the solution of the contextual situation given in the lesson. Starting from using rectangular units, the students then shifted the measurement by using square units as the standard measurement tools. The efficiency issue occurred when they did not have enough square units to cover the whole surface of tray. They began to cover “side” of tray (either row or column) and then counted how many repetitions needed to cover the tray. In the end, they utilized the strip of square units as tool for measuring area.

The variety of strategies in using models they encountered made them aware of the possibility of being different in solving area measurement problem. It did not matter whether they started from a row or a column as long as it would cover the whole surface.

c. The use of students’ own constructions and productions

There was a session in which the students could choose by themselves a surface they would measure. This chance of producing their own problem could help them constructing the concept of area measurement. In addition, it was expected that they might develop their sense of creativity and innovation because they were asked to propose their “original” problem.

d. Interactivity

As in the other design research which used RME, this research gave emphasize to the students’ interaction in the classroom. Some students proposed the idea of fully covering the whole tray with units. Later on, some other students argued that it was better if they only cover a column or a row and find how many repetition they need to fully cover the whole surface. The discussion could bring them to the conclusion of using multiplication to find the area. The numbers they multiplied were based on the number of units in column and its repetitions, the number of units in rows and its repetitions, or the number of units in a column and in a row.

Students’ interaction in group encourages them to be brave sharing their thought to their partner since they must present the result as group work. Moreover, the class discussion could motivate them in arguing, reasoning, and compromising their ideas to other.

e. Intertwining of various mathematics strands or units

Area measurement closely related to the concept of multiplication. The repetition of unit measurement by columns or rows reflected the multiplication model of the array. In fact, the students came up with the idea of covering column with square units and then count the number of column to find the area of the tray. They could conclude that it was also applied to the repetition of rows. In the end, they could find the relation between a column and a row of units as tool for finding the area. It indicated that the students could critically build the reason why multiplication applied in measuring area.
All of the indication from those three design researches was summarize in table 1. It described the relation between RME principles and students’ development of 21\textsuperscript{st} century skills.

Table 1. The relation between RME principles and students’ development of 21\textsuperscript{st} century skills

<table>
<thead>
<tr>
<th>RME Principles</th>
<th>The development of 21\textsuperscript{st} century skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>The use of context in phenomenological exploration</td>
<td>Students might develop critical thinking skills during solving the given problem embedded in the context. At the same time, they might get an opportunity to sharpen their problem solving ability by challenging themselves to face non-routine contextual problem.</td>
</tr>
<tr>
<td>The use of models for progressive mathematization</td>
<td>Students’ effort to build a model and formulate solutions when they encounter the contextual situation might give them chances to build their problem solving skills. It was also supported by their attempt on developing mathematical models to solve the problem.</td>
</tr>
<tr>
<td>The use of students’ own constructions and productions</td>
<td>Throughout the lesson series, students might come with various responses toward the problem. They might also come with different solution or even additional problem. This could be a bridge for them start their initial level of creativity and innovation.</td>
</tr>
<tr>
<td>Interactivity</td>
<td>Students could work collaboratively with each other during the whole learning process since RME gave much attention on it as a social process. They might interact with others and communicate their ideas. Moreover, in this interaction students could clarify their own solution and critically learn the others’ solution.</td>
</tr>
<tr>
<td>Intertwinement</td>
<td>The intertwinenment among mathematics topics seemed irrelevant to the development of 21\textsuperscript{st} century skills. However, students could be more critical when they learn a new topic from different perspective (mathematics strands).</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

Based on the previous section, it could be concluded that Indonesian realistic mathematics education has potential contribution on the students’ development of 21\textsuperscript{st} century skills. Mathematics instruction that used this approach and applied its principles showed some significant marks on the existence of the skills during the lessons. Students were indirectly trained to be problem solver who accustomed to face new challenging problem. In addition, contextual problem given in the beginning of lesson encouraged them to propose various informal strategies. It implied that the context might boost their creativity and innovation skills. Working in a group supported them to learn how to work collaboratively with others. Moreover, they were also supported to think critically toward their friends’ solution and communicatively shared their thought. The class discussion made them aware of respect and appreciation to the others’ opinion.

However, we must realize that this conclusion was merely a review of research-based literatures. It was limited by the fact that the researches cited here did not initially investigate the issue of 21\textsuperscript{st} century skills and the contribution of PMRI. Therefore, a specific research on this discourse was demanded to carry out. This research should be focused on the development of 21\textsuperscript{st} century skills during mathematics lesson by using PMRI approach.

REFERENCES


ABDUCTIVE-DEDUCTIVE STRATEGY: HOW TO APPLY IT IN IMPROVING STUDENT MATHEMATICS LITERACY IN JUNIOR HIGH SCHOOL?

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ABSTRACT

Mathematical literacy in this period is one of the important issues in the world of International education in developing countries, including Indonesia. Have been many efforts including the development of learning methods or strategies to improve students’ literacy skills, but still showed unsatisfactory results. Abductive-deductive strategy is an approach to learning that has been developed and applied in order to develop the ability to prove and overcome difficulties in understanding the concept of mathematics essential topics at the level of the student pre-service teacher. The results showed that students pre-service teacher who acquire learning abductive-deductive strategies mathematical reasoning ability have shown better in every essential topic of school mathematics, as well as the proving ability. These are required competencies in mathematical literacy. The next question is “Is this strategy can be applied in mathematics teaching junior high school? How mathematics learning framework with this strategy? How the application of this strategy in a junior high school student’s mathematics learning? How much is the potential to improve students’ mathematical literacy? What is a mathematical literacy? What is the main core of mathematical literacy? What competencies are needed in mathematical literacy? The answers to these questions are the focus of discussion in this article.

Keywords: Strategy, Abductive, Deductive, Mathematics Literacy, Junior High School

1. INTRODUCTION

Recently, we often heard news about socialization and implementation of Curriculum 2013. It is not in spite of government efforts to improve the state of education in Indonesia. Education has an important role in creating quality human beings. Education is also seen as a means to deliver beings who are intelligent, creative, skilled, responsible, productive and virtuous. Various attempts have been made by the government to make innovations in the world of education, for example, complements the learning infrastructure, educational training even until a replacement curriculum. Effort to improve the professionalism of teachers is also done, for example through a scholarship program to teachers and educators for continuing education.

However, these efforts seem to have failed to improve the quality of education the country. One indicator that shows the quality of education in Indonesia is said to be likely to lower the assessment on International student achievement especially mathematics. Research and Development Corporation in 2011 reported the results of a survey Trends International Mathematics and Science Study (TIMSS) in 2003 showed the class VIII student achievement (eight) Indonesia ranks 34 of 45 countries. Although the mean score become 411 compared to 403 in 1999, Indonesia is still below the average for the region. Indonesian student achievement in TIMSS 2007 more worrying, because students achievement’ scores dropped to 309, far lower than the international average score of 500. Indonesia’s performance in TIMSS 2007 was ranked 36 of 49 countries. Even worse results are shown from the results of recent research on the TIMMS 2011 ranked 39 of 43 countries. For further information please see [1], [2].

Not far from the TIMSS survey, the Programme for International Student Assessment (PISA)
achievement of Indonesian children aged about 15 years is still low. In the PISA 2003, Indonesia was ranked 38 of 40 countries, with a mean score of 360. In 2006 the average score rose to 391, which is ranked 50 of 57 countries. Meanwhile, in 2009, Indonesia ranks only 61 of 65 countries, with a mean score of 371, while the average score is 496 International [1], [3].

TIMSS and PISA results are lower on the learning achievement of Indonesian children must be caused by many factors. One is Indonesian students generally lack trained in solving problems with characteristics such as the TIMSS and PISA questions. Ability as measured by the TIMSS and PISA is often understood as literacy skills, or in mathematical terms absorbed as mathematical literacy.

One example of math tested in PISA is as follows:

"A pizzeria serve two round pizzas of the same thinkness in different sizes. The smaller one has a diameter of 30 cm and costs 30 zeds. The larger one has a diameter of 40 cm and costs 40 zeds. Which pizza is better value for money? Show your reasoning."

On the question, students are required to understand the intent of a matter, then is able to calculate the area or size of a pizza, the pizza magnitude obtained with 1 zed price or price per cm² pizza in zed, and then conclude the pizza which is cheaper.

The questions aim is to apply an understanding of the area and value for money through a problem. Of all students in the world who take the test, only 11% answered correctly. It possible cause of this is that students have not been able to make a connection between the topics contained in the question. The topics that contained in the above problem are to calculate the area of a circle, do the arithmetic operations of multiplication and division of whole numbers, and comparing the two fractions. Other possible causes are students less accustomed to the process of mathematical literacy. On the matter of the actual context of the problem seems simple and does not require a high reading ability, but if students are not accustomed to solving problems with the correct stage of the process the students will be likely to have difficulty in solving the problem. This is because, in matters similar to this has been to apply the essential topics that require mathematical literacy.

Therefore, importance discuss about learning strategies to improve mathematics literacy skills in students. In this article will discuss Deductive-Abductive Strategy is an approach to learning that has been developed and applied in order to develop the ability to prove and overcome difficulties in understanding the concept of essential math topics at the level of the students in campus is done by Kusnandi. The results showed that students in campus who acquire learning abductive-deductive strategies have shown mathematical reasoning ability better in every essential topic of school mathematics, as well as the ability to prove. These are required competencies in mathematical literacy. Furthermore, we also present the possibilities of application of this method at the level of junior high school students and how the implementation strategy. This is going to be a major study in this article. But earlier, will be explained about the meaning, the main core, and the competencies in mathematical literacy.

2. LITERATURE REVIEW

In this literature review will be discussed further on mathematical literacy are presented in the form of points question.

2.1. What is a mathematical literacy?

In the initial discussion, Ojose [4] have explained about understanding mathematical literacy. Mathematical literacy is defined simply as the knowledge to know and apply the basic math in every daily. This interpretation is not without reason, but based on the terms described by one of the agencies that measure the literacy skills of The Organization for Economic Corporation and Development [5] which defines mathematical literacy as "an individual’s capacity to identify and understand the role that mathematics plays in the world, to the make well-founded Judgments, and to engage in mathematics in ways that meet the needs of that individual’s current and future life as a constructive, concerned and reflective citizen." From this sense literacy mathematics is defined as an individual’s ability to identify, understand the role of mathematics in the world, making an accurate assessment, and involves the use of mathematics in a variety of ways to meet the needs of the individual as citizen reflective, constructive and dutiful. Based on this definition anyway, mathematical literacy is defined by PPPPTK Mathematics [6] as a person’s ability to formulate, implement and interpret mathematics in a variety of contexts, including the ability to perform mathematical reasoning and use of concepts, procedures, and facts to describe, explain or predict phenomena events. Mathematical literacy helps one to understand the role or usefulness of mathematics in everyday life as well as use them to make the right decisions as citizens who build, care and thought. It is also supported by the definition developed by the National Adult Literacy Study [7], which defines literacy as "using printed and written information in society, to achieve a particular goal and to develop one’s potential." According Kusumah [8], mathematical literacy is ability to formulate a series of questions (problem posing), formulate, solve and interpret problems based on an existing context. It is no different with the opinion Isnaini [9] which defines literacy as the ability of learners to be able to understand the facts, concepts, principles, and solving mathematical operations.
Furthermore, the essence of mathematical literacy is more than just running the procedure. Someone mathematical literacy should be able to estimate, interpret the data, solve everyday problems, making excuses in situations of numerical, graphical, and geometric, and communicate using mathematics. So explicitly, mathematical literacy someone said if you have these abilities

2.2. What is the main core of mathematical literacy?

For a better understanding of mathematical literacy, it is important also to understand the subject of mathematics. Concepts, structures, and mathematical ideas have been created as a tool to organize phenomena in the natural world, social, and mental. In the real world, the mathematical phenomenon occur not come in an organized structure as studied in the school curriculum. Rare real-life problems that arise in the appropriate manner and context understanding and solutions through the application of knowledge of only a single content only. If we look at mathematics as a science that helps us solve the problem, it makes sense to use a phenomenological approach to explain mathematical concepts, structures, and ideas. This approach has been followed by Freudenthal [10] and Steen [11], which states that mathematics is a human activity and should be associated with reality. Steen then advised to develop a deep strength in five mathematical ideas: dimension, quantity, uncertainty, shape, and change. As explained by the PISA mathematics five ideas are divided into four categories which include quantity, space and shape, change and relationship, and the uncertainty of the data.

2.3. What competencies are needed in mathematical literacy?

What competencies are needed in mathematical literacy is described as quoted from the Program for International Students Assessment (PISA) further described by Steen [12], covering eight areas: mathematics thinking and reasoning, mathematical argumentation, mathematical communication, modeling, problem posing and solving, representation, symbolizing, use tools and technology.

Furthermore, as described in PPPPTK Mathematics [6] framework for assessing mathematical literacy in PISA 2012 states that the ability of the process involves seven points as follows.

a. Communication. Mathematical literacy involves the ability to communicate the problem. Someone sees a problem and then challenged to recognize and understand the problem. Model is a very important step to understand, clarify, and to formulate a problem. In the process of finding a solution, while the results may need to be summarized and presented. Furthermore, when the settlement was found, the results also need to be presented to others with an explanation and justification. Communication skill is necessary to be able to present the results of problem solving.

b. Mathematising. Mathematical literacy also involves the ability to transform real-world problems into mathematical form or just the opposite, namely interpreting the results or mathematical models to the original problem. The word ‘mathematising’ is used to describe the activity.

c. Representation. Mathematical literacy involves the ability to restate (representation of) a problem or a mathematical object through things like: selecting, interpreting, translating, and using graphs, tables, pictures, diagrams, formulas, equations, and concrete objects to observe the problem so that more clear.

d. Reasoning and Argument. Mathematical literacy involves the ability to reason and give reasons. This ability is rooted in the ability to think logically to perform an analysis of information to produce a reasoned conclusion.

e. Devising Strategies for Solving Problems. Mathematical literacy involves the ability to use strategies to solve problems. Some problems may be simple and obvious solution strategy, but there is also a problem that needs solving strategy is quite complicated.

f. Using Symbolic, Formal and Technical Language and Operation. Mathematical literacy involves the ability with the language of symbols, formal language and technical language.

g. Using Mathematics Tools. Mathematical literacy involves the ability to use mathematical tools, such as measurement, operations and so on.

3. RESEARCH METHOD

This article draws on the results of a literature review and a review of relevant research. The method used consists of four things, namely problem formulation, data collection, discussion, and conclusion and suggestions. Problem formulation is needed in order to issues discussed in this article to be clear and wide. In collecting the data, the technique used consists of two things: literature study and documentation. Literature study performed by searching the literature sources relevant to the issues in the form of books, articles and so on then studied and reflected in the literature review. While documentation with data collection is closely related to documents originating from the records, magazines, newspapers, bulletins relevant scientific. Discussion of the activities carried out by the theoretical approach based on the literature study. The process of analysis and synthesis of data is done in the writing of this article includes data reduction and data presentation. Data reduction is done by selecting focusing, simplifying and abstracting the data that have been obtained. The next presentation of data is done by compiling information on the results of the data reduction phase is complete and then present both the data obtained from the literature. At this stage of conclusions and
suggestions, the author uses an induction technique is based on the description in the discussion. Based on the discussion as well, the authors formulate some suggestions to make recommendations that enable the development and research carried out in the next period.

4. RESULTS AND DISCUSSION

Mathematical literacy (PPPPTK Mathematics, in [6]) is defined as one’s ability to formulate, implement and interpret mathematics in a variety of contexts, including the ability to perform mathematical reasoning and use of concepts, procedures, and facts to describe, explain or predict phenomena or events. Mathematical literacy helps one to understand the role or usefulness of mathematics in everyday life as well as use them to make the right decisions as citizens of a building, care and thought.

Mathematical literacy problem as can be seen from the poor ranking of Indonesia is the responsibility of every element of education to fix it. This issue has also been the subject of study PPPPTK (Centre for Development and Empowerment of Teachers and Education Personnel) program through the Math Composer (Better Education through Reformed Management and Universal Teacher Upgrading) to deal with this education (PPPPTK Matematics, in [6]). However, the program was limited to the professional development of teachers, not the students directly. Surely this is a distinctive gap. Despite this factor of teacher quality has become one of the decisive factors in addressing this issue. Reflecting on some of the top-ranked countries in mathematical literacy of students such as Finland, Japan and the United States did so to give special attention to the quality of its teachers. Quality teachers here including the ability to explain, manage classes, the ability to create a climate conducive to learning, and learning to do variations. Learning mathematics with Abductive-Deductive strategy has been proven in research that has been applied in developing the ability to prove and overcome difficulties in understanding the concept of essential math topics at the level of the students. Results of research conducted by Kusnandi [13] showed that students acquire learning strategies have demonstrated abductive-deductive mathematical reasoning skills are better in every essential topic of school mathematics, as well as the ability to prove [14]. These are required competencies in mathematical literacy.

4.1. How mathematical learning framework with this strategy?

Based on preliminary analysis of the difficulties of understanding the essential concepts or mathematical literacy as described above, it is necessary to develop a learning model that can improve the understanding of the essential concepts. As a general framework in solving a problem in mathematics is the ability to identify the given facts (data), and formulate what is asked in the problem (the final target). In order to determine the final target is based on data provided, it is necessary to elaborate the ability to apply the essential concepts that are relevant to the given data, to gain the between target, before finding the answer to the final target. Not a few problems in mathematics can be more easily solved by adding the ability to formulate a condition (the between target) that is based on a concept relevant essential to arrive at the final target in question.

General framework as described above has been developed Kusnandi [13] in a model of learning by abductive-deductive strategy or in Indonesian Language be known as Pembelajaran dengan Strategi Abductive-Deduktif (PSAD) to develop the ability to prove the beginner student in campus learning of proof. PSAD framework that presents the not formal proof is very suitable for student teachers, and the results showed that the student teachers who learn with PSAD have the proving ability better than students studying expository. PSAD framework has been studied theoretically by Kusnandi [14] for the applicability of the proving problem is more abstract in the subject field of real analysis and abstract algebra. So expect student unpreparedness in dealing with the subject demands higher mathematical skills such that real analysis can be overcome [15].

One lesson that can be done to help overcome difficulties in students’ mathematical literacy is the process of learning abductive-deductive strategy. This strategy is a learning strategy that starts by presenting the problem to the students. Then, they are required to be able to elaborate on any information or facts provided. Through this strategy, given the problems that has to be delivered students to comprehend mathematical objects and connection between the mathematical objects with other objects. Teachers encourage students to do transactive reasoning as to criticize, explain, clarify, justify and elaborate on an idea put forward, whether initiated by students or teachers. To be involved in transactive discussion, early math skills of students plays a very important. An idea which appears can develop gradually so as to build a comprehensive mathematical concepts from information obtained.
Within the framework of the theory of Action Process Object Schema (APOS), abductive-deductive strategy can be illustrated as in Figure 1. Operationally of proving problems can be reduced become a problem of how to demonstrate the truth of expected the final target based on the information provided in the data. Data and the final target are two mental objects that were introduced to the students. In general, there are two actions that can be done when dealing directly with the problem of proof. First, analyze any information provided in the data, and then compile them to produce between targets, and these targets from longer synthesized so as to obtain the next targets, and so on. The between targets are another mental object that may have been previously owned by the student.

The process of obtaining the between target the data provided as it is a deductive process in PSAD. While the second action is to analyze the expected final target, and formulate a between target that is based on a certain rule (definition or theorem) will arrive at the final target. Condition process from the between target to the final target is the abductive process in PSAD. Other stages in the process of PSAD are doing mental actions so as to bridge the between target of deductive process results with a between target of abductive process results. Because this process is crucial in the proof of success in mathematics, then this process will be named with the key processes.

In the learning process, the intervention teachers in encouraging students to perform mental actions so that they can perform the above three processes is crucial. Transactive expressions and facilitative or scaffolding techniques can be applied so that transactive discussion can occur leading to the formation of new mental objects primarily related to an increase in the ability to prove.

The constructed process of proofing scheme that is built through abductive and deductive process as above will continue to evolve in line with the level of complexity of the interaction between mental objects of proofing problems presented. The development of this scheme will lead to the actual development and potential student progression to higher levels.

Not a few problems in mathematics are very difficult to be solved with a deductive process (start from the premises that given to obtain the final target is based on the rules that have been vouched for). As an example consider the following example [16]:

Show that $3a^2-1$ integers never form a perfect square for any integer $a$!

Note the following sequence of integers.
$11, 111, 1111, 11111, ...$

Prove that no integer in the row which is the perfect square number?

ABCD is a square and a shaded area formed by the four circles centered at A, B, C, and D. If the area of the square area is 4 area unit, find specify the area being shaded!

However, the difficulty can be reduced by formulating a condition most likely to be the final target can be achieved. The formulation of the conditions most likely to bring up an idea of what to do from premise faced. Such a thought process called abductive thinking process.
Abductive thinking is the process of formulating information with the best possible condition so that the conclusions are expected to be achieved through a rule that can be accounted for righteousness.

Abductive thinking process the thought above can be mathematically formulated as follows: Suppose Q is the conclusion/ final target is expected, and we look for a rule that is guaranteed truth that “if P then Q”. The condition is most likely that the conclusions (target Q) can be achieved is the target we must condition P. Thus, the mathematical formulation is

\[ Q \rightarrow P \]

The condition most likely is P.

Abductive thinking process is not a process of conclusion. It is only one way that the target is expected to be achieved. So that the expression used is “condition most likely”, because it is possible there are other conditions that can allow it to reach targets can be accounted for by the rules. Framework abductive thinking process is presented as

\[
\begin{align*}
Q & \Rightarrow P \\
\text{condition} & \text{most likely is } P
\end{align*}
\]

Figure 2. Framework abductive thinking process

To understand the thought process of abductive framework above, the following will be presented some illustrations.

We have introduced what deductive process framework to show that the set C(G) is a subgroup of the group G by using rule 2, as follows.

\[
C(G) \text{ satisfy (i) and (ii)}
\]

Figure 3. Deductive process framework to show that the set C(G)

Formulation “C(G) satisfy (i) and (ii)” may be the most likely condition that the ultimate target “C(G) is the subgroup” can be achieved. But the formula is not the only condition to reach the final target. Other conditions can be seen from rule 1 and rule 3. Next, we will look at the formula “C(G) satisfy (i)” as a target. The condition is most likely that the target is reached is presented below.
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Figure 4. Condition is most likely that the target Formulation "\(ab \in C(G)\); \(a, b \in C(G)\)" is the condition most likely to target “\(C(G) \text{ satisfy (i)}\)”. While formula “\((ab)x = x(ab) \; \forall x \in C(G)\)” a condition most likely to target “\(ab \in C(G); a, b \in C(G)\)".

Figure 5. Condition for \(ab \in C(G); a, b \in C(G)\)

The condition is most likely “\((ab)x = x(ab)\); \(x \in C(G)\)” as a result of the aductive process is supposed to bring inspiration how to develop “\(a, b \in C(G)\)” in the direction of the between target. At least four methods can be passed to the target’s direction. Thus, the trajectory to reach the target “\(C(G) \text{ satisfy (i)}\)” is presented below.

Figure 6. The trajectory to reach the target “\(C(G) \text{ satisfy (i)}\)”

Then, we will look at the formula “\(C(G) \text{ satisfy (ii)}\)” as a target. The condition is most likely that the target is reached is presented below.
Formulation of “$a^{-1}x = xa^{-1}; \ x \in G$” as a result of this abductive process can bring inspiration how to develop “$a \in C(G)$” in the direction of the target. Least, there are 3 methods that can be traversed in the direction of the target.

Therefore, the structure of arguments proving that the set $C(G)$ is a subgroup of the group $G$ are as follows:
Figure 9. Complete process arguments proving that the set $C(G)$ is a subgroup of the group $G$

Now we consider again the widespread problems in the area of circles.

**ABCD is a square and a shaded area formed by the four circles centered at A, B, C, and D. If the area of the square area is 4 area unit, find specify the area being shaded!**

![Figure 10. Problem (2)](image)

We will apply abductive thinking process to determine the area of the shaded region I. So that the target of the area I can be determined, then we must have area II. While the area II can be find if the area III is known. Therefore, abductive process to calculate the area I is as follows.

![Figure 11. Abductive thinking process for problem (2)](image)
Now we look at the area III (L III):
\[ L III = L BCD - L BCP = L BCD - (L ABC - L ABP) = L BCD - (L ABC - (2L ABC - L ΔABP)) = (4 - π) - \left[ π - (2 \cdot \sqrt{3} - 4) \right] = 4 - \sqrt{3} - \frac{π}{2} \]

From here we obtain that
\[ L II = L ACD - 2L III = (4 - π) - (8 - 2\sqrt{3} - \frac{π}{2}) = \frac{π}{2} + 2\sqrt{3} - 4. \]

Thus, extensive shaded area is
\[ L I = L CDB - 2L II = (2π - 4) - 2\left(\frac{π}{2} + 2\sqrt{3} - 4\right) = \frac{π}{2} + 4 - 2\sqrt{3}. \]

4.2. Whether this strategy can be applied in learning mathematics junior high school student?

This is the next question arises in our minds. In this discussion, PSAD framework will be adapted to the essential topics in high school mathematics with aspects of the ability mathematical competencies that mathematical literacy. In simple terms there are two things that will be a gap that previously measured learning ability with this strategy is the mathematical reasoning competencies and proving ability become mathematical literacy competencies measured more. Furthermore, research objectives have also changed, from student in campus to student in high school, more particularly the junior high school that base age level and maturity of the development of cognition may be said to be lower. In this case we will discuss about the differences prior research objectives.

Starting from the fact of previous studies, Indonesian students’ weak math skills can not only be seen from the results of TIMSS and PISA survey, but also can be seen from the research and survey conducted by educator in Indonesia. Based on a survey conducted Suryadi [17] in the city of Bandung, Yogyakarta and Malang obtained information that most students have difficulty in proving theorems, use mathematical reasoning to problem solving, process generalize, settlement issues geometry, mathematical modeling, as well as the discovery of relationships among the known data. Furthermore Suryadi revealed that for most teachers, problem-solving activities is one of the activities that are considered difficult to teach to students.

In this case obtained a finding that the difficulties experienced by students in the proof of the theorem, use mathematical reasoning to problem solving, process generalize, settlement issues geometry, mathematical modeling, as well as the discovery of relationships among the data. Abilities as measured above are nothing but a mathematical literacy. As it is known that the above mathematical literacy include: communication, mathematising, representation, reasoning and argument, devising strategies for problem solving, using symbolic, formal and technical language and operation, using mathematics tools.

Although the study of student difficulties occur in mathematical literacy, but also the difficulties that a bright spot that students at the middle school level already has the basic skills of literacy metematis, such as mathematical connections and mathematical reasoning on which the abductive-deductive strategy. This is the basis that in terms of basic skills students, learning with abductive-deductive strategy can be performed. In this case, differences in the ability of the research objectives are not a problem anymore.

As disclosed in the theory of learning, that learning is an activity which proceeds, of course happening in it changes gradually. Such changes arise through stages from each other in sequence and functionally related. In the concept of discovery learning by Jerome Bruner in [18], there are three steps to be taken by students, namely: information stage (stage receiving material), phase transformations (phase change material) and phase evaluation (assessment phase material). Of the three stages of the concept of the invention Jerome Bruner are interrelated.

In the learning process, the third stage always found. What matters is how much information is needed in order to be transformed. Long each stage is not always the same. It is, among others, also depending on the desired results, student motivation, learning, interest, desire to learn and a stimulus to find alone. This concept also explains that the principle of learning should pay attention to changes in the internal conditions that occurred during the students are given in-class learning experience. Experience must be provided in discovery learning which allows learners to acquire new information and skills from previous lessons.

Educational theorists such as Bruner and Piaget have emphasized the importance of student involvement in the learning process. Constructivist view of learning rejected the idea that students could effectively acquire new knowledge by absorbing the information provided by the teacher. Students should become active seekers and processors of information, not passive recipients (Schunk, [20]; Davis & Murrell, [21]). In other words, students are given the opportunity to learn independently and connect the concepts that
have been previously owned, and become involved in meaningful learning. It is therefore necessary to create a condition that allows the child to learn self-learning (Rifa’i [22]). Meaningful learning to occur, students should also be able to construct their own knowledge to assimilate and integrate new concepts. Constructivism teaching approach “puts students in the driver seat” (Perkins [23]) and emphasizes the importance of the active involvement of students.

Jerome Brunner as quoted by Suherman [24] revealed that “in the process of learning the child should be given the opportunity to manipulate objects (props)”. Through props students will see first hand how the regularities and patterns in the structure of objects that are being considered. In other words objects displayed are concrete objects. However there are a few events and objects that can not be carried in the classroom. So to keep teaching the knowledge, concepts, skills, and understanding needed by students about a problem fixed maximum, where the presentation of instructional CDs and LKPD can be very instrumental.

According Suherman [24], media is the plural of the word medium means a channel for communication. The use of media is one attempt to provide variety in learning activities. Learning media can also provide a lot of benefits of which will make learning more interesting for students. With the media to learn the material being taught to be more cryptic so indirectly learners become more widely studied because it does not just listen to a description of the teacher.

Students Worksheet (LKPD) is the print media in the form of a sheet of paper containing information or questions or questions that must be answered learners (Suyitno [25]). LKPD a good medium to engage learners in the learning process. LKPD usage is not matched by the delivery of a mature concept will make the students understanding of the material is weak. Learning outcomes are achieved with more LKPD temporary. Learners easily forget the material that has been obtained, although at the time the test results are good.

According Rustiyah [26], which is a kind of teaching media LKPD have several benefits for learners, among others: (1) increase or enhance learners’ attention, (2) prevent verbal, (3) help develop a systematic understanding; (4) develop explorative attitudes, and (5) can be oriented directly to the environment and to give unity to the comment. Problem is packaged in LKPD in the form of research is a matter of description. According Arikunto [27], merits a description about the form, among others, provide opportunities for learners to express purpose with its own language style and determine the extent of understanding of the issues tested.

If viewed from the thinking stages of children, as described by Brunner, that children with more than 8 years of age has been entered in the developmental stage symbolic. This stage is the stage of development where children already understand the symbols and concepts such as language and symbols to represent numbers. Communication is done by using a lot of system symbols. The more mature a person in his thinking process, the dominant symbol system. So that, children with age in junior or senior high school has been enter in this period. Even so does not mean he is no longer using the system enaktif and iconic. The use of media in learning activities is one proof still needed enaktif and iconic in the learning process. From the description of the meaning and potential in the learning process must be supported with media facilities, it is possible to apply learning abductive-deductive strategy for junior high school and senior high school students.

4.3. How the application of this strategy in a junior high school students’ mathematics learning?

This strategy is a learning strategy that starts by presenting the problem to the students then they are required to be able to elaborate on any information or facts provided. Through this strategy, given the problems that has to be delivered students to comprehend mathematical objects and connect between the mathematical objects with other objects. Teachers encourage students to do transactive reasoning as to criticize, explain, clarify, justify and elaborate on an idea put forward, whether initiated by students and teachers.

From the description of learning abductive-deductive strategy, the authors develop a customized syntax to support the implementation of learning based learning problem solving. The syntax is as follows developed.
Table 1. Syntax learning mathematics with abductive-Deductive Strategy

<table>
<thead>
<tr>
<th>Phase</th>
<th>Teacher Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>Providing orientation about the problem to students. Teacher discusses the learning objectives, describe various important logistics needs, and motivate students to engage in activities to overcome the problem.</td>
</tr>
<tr>
<td>Phase 2</td>
<td>Organizing students to examine or understand the problem and plan a solution. Teacher helps students to define and organize learning tasks related to the problem.</td>
</tr>
<tr>
<td>Phase 3</td>
<td>Helping independent investigation or group Teachers encourage students to do transactive reasoning as to criticize, explain, clarify, justify and elaborate on an idea put forward, whether initiated by students and teachers.</td>
</tr>
<tr>
<td>Phase 4</td>
<td>Developing and presenting findings. Teachers assist students in planning and preparing materials for presentations and discussions.</td>
</tr>
<tr>
<td>Phase 5</td>
<td>Generalizing the findings obtain Teachers help generalize the findings obtained.</td>
</tr>
<tr>
<td>Phase 6</td>
<td>Conducting discussions of strategy thought processes to more problems Teachers assist students in finding strategies to the problems that much more.</td>
</tr>
<tr>
<td>Phase 7</td>
<td>Implement training and test evaluation Teachers provide training and test evaluation.</td>
</tr>
</tbody>
</table>

Furthermore, answering questions about: “How is the potential to increase students’ mathematical literacy?” Certainly, needs further research studies to support this study of theory. But the study of theory based on the above discussion, the potential application of learning with abductive-deductive strategy capable of providing a great impact in improving the mathematical literacy skills of students in middle school, junior high school or senior high school either.

5. CONCLUSIONS

As explained in the introduction about the purpose of this article which is the study of how the application of abductive-deductive strategies in mathematics learning to improve mathematical literacy in the junior high school students. Based on the discussion made above, it can be concluded that the implementation of this strategy for junior and senior high school students is feasible. The extent of a learning model with abductive-deductive strategy is the improvement of mathematical literacy of students must not only be seen from a review of theory. Further research is needed, especially in its application to middle school students, both at the junior and senior high school. However, based on the above discussion, the application of learning with this strategy it is possible to effectively in improving students’ mathematical literacy.

Furthermore, it can be recommended for further research is the application of learning with abductive-deductive strategy at the level of junior or senior high school students with coverage of mathematical literacy skills, such as: problem solving skills, mathematical reasoning, mathematical connections, verification, modeling, to the use of instructional media. In fact, research also needs to be done as well in terms of student acceptance or disposition of student attitudes toward these learning strategies, such as: the level of persistence, self-confidence, perseverance, curiosity, inventiveness and appreciation the role of mathematics in culture and value, mathematics as a tool, and as a language.

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USING METAPHORICAL THINKING APPROACH IN ENHANCING JUNIOR HIGH SCHOOL STUDENTS’ MATHEMATICAL REASONING

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ABSTRACT

The aim of this research is to examine junior high school students’ mathematical reasoning enhancement by metaphorical thinking approach. The research utilized a quasi experimental design. The population in this research was students in eight grader students of a junior high school in East Jakarta. The sample consists of 27 students in control group and 31 students in experiment group. The research comprises instrument mathematical reasoning test. The quantitative analysis used a two-way ANOVA, while qualitative analysis used a descriptive one. The result of qualitative analysis indicates the students were either able to metaphor a concept or to find connection between concepts. Furthermore, the result of quantitative analysis indicates higher enhancement mathematical reasoning ability under metaphorical thinking approach compared to conventional teaching. There is no interaction effect between students’ prior mathematical ability and teaching approach towards students’ mathematical reasoning.

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1. INTRODUCTION

Liquid waste is the waste water from tofu production. According (1) generally organic content in the liquid waste, is a protein (40% - 60%), carbohydrates (25% - 50%), and fat (10%). Looking at the content of organic matter, the liquid waste can be used as a nutrient source for the growth of microorganisms. In addition, the liquid waste is very easy to obtain, because the majority of manufacturers will discard the liquid waste through aquatic ecosystems that located around the industrial area.

The problems related to reasoning ability have to be acquainted as early as possible, including some problems in mathematics subject. This is based on [1], who stated that reasoning can help children see that mathematics is logical and supposed to make sense, and also can foster children’s belief that mathematics is something they can comprehend, think thoroughly, justify and evaluate. This statement is in line with one of mathematics teaching and learning objectives [3], namely to use reasoning toward mathematical pattern and trait, to manipulate mathematics in generalization, compiling proof, and explaining mathematical ideas and statement. Besides, [4] in Assessment Frameworks were revealing that there are three mathematics cognitive domain, namely knowing, applying, and reasoning. In addition, Mullis, et al. [4] proposed an argued reasoning is included in mathematics cognitive domains because reasoning mathematically involves the capacity for logical, systematic thinking, and also includes intuitive and inductive reasoning based on pattern and regularities that can be used to arrive at solutions to non-routine problems, that requiring reasoning may do in different ways because any solution to the problem must involve several steps, perhaps drawing on knowledge and understanding from different areas of mathematics.

In fact, the importance of reasoning ability is not in line with students’ achievement now a days. The results are indicated in some previous studies ([7], [9], and [11]). Based on their studies, score postest of reasoning ability that students achieved less than 60% from ideal score. Besides, the result of TIMSS survey on 2011 [5], indicated that the reasoning ability of junior high school students in Indonesia, particularly grade eight, were below average, that was about 17% students who answered correctly, whereas international average showed 30%.

One of the factors which result in situition above is implementation of the approach not suitable for students. [2] stated that choosing teaching strategy and managing learning environment have significant
Influence for success in mathematics subject. Furthermore, there are three important things if someone wants to be successful in mathematics learning. One of them is knowledge how mathematics is created, that cannot be separated from mathematical characteristic. Basic mathematics are abstract concepts. This contradicts to meaningful mathematics teaching. [10] stated that in mathematics teaching and learning, students should be invited in daily contexts because it can make an impression that mathematics is useful in daily life.

Metaphorical thinking approach is teaching approach for understanding and explaining abstracts to concrete concepts through visualisation and analogy. There are two elements in metaphorical thinking. The first element is conceptual metaphors which are in fact fundamental cognitive mechanisms which project the inferential structure of a source domain onto a target domain [6]. In addition, Lakoff and Nunez [6] distinguish, three important types of conceptual metaphors: grounding metaphors; which are ground our understanding of mathematical ideas in terms of everyday experience, redefinitional metaphors; which are metaphors that impose a technical understanding replacing ordinary concept, linking metaphors; which are metaphors within mathematics itself that allow us to conceptualize one mathematical domain in terms of another mathematical domain. The second element is images schemas which are basic dynamic topological and orientation structure that characterizes spatial inferences and link language to visual-motor experience [6]. Through gesture, students can express unspeakable terms correctly. Based from the explanation previously, it can be concluded that metaphorical thinking approach plays important role in learning teaching process, because metaphors are part of daily life. Through process of metaphors, students are trained to see the relationship between their own knowledge or experience and the new knowledge in their daily life. Students are also trained how to analogy a model and interpret the knowledge which they build.

Besides the implementation of metaphorical thinking approach and mathematical reasoning ability that will be measured, there is one thing to be considered in learning process, which is Prior Mathematical Ability (PMA). This is because mathematics is hierarchy science and interconnected concepts. Students are expected to connect between their own knowledge and the new knowledge, so that it happens meaningful learning process.

Based on the arguments and explanations previously, it motivates researcher to conduct an experiment by using metaphorical thinking approach in enhancing junior high school students' mathematical reasoning ability. Thus, the purposes of this study are to examine junior high school students' mathematical reasoning enhancement between experiment group and control one, and also to find whether or not interaction effect between students' prior mathematical ability and teaching approach towards students' mathematical reasoning.

### 2. RESEARCH METHOD

This study utilized Quasi Experimental using Nonequivalent Control Group Design. It involved 58 grade-8 students in a junior high school in East Jakarta. The instrument in this study is mathematical reasoning which consisted of 6 items. There are three indicators in mathematical reasoning instrument, namely analogy, generalization and do computing utilized certain rules or formulas.

Before experiment was conducted and data were analysed, students were classified according to rule as in Table 1.

<table>
<thead>
<tr>
<th>Classification</th>
<th>PMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>PMA ≥ 80.29</td>
</tr>
<tr>
<td>Medium</td>
<td>74.84 ≤ PMA &lt; 80.29</td>
</tr>
<tr>
<td>Low</td>
<td>PMA &lt; 74.84</td>
</tr>
</tbody>
</table>
In the following figure, we present sample of instrument of this study.

Example 1: Analogy
Observe this figure carefully!

3-dimensional shape in the left had relationship with this 2-dimensional shape below.

[Figure showing 3D and 2D shapes]

Similar to

Draw 2-dimensional shape had relationship with this 3-dimensional shape in the right!

Give your reason!

Example 2: Generalization
Observe this figure carefully!

[Figure showing 2D and 3D shapes]

If length, width, and height of small box in the 1st pattern are 20 cm, 10 cm, and 15 cm respectively, then find the length of ribbon needed to tie the surrounding area of box on 7th and 10th pattern. Can you find the formula for n-th pattern? Explain your answer!

3. RESULT AND ANALYSIS
The two-way ANOVA test was used to determine whether or not a difference mathematical reasoning ability between experiment and control group, and also to determine whether or not interaction effect between students’ prior mathematical ability and teaching approach towards students’ mathematical reasoning. The results of two-way ANOVA are illustrated in table 2.

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
<th>H₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>1.702*</td>
<td>5</td>
<td>.340</td>
<td>7.545</td>
<td>.000</td>
<td>-</td>
</tr>
<tr>
<td>Intercept</td>
<td>18.206</td>
<td>1</td>
<td>18.206</td>
<td>403.422</td>
<td>.000</td>
<td>-</td>
</tr>
<tr>
<td>PAM</td>
<td>.064</td>
<td>2</td>
<td>.032</td>
<td>.704</td>
<td>.499</td>
<td>Accepted</td>
</tr>
<tr>
<td>Approach</td>
<td>1.489</td>
<td>1</td>
<td>1.489</td>
<td>33.002</td>
<td>.000</td>
<td>Rejected</td>
</tr>
<tr>
<td>PAM * Approach</td>
<td>.118</td>
<td>2</td>
<td>.059</td>
<td>1.305</td>
<td>.280</td>
<td>Accepted</td>
</tr>
<tr>
<td>Error</td>
<td>2.347</td>
<td>52</td>
<td>.045</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23.491</td>
<td>58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>4.049</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. R Squared = .420 (Adjusted R Squared = .365)

Table 2 reveals that there is difference mathematical reasoning ability between experiment and control group, so it can be concluded that the metaphorical thinking approach ables to enhance students’ mathematical reasoning ability than conventional one. This is assumed because of the process of metaphors,
through metaphors students trained to see the relationship between their own knowledge or experience and the new knowledge in their daily life, so students more understand interrelated concepts. This is in line with statement of Davis [8], metaphors allow students to work with abstract ideas by mapping them strongly.

Based on table 2, it obvious that there is no interaction effect between students’ prior mathematical ability and teaching approach towards students’ mathematical reasoning, or in other words, implementation of metaphorical thinking approach can be implemented in all cognitive level. This is assumed because of using of metaphors which relates students’ daily life. This phase, students discussion either in small or class group, then based from the result of discussion, they analyse which one of the correct metaphors, so all students able to understand what they have been learned. The graphics of interaction effect can be illustrated in figure 1.

4. CONCLUSIONS
The results indicate that the higher enhancement mathematical reasoning ability under metaphorical thinking approach compare to conventional teaching. It means that metaphorical thinking approach has a good contribution towards students’ mathematical reasoning. Based on the results, we also found that there is no interaction effect between students’ prior mathematical ability and teaching approach towards student’s mathematical reasoning, or in other words, the implementation of metaphorical thinking approach can be implemented in all cognitive level.

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THE DEVELOPMENT OF MATHEMATICS WORKSHEET BASED ON CRITICAL ACTIVITIES FOR YUNIOR HIGH SCHOOL STUDENTS

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ABSTRACT
Liquid waste of tofu can be used as an alternatives medium in the production of protease. This research aims to develop the mathematic worksheet. This research focuses on: (1) the development of the mathematic worksheet based on critical activities; (2) the difference between the critical thinking skills of students before and after using the mathematic worksheet based on critical activities; (3) the attitude of students towards learning by using the mathematic work sheet based on critical activities. This research uses ADDIE model which consists of 5 steps: Analysis, Design, Development, Implementation and Evaluation. The instrument used in this study were: (1) validation sheet and questionnaire test on a limited scale; (2) pretest and posttest question; (3) the scale of the students’ attitude. The results of this study are as follows: (1) based on the validation result of the experts’ test which consists of the teaching-learning aspect, the substance of the material, linguistic and graphic, the average score of 4,25 is obtained, which is in good category, while the validation result of the limited scale test which consists of the ease usage aspect, display and features, the average score of 4,34 is obtained, which is in good category, so that the mathematic worksheet are good and the product is ready to be used in schools; (2) there are differences in students' critical thinking skills before and after using the mathematic worksheet based on critical activities; (3) based on the attitude scale data analysis, it is found that most students show positive attitude towards learning by using the mathematic worksheet based on critical activities.

1. INTRODUCTION
Teaching material is an important part of the learning process. Teaching materials are materials or subject matter systematically arranged, used by teachers and learners in the learning process. (Pannen in Prastowoto, 2011). Teaching materials can be either written or oral material. One of the teaching materials that can be developed in the teaching-learning are students' worksheet. There are many reasons for developing students' worksheet, those are the students' worksheet generally is only train students to answer the question or there is only a matter clustering the students' worksheet, and provides summary of the material also it does not require students to find their own concepts learned through process skills activities, as well as the availability of teaching materials appropriate to the curriculum.

Students’ worksheet is one alternative teaching material appropriate for students because the student activity sheet helps students to add information about the concepts is learned through systematic learning activities (Suwito, 1997). In general guide development of teaching materials (Organization of Education, 2004), students’ worksheet are sheets contains a task that have to be done by learners. Students' worksheet can be a guide for the development of cognitive practice and guidance for the development of all aspects of learning in the form of guides experiments and demonstration experiments (Trianto, 2011). Therefore teachers need to develop students' worksheet that can develop students' thinking skills, including critical thinking skills.

Critical thinking is a process of systematic, organized, which allows students to evaluate evidence, assumptions, language and logic underlying statements of others in order to achieve a profound understanding (Johnson, 2012). Critical thinking is reasoned and reflective thinking with an emphasis on making decisions about what has to be believed or be done. Therefore, indicators of critical thinking can be derived from students' critical activity (Ennis in Mulyana, 2008). In this study to develop teaching materials in the form of
mathematic worksheets based on critical activities using ADDIE Model (Analysis, Design, Development, Implementation, and Evaluation).

2. RESEARCH METHOD

The method used in this research is a method of research and development (R&D). In this study, the mathematic worksheets based on critical activities were developed by using the ADDIE Model (Analysis, Design, Development, Implementation, Evaluations).

In the analysis phase, the researcher does needs analysis, curriculum analysis, and analysis of student characteristics. In the design phase, the researcher obtained content design, in the development phase, the preparation of the mathematic worksheets and expert testing. In the implementation phase, the teaching-learning process was determined, and the effectiveness of the mathematic worksheets based on critical activities was evaluated. The results of the evaluation phase included the implementation of the mathematic worksheets based on critical activities.

The data was obtained from this study is quantitative data and qualitative data. Quantitative data in the form of: 1) the percentage of the results of expert validation, test and attitude scale limited scale, 2) value test students' critical thinking skills obtained from the written test or essay in narrative form by 5 questions. Descriptive qualitative data in the form of mathematic worksheets based on critical activities and students' attitudes towards the mathematic worksheets based on critical activities.

3. RESULT OF DEVELOPMENT AND DISCUSSION

3.1. Overview Development of Mathematic Worksheets Based on Critical Activities Using ADDIE Model

The analysis phase includes analysis of curriculum, analysis of student characteristics, and analysis of teaching materials. Curriculum analysis was done by the literature study include: analysis of the subject matter, competence and basic competences. Characteristic of student analysis is to identify the characteristics of students who will use the mathematic worksheets based on critical activities that is student. Analysis of teaching materials is done with analysis teaching material whatever is used in studying mathematic at school which will be an object of research. At the design phase, the researcher did design of outline the contents of the mathematic worksheets based on critical activities, which consists of four mathematic worksheets such as the surface area of cubes and blocks, cubes and blocks volume, surface area of the prism and pyramid, and pyramid volume. At this stage of development, there are three activities performed after the preparation of the mathematic worksheets based on critical activities that are teaching and learning, testing, and product revision.

Based on the results of the validation by expert lecturers and teachers of mathematics and the trial is limited to the 10th graders at a good qualification. So that the actual product is ready to be used for activities in the field of learning not revised. Results of analysis of expert validation and limited test presented in Table 1 and 2.

<table>
<thead>
<tr>
<th>No</th>
<th>Aspect and Indicator</th>
<th>Experts E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Suitability of the material with Competency Standards and Basic Competence</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>Suitability of the material with needs of students</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>Relevance between evaluations and learning objectives</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>Material truth in theory and concepts</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>Correct use of the term appropriate scientific field</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>Depth of the material</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>Conformity with the order of the material, the students' ability level</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>17</td>
</tr>
</tbody>
</table>
Based on table 1, which presents the results of the experts’ validation of teaching learning aspect, substance of the material, linguistic, and graphic, the average score of 4.25 is obtained, which is in the good category.

**Table 2. The Result of Questionnaire Limited Trial**

<table>
<thead>
<tr>
<th>No</th>
<th>Aspect and Indicator</th>
<th>Student S1</th>
<th>Student S2</th>
<th>Student S3</th>
<th>Student S4</th>
<th>Student S5</th>
<th>Student S6</th>
<th>Student S7</th>
<th>Student S8</th>
<th>Student S9</th>
<th>Student S10</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ease of use</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>Mathematical problems and solutions presented in mathematics worksheets</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>41</td>
</tr>
<tr>
<td>3</td>
<td>Display</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>44</td>
</tr>
<tr>
<td>4</td>
<td>Writing and images</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>5</td>
<td>Features</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>43</td>
</tr>
<tr>
<td>6</td>
<td>The images in the “find out” easily understood</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>38</td>
</tr>
<tr>
<td>7</td>
<td>Concepts discovery</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Number</td>
<td>304</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>4.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
category: 5 = very good, 4 = good, 3 = enough, 2 = not good, 1 = is not very good

Based in table 2 shows that validation result of the limited scale test which consists of the ease usage aspect, dispaly and features, the average score of 4,34 is obtained, which is in good category, so that the mathematic worksheets based on critical activities are good and the product is ready to be used in schools In the implementation phase, the learning process is carried out to determine the effectiveness of student mathematic worksheets based on critical activities who becomes the object of this study was eighth grade students of SMP Negeri 17 in Bandung.

In this research, meeting six times that at first meeting conducted a pretest, the second meeting up with the fifth meeting of the teaching-learning process is carried out using mathematics worksheets based on critical activities, where as the sixth meeting did post test. This study was conducted on May 3, 2013 until May 21, 2013. At this stage of the evaluation, revision of mathematic worksheets based on observations during the teaching-learning process. It can be seen from the attitude scale question are given at the end of the teaching-learning process using mathematics worksheets. Revision of the results obtained for the final product in the form of mathematics worksheets based on critical activities for junior high school students of class VIII semester.

3.2. Critical Thinking Ability Differences Before and After Using Mathematic Worksheets Based on Critical Activities.

Descriptive data about students' critical thinking ability before and after use mathematic worksheet based on critical activity can be seen in Table 3.

Tabel 3. Descriptive Data of Students' Critical Thinking Ability

<table>
<thead>
<tr>
<th></th>
<th>Using Mathematic Worksheets Based on Critical Activities</th>
<th>Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>Mean</td>
<td>30,3871</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>30,0000</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>79,712</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>8,92815</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>10,00</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>50,00</td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>40,00</td>
</tr>
<tr>
<td>After</td>
<td>Mean</td>
<td>78,8065</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>80,0000</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>94,828</td>
</tr>
<tr>
<td></td>
<td>Std. Deviation</td>
<td>9,73796</td>
</tr>
<tr>
<td></td>
<td>Minimum</td>
<td>60,00</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>100,00</td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>40,00</td>
</tr>
</tbody>
</table>

Furthermore, to see whether there is a difference in students' critical thinking skills before and after use mathematic worksheets based on critical activities, the mean difference in the two tests used in this case paired sample test. The hypothesis is:

\( H_0 \) = There were no differences between the students' critical thinking skills before and after using mathematic worksheets based on critical activities

\( H_1 \) = There are differences between the students' critical thinking skills before and after using mathematic worksheets based on critical activities

Calculation results can be seen in Table 4.
Tabel 4. Paired Samples Test

<table>
<thead>
<tr>
<th>Pair</th>
<th>Paired Differences</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1</td>
<td>After - Before</td>
<td>48,419</td>
<td>10,375</td>
<td>1,863</td>
</tr>
</tbody>
</table>

Determination of hypotheses based on: if the Sig > 0.05 then H₀ is accepted and if the Sig < 0.05 then H₀ is rejected.

Based on Table 4 shows that the Sig is 0.000. Because the Sig < 0.05, then H₀ is rejected or it can be said that the students' critical thinking ability before and after using mathematic worksheets based on critical activities differ significantly. So the use of mathematic worksheets based on critical activities highly effective in improving students' critical thinking ability.

3.3. Student Attitudes Toward Mathematic Worksheets Based on Critical Activities.

Based on the results of attitude scale data analysis showed that students' attitudes are positive towards mathematic worksheets based on critical activities.

From the results of the students' attitude scores of 3.68 and 2.5 indicates a neutral attitude that students positive attitudes toward mathematic worksheets based on critical activities on the material flat side up space.

4. CONCLUSIONS

Based on the results of research on the development of teaching material such as mathematic worksheet based on critical activities, it is concluded as follows:

a. based on the validation result of the experts’ test which consists of the learning aspect, the substance of the material, linguistic and graphic, the average score of 4.25 is obtained, which is in good category, while the validation result of the limited scale test which consists of the ease usage aspect, look and features, the average score of 4.34 is obtained, which is in good category, so that the mathematic worksheet are good and the product is ready to be used in schools;

b. there are differences in students' critical thinking skills before and after using the mathematic worksheet based on critical activities;

c. based on the attitude scale data analysis, it is found that most students show positive attitude towards learning by using the mathematic worksheets based on critical activities.

Advice can be given: the development of student activity sheet can cover wider material or on other subjects as well as more creative in making math student activity sheet.

REFERENCES


IDENTIFYING STUDENTS’ MISTAKES IN SOLVING THE PROBLEM ABOUT SET THEORY

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FKIP UPGRI Palembang

ABSTRACT
Set theory is one of the important and fundamental concepts in modern mathematics. The study of student errors in solving the problems of set theory is very important and useful but not yet fully implemented. This study was conducted to analyze the fault (location, type, and cause errors) students in solving problems of set theory. This research is a qualitative descriptive study. Subjects were students of Mathematics Education UPGRI Palembang, amounting to 120 people. Data collected through tests and interviews. Identification of student errors in solving the problems of set theory is focused on the errors in the mathematical objects, namely concepts, principles, and procedures. The results could be used as a reference to improve the understanding of the set concept appropriately.

1. INTRODUCTION
Mathematics is one of knowledge that is suitable with the development era. The development in mathematics is happened as changing human thinking in society nowadays. For instance, set theory is one of branch in mathematics. Set theory is one of the important and fundamental concepts in modern mathematics. Therefore, the study that is related to set theory is very useful. A set theory is considered as a basic to develop others aspects in mathematics. In addition, it is also source of derivative mathematics in its development.

A set theory is one of important topic that is given for university students of “Statistics Mathematic 1” course at the beginning semester in Faculty of education and teacher training PGRI Palembang. The aim of this course is that students are able to understand a set theory and to solve the problem related to set theory. To achieve these goals, students should know the meaning of symbols and terms in set theory. In addition, students should find out about objects in set theory including concepts, principle, and procedure.

Learning is a process of study that is purposed to achieve such a learning goal. The learning process can be done through classroom activity or giving a task by lecturer. We can see the learning outcome by looking at the result of task or test. We can also observe the students’ thinking by analyzing the process of students’ answer. From the result of analysis test or task, we can find out some errors or mistakes that appear in students’ answer.

The mistakes in solving the problem can be happened because of difficulty in learning. There are some factors that might contribute in unsuccessful mathematics learning. Therefore, students with learning difficulty sometimes make mistakes or errors in answering the question. According to Isgiyanto (2011:114), learning difficulty is defined as inability of students to understand certain competency related to the specific content, cognitive processes, or skills in problem solving.

To identify the mistake of students, it can be done through attribute. The attribute is defined as the content, processes, skills or required competencies of students to solve the problem (Isgiyanto, 2011). According to the content aspect, attribute is the material used in the test framework. In processes view, attribute is the expected skill of student after they learned the given material of the content. Meanwhile, attribute on skills is the specific skills.

Besides, Subaidah (2006:172) proposed three types of errors in solving the problem of mathematics, namely misconceptions, error in principle and mistake in operation. Misconception is related to error in using the concept of mathematical content. Error in principle means the misunderstanding of the relation among objects in mathematics. And mistake in operation is happened because of calculation errors. Students’ mistake in answering the question is defined as an unexpected response in solving the problem.

In this study, the types of students’ mistake in solving the problem about set theory is divided by three types: (1) Misconception is that the error in using the concept related to the concept of set theory such as variable concept in set theory or misunderstanding the meaning of the question about set theory. (2) Error in
principle is that the errors associated with the relation among objects of the set, such as using a mathematical formula, interpret the question, or the mistake in final step. (3) Mistake in operation is that the errors in calculating process such as students are not using the correct operating rules.

2. RESEARCH METHOD

This study is a qualitative descriptive study to describe the types of students’ mistake in solving the problem about set theory. Research subjects were all students of the third semester of Mathematics program in Faculty of education and teacher training UPGRI Palembang academic year 2013/2014 amounted to 235 students as population of this study.

Considering the effective and efficient study, the researchers need to conduct sampling of the population. The researchers did the stratified random sampling which is the sampling technique gradually. The first step is to classify the sample into sex and the second step is to classify the sample group on the semester grade IIIA, IIIB, IIIC, IIID, IIIE and IIIF. Characteristic of these samples will be taken 50% of each group. From the calculation of the number of samples to be representative of the population in this study were 120 students. The following table 1 provides the detail number sample of this study.

<table>
<thead>
<tr>
<th>No</th>
<th>Sex</th>
<th>Class</th>
<th>A number of population</th>
<th>50% Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Man</td>
<td>IIIA</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIB</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIC</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIID</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIE</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIF</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Woman</td>
<td>IIIA</td>
<td>39</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIB</td>
<td>36</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIC</td>
<td>32</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIID</td>
<td>37</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIE</td>
<td>23</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>IIIF</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jumlah</td>
<td>235</td>
<td>120</td>
</tr>
</tbody>
</table>

There are 24 objectively reasoned questions for the test and the question is valid were specified by mathematics education experts. Question of the form data item and every item responses were analyzed to find some mistakes of students in solving the problem about set theory. To find some errors that lead students to answer incorrectly, the researcher analyzed based on the students’ answer.

The question of set theory includes types of set, the relation among sets, operation in set, and the property of set (syllabus Mathematics program Faculty of education and teacher training UPGRI Palembang, 2006).

Data analysis was carried out immediately after the subjects completed a written test questions with the following steps:

a. Analyze the students’ answer to determine the ability of students and some mistakes in solving the problem about set theory.
b. Interview the subject of research to get more detail students’ answer in solving the problem about set theory orally.
c. Analyze the result of interview
d. Triangulation data to compare between the written answer and oral answer through interview. If there is a match between those data, we conclude that the data obtained is credible.

The procedures of this study are as follows:

a. Orientation field, including notification about research plan to the dean of Faculty of education and teacher training UPGRI Palembang, followed by chairman Mathematics and Science Department, then met chairman of Mathematics program UPGRI Palembang.
b. Validation
c. Revision
d. Implementation of the test and interview
e. Data analysis
f. Writing report
3. RESULT AND ANALYSIS

Qualitative descriptive approach was chosen to describe the types of students’ mistake in solving the problem about set theory. Before doing the qualitative descriptive, the study was started by quantitative descriptive approach in order to calculate a number of errors committed by each participant test. From the test results, it can be guideline to conduct the interview.

Qualitative data analysis is an analysis between the result of written answer and interview with participants. The purpose of qualitative data analysis is to search some errors and find out the types of students’ mistakes in solving the problem about set theory.

Types of students’ mistakes in solving the problem about set theory

Students’ mistakes in solving the problem about set theory are caused by learning difficulties. Mistakes happen due to no strong argument system owned student (Damawijoyo, 2010). The impact to these weaknesses, there is no structured mind set in solving math problems. According Griel (Isgiyanto, 2011), students’ mistake in answering the question is defined as a response of the question that is not in accordance with the expected response. Identifying the students’ mistakes is committed to find out the students’ answer that is not suitable with the correct answer.

To find students’ mistakes, we used rubric scoring as a guideline. Identifying students’ mistakes focused on three types of students’ mistake, namely 1) misconception, 2) errors in principle, and 3) mistakes in operation.

The finding of students’ mistake is following explanations:

a. Misconception

Misconception is defined as an error in using some concepts such as definitions, theorems, and property related to mathematics idea improper ways. The difficulty lies in understanding the concept of student. This is consistent with research conducted by Kashefi, Ismail, and Joseph (2012) with the title “Overcoming Obstacles in Multivariable Calculus Students through Blended Learning: A Mathematical Thinking Approach” is the conclusion of this study these were Difficulties with the concepts.

For instance, question number 6:
If A= \{x|x<16, x is natural number\}, B = \{x|x<16, x is prime number\},
C= \{x|2< x<16, x is even number\}, So the empty set is …

<table>
<thead>
<tr>
<th>A.</th>
<th>A − B</th>
<th>C.</th>
<th>A ∩ B ∩ C</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.</td>
<td>(A U B) ∩ C</td>
<td>D.</td>
<td>A ∩ C</td>
</tr>
</tbody>
</table>

Misconception of students appeared because students did not understand (1) concept of the empty set, (2) concept of operation in set particularly difference, (3) inaccuracy the concept of inequality. Indicator misconception is that students cannot apply the concepts, formulas, and properties in correct way.

b. Errors in principle

Error in principle is that the errors associated with the relation among objects of the set, such as using a mathematical formula, interpret the question, or the mistake in final step. For instance, students did mistakes for the question number 8:

According to the data of Mathematics education program on even semester, there are 42 university students who take statistic course, 68 university students take calculus course, and 54 university students who take Algebra course. Besides, there are 22 university students who take statistics and calculus course, 25 university students take calculus and algebra course, 21 university students take statistics and algebra course, and 10 university students take three courses. The correct statement according to the data, except …

<table>
<thead>
<tr>
<th>A.</th>
<th>There are 31 university students who only take calculus course.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.</td>
<td>Total number of university students in mathematics education program that take statistics, calculus, and algebra course is 164 students.</td>
</tr>
<tr>
<td>C.</td>
<td>There are 9 university students who only take statistics course.</td>
</tr>
<tr>
<td>D.</td>
<td>If total number of university students in mathematics education program is 150 students, it means that there are 44 university students who do not take three courses above.</td>
</tr>
</tbody>
</table>

Errors in principle for the question number 8, namely (1) misinterpret the question, (2) error in using the formula of operation set particularly union, intersection, and complement.
c. Mistake in operation
Mistake in operation is that the errors in calculating process such as students are not using the correct operating rules. For instance, students did mistake in operation of set for the question number 22.

If \( A = \{x | x^2 - x - 12 < 0, x \text{ is real number}\} \) dan \( B = \{x | x > 3, x \text{ is real number}\} \), so \( A \cap B^c \) is...

| A. \( \{x | -3 < x < 3\} \) | C. \( \{x | -3 < x < 4\} \) |
| B. \( \{x | -3 < x < 3\} \) | D. \( \{x | -3 < x < 4\} \) |

To solve the problem number 22, students should be able to deal with the quadratic inequality procedure on set A. Likewise; students should be able to overcome the absolute inequality on set B. Then, students should operate the intersection between intersection two sets and complement of set. Mistakes in operation is occurred by students as respondent, namely 1) Students cannot apply the operation of number and algebra properly (especially solving quadratic inequality and absolute value), 2) Students cannot do calculation appropriately, 3) Inaccuracy at final step

Furthermore, some findings of students’ mistakes in solving the problem about set theory will be explained in detail from the following table 2.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Identifying students’ mistake in solving the problem about set theory</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sub Topic</strong></td>
<td><strong>Question Number</strong></td>
</tr>
<tr>
<td>Kinds of sets: 1. Universal set 2. Finite/Infinite set 3. Cardinal number.</td>
<td>8, 6, 2, and 22</td>
</tr>
<tr>
<td><strong>Types of students’ mistake</strong></td>
<td><strong>Concept</strong></td>
</tr>
<tr>
<td>1. Misunderstanding of diagram. 2. Misinterpret the question</td>
<td>Errors in using the formula.</td>
</tr>
<tr>
<td>1. Mistakes in relation between sets. 2. Misconception of subset. 3. Errors in notation of sets. 4. Misunderstanding of equivalence. 5. Misunderstanding the definition of disjoint set. 6. Mistakes in determining the kinds of sets. 7. Misunderstanding of universal set.</td>
<td>1. Mistakes in represent the figure. 2. Fault to determine the formula of subset. 3. Mistakes in represent the notation of set. 4. Errors in the last step/procedure. 5. Fault to determine the relation between two sets.</td>
</tr>
</tbody>
</table>
4. CONCLUSION

According to the result of the study above, we can conclude some point as the following:

a. We can classify three kinds of students’ mistakes namely misconception, errors in principle, and mistakes in operation.

b. Particularly, students’ mistakes in solving the problem about set theory is that: (1) misinterpret the question and represent the notation of sets, (2) Errors in drawing the diagram Venn, (3) errors in the final step of procedure.

c. The factor of students’ mistakes, such as less understanding of the meaning of the questions, the meaning of set definition, and the meaning of set notation.

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REFERENCES


USING GEOGRAPHIC SPECIFIC CONTEXT IN LEARNING MATHEMATICS TO ENHANCE CREATIVE THINKING SKILLS OF SECONDARY SCHOOL STUDENTS

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ABSTRACT
The use of geographic specific context in learning mathematics is still a problem because of the lack of teaching materials available. This study revealed the use of geographic specific context those are often used by teachers in the learning mathematics in secondary school through observation of the learning process and interviews mathematics teachers. The results showed that the potential of every region is rarely used in learning mathematics. The regional potency context such as coast, fishery, watershed, harbor, city, mountain, agriculture, economy, archipelago, environment, airport, and technology is rarely used on math learning. Contexts are only used at the beginning of the learning process to draw student interest. It has not been used to develop mathematical concepts and creative thinking skills. Teachers often use the context of agricultural, marine, urban life, and the objects around students. Teachers believe that the use of geographic specific context in learning mathematics can improve the mathematical skills of students to think creatively.

1. INTRODUCTION
Success in implementing learning math teacher can be seen from the success of the students to understand, apply, and develop the material taught both mathematics itself, other subjects, as well as in everyday life. The students' success can not be achieved with less well if the teacher learning meaningful and less encouraging innovation and creative thinking to every student. Learning should be an interesting and challenging mathematical thinking processes so that students can practice skills students creative thinking skills. The use of geographic specific related to students' daily lives and the development of technologies that can be understood at every lesson students will affect the students in learning activities.

In facts, there is no formal definition of a territory with specific geographical features. But, the report therefore distinguishes five types of specific regions: border regions, mountains regions, island regions, sparsely populated regions (SPRs), and outermost regions. These types of regions can be considered to have easily identifiable geographical features, some of which imply particular development challenges, notably regarding demographic change and migratory phenomena, accessibility, or region integration [1]. Using geographic specific as a context in teaching mathematics is rarely used. The teaching mathematics in a variety of educational units use textbooks as the only referral and very rarely associate mathematics with its use in the geographic specific especially in real world or to solve everyday problems both on the economic, social, environmental, fishery, forest, mountains, rural, city activities, port, river, lake, culture, artistic, buildings, and technology. The questions used by teachers in general just use simple math problems. Such questions can not train potential students thinking at a high-order mathematical thinking skills. Simple math problems can only be used on certain aspects of mathematics.

The results of study [2] shows that the problems are still the dominant use regular junior high school mathematics teacher. The results showed that mathematics teachers less capable of designing or modifying the mathematical problems that exist in textbooks into geographic-contextual issues that are more related to the daily lives of students or more challenging students' thinking processes. In fact, questions like these are very attractive to enable students in learning and to challenge students' mathematical thinking.

The use of contextual issues are very interesting and can provoke communication skills to interact with other students or the teacher. That is, the presentation of geographic-contextual issues and problem
solving will attract students to follow the learning process and challenge the thinking of students to creatively solve a given problem or faces either in individu or as a group through effective communication and facilitated by teachers in the classroom. Effective mathematics learning situations-interactive teacher must always be pursued that potential problems be known and where possible be solved by mathematical method studied.

The lack of use of every day contexts students when teachers teach mathematics increasingly distanced impact on student sofmathematics and the low ability students in solving mathematical problems shaped the story or problem solving questions. The low ability secondary school students' mathematical problem solving can be seen from the results of research [3], [4]and [5] in class VIII and class IX secondary school in Southeast Sulawesi province. Although the research [2] and [6] show that the results of mathematical problem-solving abilities of students in the coastal city of Kendari has to be improved but the materials used are based teaching materials that have not touched the potential of the geographic-specific generally. Even in the course of a study [6] concerning the context of use of palmia such ascoconut, palm, sago, rattan, betel nut, palm oil, and palmplantin mathematics showed that palmcont exis less well known that students needed other contexts. Though Indonesias theworld’s scenter of diversity palm [7]. The results [6] also showed that mathematics teachers in desperate need of contextual teaching materials that directly relate to the placed and potential problems. Mathematics textbooks that have been used by teachers are not directly related to the issues and potential impact on the lack of student participation in the learning mathematics.

This list indicates the need for designing a math lesson that can directly benefit the formation of the mindset of students on a regular basis, logical, and creative. Such learning can be realized if teachers use teaching materials that can facilitate and encourage students to think. The instructional materials should be interesting and challenging students' thinking processes. Such materials can be packaged through better utilization of various issues related to geographic-specific daily lives, as well as those related to mathematics itself and other knowledge in geographic-specific. Using geographic-specific as a context and problems in learning mathematics instructional materials packaged in an appropriate step to train creative mathematical thinking skills and instill the importance of knowledge about the issues every context.

Learning math curricula a struuctee should be able to realize that learners problem solver, logical thinking, systematic, productive, reflective, critical, creative, character, and skilledbersosial. Generations such this can only be realized through meaningful learning supported by appropriate teaching materials, teachers are professional, and quality education system.

Various studies have shown that the activities of integrating the values of life and cultural and natural resources in the learning of mathematics is an activity that must be done in-class teacher. According to [8], the classroom is part of a community that defines cultural practices. When a student enters a school, they carry a variety of values, norms, and concepts that are part of their development. According to Bishop, some of it is that they bring mathematics [8]. In [8] went on, unfortunately, the mathematical concept of the school curriculum is presented in a way that is not related to students' mathematical culture. Though aspects of culture an important contribution to improving students' math skills in the classroom. This opinion is in line with the opinion of Bishop, Boaler, and Zavlasky, that contribute to the cultural aspects of mathematics known as a part of daily life, develop skills in meaningful connections, and deepen understanding of mathematics [8].

Creative thinking in mathematics refers to the notion of creative thinking in general. Bishop explained that a person requires two complementary models of different thinking in mathematics, the creative thinking that is both intuitive and analytical thinking that is logical [9]. This notion suggests that creative thinking is not based on logical thinking but more as a thought that suddenly appears, unexpected, and unusual. [9] looked at creative thinking as a combination of logical thinking and divergent thinking which is based on intuition but still in consciousness. When a person applies creative thinking in solving a practical problem, then the intuitive divergent thinking to generate new ideas. It is useful to find a solution to a problem. This understanding explains that creative thought logical and intuitive attention to generating ideas. That is, the two parts of the brain creative thinking will be required so that the balance between logic and intuition are very important. Placement of logical deduction would ignore too many creative ideas.

To bring creativity thinking needed freedom of thought. Someone who is free thinking means thinking not under the control or pressure. According to [10], creative thinking is the thinking that is original, reflective, and produces a complex product. Thinking involves synthesis of ideas, build new ideas and determine its effectiveness. Moreover, it also involves the ability to make decisions and produce new products.

In this research, creative thinking is viewed as a single unit or a combination of logical thinking and divergent thinking to produce something different (novelty), which is one indication of the emergence of mathematical creative thinking abilities of students. Another indication is associated with the ability to think logically and divergent thinking. In addition to the novelty, someone who thinks creatively also bring flexibility in solving a problem. According to [11], creative thinking is considered almost always involves flexibility and smoothness as well as in the context of mathematics, smoothness criteria seem less useful than flexibility. Dexterity also stressed on many different ideas being used. So in mathematics to assess the divergence of the product and the flexibility to use the criteria of novelty. Another is the appropriateness criteria. Mathematical
response may indicate that high novelty, but it is useless if it does not fit the general mathematical criteria. So, based on some opinions that creative thinking skills can be demonstrated of flexibility, fluency, originality, appropriateness or usefulness. This indicator can be simplified or viewed in common sense combined with a supleness, smoothness, and novelty. While the feasibility or utility is included in all three aspects.

Referring to the general understanding of creative thinking and creative thinking abilities mathematical indicators used by Krutetskii, Balka, Getzel & Jackson [12], [12], and [11], then thinks creative mental activity is defined as a person who is used to construct the wide or ideas that seamlessly and flexibly in solving math problems correctly.

2. RESEARCH METHOD

This study was conducted in two secondary school levels in Kendari, Southeast Sulawesi, Indonesia (medium and low). Students sample is determined based on the simple random sampling. Through technique researchers sampled year-8s of the schools under study, they were 33 students of SMP Negeri 5 Kendari (medium level) and 29 students of SMP Negeri 11 Kendari (low level) Southeast Sulawesi.

The research instruments used were mathematical creative thinking skill test, teacher’s during-learning observations, and guided interviews with teachers to elicit more information about using geographic specific as a context in learning mathematics. The data were analyzed by means of descriptive-qualitative analysis, t-test, and One-way ANOVA, employing SPSS-20 program for windows at α = 0.05 significance level.

3. RESULT AND ANALYSIS

3.1. Interviews and Observation

Based on the analysis of observations and interviews with teachers in two SMP found some condition studied mathematics learning process. Mathematics teachers who were interviewed as many as five people qualified scholars. Observation and interview results are presented as follows.

a. Teachers have not been to use geographic specific as a context in learning mathematics.

b. Contextual problems only arise spontaneously without any planning.

c. Quality of interactions and activities of students in learning to be better when teachers use contextual problems.

d. Teachers only use contextual issues to attract the attention of students, not to take advantage of contextual issues to further develop thinking skills.

e. Teachers used context is the problem of fish, guava, mango, and coconut. Geographic specific as agriculture, fisheries, city activities, flooding, street accident, fields, plantations, mountains, tourist attractions, cultural, artistic, historical buildings, lakes, rivers, ports and their problems have not been used in the learning of mathematics.

f. The use of contextual problems was realized very helpful teachers to attract students but low proficiency students so that students have difficulty understanding or translating the problem into a mathematical model.

g. Students is still difficult to solve the mathematics problem presented in world or real problem.

h. Students are less able to perform basic matharithmetic cooperations especially with regard to fractional and negative integers.

i. Low-ability students so that the learning process is slow and the students have difficulty in solving math problems.

The results above indicate that the teacher has work towards effective learning though it still needs a lot of improvement. It is recognized that learning mathematics better if presented in the form of contextual examples problems related to issues or objects that often students have encountered or use. However, such context is simply to draw attention to students as simply appear spontaneously without any planning. The books used by teachers also have a maximum load range of contexts and permasahannya that can help teachers plan lessons, make an example or problem that can make students interested in learning, actively interact, communicate mathematics, build character, and train alternative thought patterns. The material on mathematics textbooks is still the main reference. Some of the issues presented are not contextual. This reduces the quality of the interaction between the components of the learning in the classroom so that also lack the social skills train students. This means that the use of context to draw the attention of students in the classroom is very important in learning mathematics.

3.2. Students’ MCTS Based on School Level and MCTS Types

The results of descriptive analysis of the data students’ mathematical creative thinking skills (MCTS) based on school level and MCTS Types are presented in Table 1
Table 1 Description of Students’ MCTS Based on School Levels and MCTS Types

<table>
<thead>
<tr>
<th>School Levels</th>
<th>MCTS Types</th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
<td>Elaboration</td>
<td>33</td>
<td>52.36</td>
<td>13.57</td>
</tr>
<tr>
<td></td>
<td>Fluency</td>
<td>33</td>
<td>43.03</td>
<td>15.74</td>
</tr>
<tr>
<td></td>
<td>Flexibility</td>
<td>33</td>
<td>39.58</td>
<td>14.29</td>
</tr>
<tr>
<td></td>
<td>MCTS Total</td>
<td>33</td>
<td>44.97</td>
<td>7.91</td>
</tr>
<tr>
<td>Low</td>
<td>Elaboration</td>
<td>29</td>
<td>49.48</td>
<td>16.93</td>
</tr>
<tr>
<td></td>
<td>Fluency</td>
<td>29</td>
<td>35.24</td>
<td>12.81</td>
</tr>
<tr>
<td></td>
<td>Flexibility</td>
<td>29</td>
<td>26.48</td>
<td>8.93</td>
</tr>
<tr>
<td></td>
<td>MCTS Total</td>
<td>29</td>
<td>36.90</td>
<td>8.99</td>
</tr>
<tr>
<td>Total</td>
<td>Elaboration</td>
<td>62</td>
<td>51.02</td>
<td>15.17</td>
</tr>
<tr>
<td></td>
<td>Fluency</td>
<td>62</td>
<td>39.39</td>
<td>14.85</td>
</tr>
<tr>
<td></td>
<td>Flexibility</td>
<td>62</td>
<td>33.45</td>
<td>13.68</td>
</tr>
<tr>
<td></td>
<td>MCTS Total</td>
<td>62</td>
<td>41.19</td>
<td>9.30</td>
</tr>
</tbody>
</table>

Remarks: Data in the scale of 0-100

Table 1 shows that students’ MCTS in medium school level is higher than students’ MCTS in low school level. The students’ MCTS in medium school level is 44.97 and students’ MCTS in low school level is 36.90. Table 1 also shows that from four types of MCTS, elaboration type is highest type that students got with the average score 51.02. In other MCTS types, students have score lower such that fluency 39.39 and flexibility 33.45. In general, students’ MCTS is in low level category based on average 41.19. Therefore students MCTS is still be enhancing.

The result of significance test of the difference of students’ MCTS based on School Level is presented in Table 2.

Table 2 Significance Test of the Difference of Students’ MCTS Based on the School Level

<table>
<thead>
<tr>
<th>Levene’s Variance Homogeneity Test</th>
<th>t-test for Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>Sig.</td>
</tr>
<tr>
<td>0.166</td>
<td>0.685</td>
</tr>
</tbody>
</table>

Table 2 shows that the probability values (sig.) from the Levene’s test of homogeneity of variance are greater than 0.05, so the variances are homogeneous. So, to test for differences of both mean values, the t-test can be used to show that the probability value (sig.) is smaller than 0.05. This means that students in medium school level obtain a significantly greater MCTS compare to those students in low school level. This is understandable because student in medium school levels has got learning by means of the more completely tools of learning than students learning in low school levels. Teacher in medium school levels use this tools of learning to teach all of subject matter especially mathematics. Students in low school level did not get teaching by using those tools of learning effectively. So, they have not got experience to solve all of kinds mathematical problem.

Based on the interview and observation learning process it can be say that teacher did not use contextual problem in learning mathematics. Whereas, context in learning mathematics can make students interested and challenged to solve the problem. When students solve contextual problems, enthusiasm, attention, motivation, and knowledge used to solve the problem with understanding, compare, describe, analyze, create a mathematical model, complete model, answer the problem, discuss the answers, maintaining an answer, and to negotiate the process and the results of solving the problem [2]. Familiar contexts are encoded internally as representational configurations in common words, images, formal notations, strategies, and operation, and (ideally) comfortable affect [13]. This opinion shows that through the use of proper context, students may be interested in following the process of learning, challenged to solve a given problem, and can represent words, pictures, formal notation, strategies, and operations comfortably in solving any given problem.
The use of proper context, in accordance with the conditions of the students and the location where the students come from, students can construct knowledge into a mathematical representation that is suitable and can be used to slowly spontaneously creative or flexible process of solving a creative problem. Through contextualization, students learn to construct special cases, to see particular in the general, to more toward the concrete in a new representational situation, and to take these steps spontaneously and flexibly. Through abstraction, they learn to generalize, to see the general in the particular, to move away from inessential details of the concrete representational situation, and to do these things also spontaneously and flexibly [13]. If people from different cultural backgrounds can view such things as optical illusions differently, it is no wonder they perceive so much of the rest of the world differently as well. When this information is coupled with information concerning other basic psychological processes such as attribution, emotion, and personality, the effect of culture on individual psychology is amazing [14]. This means that using geographic specific in learning mathematics can enhancing students’ creativity thinking.

Using geographic specific as a contextual problem has inspired interest of student to solve problem presented. This interest has lead them to make discussion with the teacher and other students to become better understood about the mathematical concept. According to [15], through their active discussion with the teacher and peers, students are expected to gain a greater understanding of the conceptual under pinnings of mathematics and problem-solvers become better. Using appropriate context, students can be train to make an abstraction or generalization logically. This what McGregor called Bridging, that is the phase of a cognitive acceleration lesson where abstracting and generalizing the reasoning or logical thinking applied in the lesson is related to real life contexts [16]. Real life context can be take from many sources related with students real life or location where the learning process is done. Geographic specific is one of the solution to solve this problem of learning mathematics contextually. Geographic specific has many contexts and problems that should be given to the students so that they recognize, understand, and be aware of the various conditions of the geographical context. Consciousness is meant is that mathematics has many benefits in life because it can be used to solve a variety of problems, students also can find out the development of existing natural resources, problems, and the importance of preserving these resources for the sustainability of life and economic development. Nevertheless, the use of the geographic context also in influenced by the readiness of students to learn mathematics properly. That is, the effectiveness of the learning processes also influenced by students' prior ability.

Effective learning and knowledge constructions involve the cognitive processes of selecting, organizing, and integrating information [17]. It means that the learning process goes well, then the student should be able to perform cognitive processes selectively, organized, and able to integrate all the information. This cognitive process an be run if the student is able to understand and relate what is learned. Use the appropriate context and prior knowledge about the ownership of a sufficient mathematics will be able to make the students capable of learning mathematics effectively. The reason why students didn’t learning effectively that is students do not have sufficient prior knowledge of the content they are studying to determine what information is important or what sorts of questions they should ask themselves about the content [18]. Mastery of mathematics prior knowledge affect students’ mathematical creative thinking skills. This explanation is presented in Table3.

<table>
<thead>
<tr>
<th>Test of Homogeneity of Variances</th>
<th>ANOVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levene’s Statistics</td>
<td></td>
</tr>
<tr>
<td>Sig.</td>
<td></td>
</tr>
<tr>
<td>Sum of Squares</td>
<td></td>
</tr>
<tr>
<td>df</td>
<td></td>
</tr>
<tr>
<td>Mean Squares</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
</tr>
<tr>
<td>Sig.</td>
<td></td>
</tr>
<tr>
<td>Result</td>
<td></td>
</tr>
<tr>
<td>1.427</td>
<td>.248</td>
</tr>
<tr>
<td>1832.187</td>
<td>2</td>
</tr>
<tr>
<td>916.094</td>
<td>15.714</td>
</tr>
<tr>
<td>0.000</td>
<td>Significant</td>
</tr>
<tr>
<td>3439.490</td>
<td>59</td>
</tr>
<tr>
<td>58.296</td>
<td></td>
</tr>
<tr>
<td>5271.677</td>
<td>61</td>
</tr>
</tbody>
</table>

Table 3 shows that the probability values (sig.) from the Levene’s test of homogeneity of variance are greater than 0.05, so the variances are homogeneous. So, to test for differences of the third mean values, the one-way ANOVA can be used to show that the probability value (sig.) is smaller than 0.05. This means that there is a significant difference among of students MCTS based on their mathematics prior knowledge. Based on the results of post hoc test we have conclude that mathematical prior knowledge has an effect to students’ MCTS. The students who have high mathematical prior knowledge is got higher MCTS than the students have medium or low mathematical prior knowledge. This is the important thing that students must be know about mathematical prior knowledge or mathematical based knowledge. In learning mathematics, every subject matter is connected. If the students want to learn the continuous subject of mathematics, they must be have learn the previous or the
prerequisite materials of that subject mathematics. Using geographic specific is most important to help students interest in learning mathematics. By using geographic specific, students can be known what, why and how to use mathematics in helping their life problem. According to [19] the single most important reason that students do not use effective learning strategies is because they have not been taught which ones to use, how to use them, and when to use them. Students try to learn through practical activities that are intrinsically interesting, giving the mathematical problem to be solved, and choosing subject matter that has a natural appeal to their interests [20]. This can be realized through the use of various learning approaches such as model approach to CTL (contextual teaching and learning) and the model of problem-based learning (problem-based learning) through discussion activities. According to [15] through activities and discussions with partner teachers, students are expected to gain a better understanding of the basic concepts of mathematics and become creative problem solvers. According to [21], after completing the task group, the results need to be represented to the entire class and a debriefing that focused on group process should be implemented. It means that if students know why they learn mathematics, they can be use effective learning strategies to learn mathematics effectively and to think mathematics creatively.

Based on the explanation above it can be said that using geographic specific as a context in learning mathematics is the solution to enhance students’ MCTS. The context is used as a problem to be analyzed, discussed, and solved step by step from easy or simple to difficult or complex, making it more challenging. Further discussion of these issues will bring more students able to develop the capacity to think and think will be a habit. In this group activity, the cultural aspects should be noted. According to [8], contributing to the cultural aspects of mathematics known as a part of daily life, develop skills in meaningful connections, and deepen understanding of mathematics. The communication can facilitate the process of solving problems and planting mathematical concepts to students.

The results of this research supported to [22] results that a need to engage with teachers in conversations about the teaching and learning of mathematics with non-dominant students; a need to continue developing narratives of successful participation of non-dominant students in mathematical discussions to counter the pervasive deficit narratives that so often surround us. It means that to improve creative thinking skills, teachers must use geographic context in learning mathematics so the students know what, why, and how to learn or to use mathematics that can solve their problem.

4. CONCLUSIONS

Based on the results of this study concluded that: the ability of teachers to use geographic specific as a context in learning mathematics is still lacking. Context is only used at the beginning of the learning of mathematics to attract students’ attention, but has not been used to build a mathematical concept and has not been used to improve students’ mathematical thinking skills.

Based on these conclusions suggest that: more creative teachers prepare teaching materials contextual problems or innovate either with itself or modifying the model or the problem or problems that exist in textbooks creative to make learning becomes more interesting and challenging process of students’ mathematical thinking. One alternative learning model that can be used by teachers is a contextual problem-based learning model.

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INSTRUMENT OF MATHEMATICS HIGH-ORDER THINKING SKILLS OF JUNIOR HIGH SCHOOL STUDENT

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ABSTRACT
Mathematics high-order thinking skills are one of the goals and needs attention in teaching mathematics in Junior High School. The low of student’s mathematics high-order thinking skills indicates that students have difficulties in learning mathematics. The purpose of this study was to obtain information about the difficulties of Junior High School students and teachers in Bandar Lampung City, followed by validating an instrument to measure the mathematics high-order thinking skills based on learning material developed. The subjects were students of eighth grade and math teacher from Junior High School in Bandar Lampung City, which is elected from each of the rating categories that high-ranking school (SMPN 4 Bandar Lampung), middle-rank school (SMPN 5 Bandar Lampung), and low-rank school (SMPN 22 Bandar Lampung). Techniques of data collection were questionnaires, observations, interviews, sheet of instrument validation, and testing. From the analysis of the data, be concluded that the materials largely considered difficult by students and teachers of high-ranking school is Pythagorean theorem and their application; for the students and teachers of middle-ranking school is at variable system of linear equations; for students and teachers of low-ranking school is the equation of straight line. The observation of the learning system in the three schools concluded that education in general were largely traditional learning. Based on the analysis of data validation and tryout, it was concluded that the test instrument can be used to measure the students' mathematics high-order thinking skills.

1. INTRODUCTION
The analysis of mathematics learning in several schools in Bandar Lampung showed that learning mathematics is still traditionally; trained low order thinking skill. So that Mathematics High-Order Thinking skills (MHOTs) of junior high school students were generally low. One effort to do is apply the Problem Based Learning (PBL) and development of the interactive learning media based on open-ended. PBL began with the presentation of contextual issues. Issues raised and the students need to interpret the problem, collect the necessary information, evaluate alternative solutions, and present a solution. When students develop the method to solve the problem, students integrate knowledge of concepts and skills they have. Therefore the use of interactive media base on open-ended in learning mathematics can be used to improve student MHOTs.

Development of thinking skills, should receive serious attention, because a number of studies, such as [1]: Peterson, 1988; Mullis et al, 2000 in [17] suggests that the study of mathematics in general is still focused on the development of low-order thinking skills. In addition [5], [6], [7], [8], [9], [10] suggests that problem-based learning and open-ended learning can be used to enhance a variety of mathematical skills, particularly in the enhancement of MHOTs.

In order for students to be independent and have the thinking skills, learning math should be packed so that skills can be developed. Students should be accustomed to communicating ideas and concept in learning activities and in the use of mathematics. But in reality, it is often surprising to students when they are asked to give consideration or explanation for the answer. This happens because students are rarely asked to give an explanation in mathematics learning.

Reformations in mathematics education stating that the student must learn to recognize the elements of mathematics in context, applying the appropriate mathematical tools, and engage in mathematical reasoning. However, developing and implementing learning activities appropriate load thus enabling students to develop higher-order thinking skills, not an easy thing to do by most of mathematics teachers. Based on research...
conducted by [16] note that there are several ways to do this include in mathematics learning students need to face various problems which require effort to try a variety of alternative solutions, and the problems given to students should be relevant to their development.

About thinking [11] said that thinking is a mental process that involves operations such as classification, induction, deduction and reasoning. [15] said that thinking is a dynamic process that can be described by the process and the way. [4] said that the thinking process is a process to obtain information (from the outside or from the students), processing, storage and recall of information from students' memories. [2] said that thinking is the ability to analyze, criticize and reach conclusions based on inference or careful consideration.

Thus thinking is a complex process, dynamic and non-algorithmic nature because it involves mental processes for obtaining information, classifying, analyzing, reasoning; perform induction, deduction up to reach a conclusion by inference or judgment. So basically in thinking there are three steps taken are: 1) understanding the formation, 2) the formation of opinion, and 3) conclusion.

According to [3], thinking skills include highly specialized skills to the skills that are very common, of prowess in logical reasoning gave rise to the perception that smart, from capacity to decipher anything from the whole to the parts up to the capacity to put something so be a part of the well and on the whole, the ability to explain how a situation can occur up to the ability to predict how the process will end, of an ability to distinguish, recognize the uniformity and equality up to proficiency in the uniqueness , of a means to justify confidence through consideration and possible reasons to raise ideas and develop concepts, of the capacity to solve problems up to completing capacity and prevent the emergence of complex issues, from the ability to evaluate the ability to perform up to the back.

Thinking in mathematics is closely related to mathematical power. Mathematically the term implies the ability or power of a person related to the characteristics of mathematics. So the mathematical power is the ability to think mathematically or ability to perform activities and processes or mathematical tasks. Judging from the depth of mathematical activity, mathematical power can be classified into two types, namely low-level thinking (Low order thinking - LOT) and higher-order thinking (HOT).

Bloom in [12] said that a LOT covering the first three aspects of the cognitive aspects of knowledge, understanding, and application. While [4] said that the low-level thinking includes aspects of remembering, focus and gather information. [18] said that LOT includes simple arithmetic operations, applying mathematical formulas directly; follow the procedure (standard algorithm). Therefore LOT closely related to routine matters. [12] said that aspect of the analysis, synthesis and evaluation of Bloom's taxonomy, including the HOT aspects. [4] said that HOT covers aspects of organizing, constructing, investigating and evaluation. [2] said that HOT is non-algorithmic characteristics of the course of action is not fully specified in advance, tends to be complex, often generate a lot of solutions, involving judgment and interpretation, as well as a high mental activity.

[18] said that HOT includes understanding the mathematical ideas in more depth, to observe and explore the idea that implied, preparing conjecture , analogy and generalization, reason logically, solve problems, communicate mathematically and associate something mathematical ideas with other intellectual activities. [1] stated that the ability HOT is basically a non-procedural thinking skills, among others, include (1) seek and explore patterns and to understand the mathematical structure of the underlying relationship, (2) using available materials appropriately and effectively in when formulating and solving problems, (3) to make mathematical ideas significantly, (4) to think and reason flexibly, (5) develop a conjecture, (6) generalizations, (7) justification, and (8) communicate ideas mathematically.

Current issues in mathematics learning are developing HOTs, and make it a primary goal of learning mathematics. To that end, the concept of mathematical learning emphasizes meaning constructing actively as a result of linking new ideas in the understanding of the past. As such mathematics learning in the classroom must have changed, from the charge of mathematics into how students learn mathematics effectively.

This research is a descriptive study conducted to obtain information about the difficulty for eighth grade students and teachers of Junior High School in Bandar Lampung City, followed by validating an instrument to measure the high-order thinking ability based on learning material has developed.

2. RESEARCH METHOD

The study population was all Junior High Schools in Bandar Lampung City, which is divided into three categories, namely high-rank schools, middle-rank school, and low-rank school. School rankings are selected in this study are based on the acquisition value of the national exam (UN) in the last 3 years i.e in 2010, 2011 and 2012. Sampling conducted stratified random sampling, i.e by selecting a random sample of schools for each qualifying school, so the sample size is 3 schools. In the sample of this study will be sought data on a difficult materials, models and tools that teachers use in mathematics learning.

This study was conducted to obtain data on the difficult materials, teaching methods and tools are used by junior high school teacher in Bandar Lampung city and develop instruments to measure mathematical high-
order thinking skills. For that do the following steps: a) identify the materials that are considered difficult by teachers and or students, b) identify the models of learning and learning tools applied by the teacher in the learning of mathematics, c) validate the instrument to measure mathematical high-order thinking skills for learning materials will be developed at each school rankings.

The design of this study began with the observation of the learning model of media and devices used by teachers, giving questionnaires to interview the students and teachers to obtain data on materials that are considered difficult by teachers and or students at the junior high school representing high-rank school, middle-rank school, and low-rank school. Having concluded the learning materials will be developed further developed instruments to measure higher-order thinking skills. Further tests carried out on the test instrument each school rankings.

Observation activities conducted to observe models of media and learning tools used by teachers. Observation was done to the two teachers who teach eighth grade math at each school (high-rank school, middle-rank school, and low-rank school). Interviews with several students (5 people) to know where the material that they find difficult. Interviews were also conducted on teachers to identify difficult mathematical material taught to students. Results of student questionnaires, teacher interviews, and observations were analyzed descriptively to determine the mathematical material that is difficult to understand by students and teachers taught to the students, as well as identify the model of media and learning tools that are often used by teachers. Descriptive analysis was used as the basis to create models of media and learning tools that will be developed. At the end of the study to be carried out, tests to determine the ability of a high level of students’ mathematical thinking. In this study also conducted tests to determine whether viable instrument used to obtain data on the high-level mathematical thinking skills of students.

3. RESULT AND ANALYSIS

Based on research conducted on high-rank school (SMPN 4 Bandar Lampung), middle-rank school (SMPN5 Bandar Lampung), and low-rank school (SMPN 22 Bandar Lampung), note that education in general is still dominated by the teacher. When starting the lesson, the teacher gave aperception by reviewing previous material and support material. After that the teacher explains the material, provide an opportunity for students to ask, then the teacher gives exercises to students and students are discussing it to solving the problems.

Based on interviews to the teachers of SMPN 4 Bandar Lampung, note that they have difficulties in Pythagorean Theorem taught to the students, because a lot of students who do not understand the concept of a right triangle and its properties. Most students simply memorize the formulas without understanding the concept. Of the outcome of the questionnaire to the students aware that all students have difficulty doing word problems related to the application or Pythagorean Theorem in daily life when it is not accompanied by illustrative drawings. So in high-rank school will develop models of instructional media and devices for material Pythagorean Theorem.

Based on interviews conducted to the teacher of SMPN5 Bandar Lampung, note that they have difficulty in teaching materials Two Variables Systems of Linear Equations (TVSLE) taught to the students. Teachers said that in general students are still difficulties in material nonlinear equations. Most students simply memorize the formulas without understanding the concept. Difficulties experienced by students are the difficulty in modeling the problem of word problems into mathematical form. Based on questionnaires given to students known that in general students have difficulties in sub-section One Variable Linear Equations and sub-chapter non-linear system of equations. So in middle-rank school will be developed media and learning tools for TVSLE material.

Based on interviews conducted to the teacher of SMPN22 BandarLampung, note that they have difficulties in Equation of a straight line taught to the students. Teachers said that students generally do not understand the different forms of equation of a straight line. From the questionnaire to the students aware that there are four students who said that they not able to work on the problems of the equation, and there is only one person who thinks that gradient is an easy material. Students generally said that the question of the use of data fragments difficult to resolve. So in low-rank school will be developed media and learning tools for Equation of a straight line material.

Consideration of the content and face validity of the third expert judgment to test of mathematics high-order thinking skills were analyzed using the Q-Cochran test statistic. This analysis was conducted to determine the uniformity expert judgment in giving consideration to the test. Based on the analysis of face validity is obtained that Asymp. Sig = 0.306 and content validity analysis shows that the Asymp. Sig = 0.255. This means that the third consideration expert judgment against each item on the test is uniform. In this study, the validity of the items was also tested to determine the items to support a total score. To measure the correlation coefficient between the item score and total score is used Pearson product moment formula \( r_{xy} \). To test the significance of each correlation coefficient was used \( t \)-test [14] with the following criteria: if \( t_{hit} > t_{table} \) at significance level of \( \alpha = 0.05 \) and \( df = n - 2 \), then \( H_0 \) is rejected (valid). Otherwise \( H_0 \) is accepted (not
In this study, the reliability of the test was also measured to determine the level of reliability of the test. To calculate the reliability coefficient Cronbach alpha tests used formula [13]. Interpretation of test reliability coefficient \( R_{11} \) using the benchmark as follows: (1) If \( R_{11} \) is equal to or greater than 0.70 means that the test has high reliability, and (2) If the \( R_{11} \) is smaller than 0.70 means that the test does not have high reliability. Calculations about the validity, reliability, discrimination power, and level of difficulty of the test are summarized in Table 1.

**Table 1. Recapitulation of Mathematics high-order Thinking Ability Test**

<table>
<thead>
<tr>
<th>School rank</th>
<th>Item</th>
<th>Validity</th>
<th>Reliability</th>
<th>Discrimination Power</th>
<th>Level of Difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>1</td>
<td>Valid</td>
<td>0.69</td>
<td>Less</td>
<td>easy</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Valid</td>
<td></td>
<td>Less</td>
<td>easy</td>
</tr>
<tr>
<td>Middle</td>
<td>1</td>
<td>Valid</td>
<td>0.73</td>
<td>Less</td>
<td>easy</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Difficult</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Difficult</td>
</tr>
<tr>
<td>Low</td>
<td>1</td>
<td>Valid</td>
<td>0.66</td>
<td>Less</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Valid</td>
<td></td>
<td>Less</td>
<td>Easy</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Difficult</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>Valid</td>
<td></td>
<td>Good</td>
<td>Difficult</td>
</tr>
</tbody>
</table>

Based on the results presented in Table 1, shows that in general for each school rankings, test instruments that are used to measure higher-order thinking skills. There are some items that are made about the test did not show expected results in terms of discrimination power and level of difficulty. Based on interviews with students after testing, obtained information that some students have difficulty in understanding the purpose of the question. Therefore, test instruments should be revised in terms of language and image.

4. CONCLUSIONS

Based on the results of research conducted, it can be concluded that:

a. Learning at each school (high-rank school, middle-rank school, and low-rank school), were largely traditional. Learning begins with the presentation of the material, give examples, and exercises. Discussion is usually done when students work on exercises. It is therefore necessary to develop learning that is able to develop high-level thinking skills. It is therefore necessary to develop learning that is able to develop mathematics high-order thinking skills.

b. Based on teacher interviews and analysis of the questionnaire given to the students obtained information about the materials that are difficult to understand by students or be taught by the teachers for each school. At the high-rank school is being developed materials Pythagorean theorem, at middle-rank schools is being developed material system of linear equations of two variables, and at the low-rank school is being materials straight-line equation.

c. Based on a consideration of the content and face validity of the test of mathematics high-order thinking skills by three experts judgment, the results showed that they provide uniform conclusions.

d. Based on the results of the test trials for each school show that the test can be used to measure the mathematics high-order thinking skills.

**ACKNOWLEDGEMENT**

Thanks to the Directorate General of Higher Education Department of Education which has funded this research through a research grant competition in 2013 in collaboration with the Research Institute of the University of Lampung, through a Letter of Implementation Research Agreement No.: 393/UN26/8/PL/2013, also the chairman and teachers and students of SMPN 4, SMPN 5, and SMPN 22 Bandar Lampung.
REFERENCES

LEARNING GEOMETRY THROUGH PACE MODEL ASSISTED GEOGEBRA AS EFFORT TO IMPROVE MATHEMATICAL COMMUNICATION AND REASONING ABILITIES IN JUNIOR HIGH SCHOOL STUDENTS

Nurfadilah Siregar
Students of Indonesia University of Education

ABSTRACT
This study aims to determine the increasing of mathematical communication and reasoning abilities of students who received learning geometry through the PACE model of assisted GeoGebra and determine the relationship between mathematical communication and reasoning abilities of students. The type of this study was quasi-experiment with samples are students in class VII A and B on one of SMPIT in West Bandung regency. VII A class called the experimental group, while the control group VII B class. Both groups were given a pretest, learning through the PACE model of assisted GeoGebra for the experimental group and the conventional learning to control group, and last a posttest. Hypothesis were tested through parametric test (t-test) and non-parametric tests (Mann-Whitney test), and Spearman's correlation. The results showed both of the groups have the same quality, which is in the middle category of mathematical reasoning ability and low category for mathematical communication. However, after the hypothesis was tested with a predetermined formula obtained that mathematical reasoning ability and communication better in experimental group than control group. In addition, there was a relationship between the ability of students' mathematical reasoning and communication in the middle category of experimental group.

1. INTRODUCTION
According to survey results IMSTEP-JICA (Ulya, 2007), one of the causes of low quality in students' understanding in mathematics because mathematics instruction focuses only on the examples worked out by the teacher. Along this time learning of mathematics just focus on practice that similar with samples, the effect reasoning ability of students are deficient. Supported by report from TIMMS that learning mathematics in Indonesia, more emphasis on basic skills, some understanding of the concept and practice, and just little about the ability of reasoning, communication, application in real life and so forth. Whereas the ability of students' mathematical reasoning and communication is an essential part that must be developed.

Reasoning is a mental activity to improve thinking by looking at some facts or principles that resulting in mental processes such as knowledge or conclusions. According Keraf (Sadik, 2003) reasoning is thinking process that connects the facts which known to lead into a conclusion. In addition, which plays a role in improving mathematics education is communication. Both oral and written communication takes students on a deep understanding of mathematics and able to solve the problem correctly. On mathematics learning activities in the classroom, student doing communication while learning math and learn to communicate mathematically. For example, when discussing in learning mathematics, every students will ask or answer a question with an explanation about the ideas, situation, or mathematical relation by oral and writing, and denotes a situation, drawing, diagram, or tangible objects into symbols, ideas, or mathematical models.

One of mathematical material that have characteristics as above is geometry. Geometry as a field of study in mathematics has many portion to be studied by students in schools. From distribution of competency standards for secondary school education, geometry getting the largest portion (41%) compared with other materials such as algebra (29%), number (18%), statistical and probability(12%). However, students mastery to understand geometry concepts still low and needs to be improved (Abdussakir, 2009). This is not only happening in Indonesia, even though in the international (Laborde et al in Nurhasanah, 2010). Likewise with Jiang (2008) who said that one part of a very weak math absorbed by students in school is geometry. He reveal that the majority of students who enter in high school have limited knowledge on experience of geometry.
According Abdussakir (2009) geometry has a special position in the secondary mathematics curriculum, because of the concepts contained therein. From psychology point of view, geometry is an abstraction of representation visual and spatial experience, such as plane, patterns, measurement and mapping. From a mathematical standpoint, geometry provides approaches to solving problems, such as images, diagrams, coordinate systems, vectors, and transformation.

Sabandar (Mulyana, 2003) states that the purpose of teaching geometry in schools is expected to provide a systematic attitude and habits for students to be able give an idea of the relationships between models geometry and classifications among them. It is necessary to provide opportunities and adequate equipment so that students can observe, explore, try, and discover the principles of geometry through informal activities. Then continue with formal activities and apply what they learn.

To facilitate learning of geometry better than ever, the author raised a Model Project, Activity, Cooperative, Exercise (PACE) as consideration to improve mathematical communication and reasoning ability in junior high school students. According to Lee (1998) PACE model is constructivist learning model based on the following principles: (1) Students learn better by constructing their own knowledge through a guided process, (2) Training and feedback is an important element to understand new concepts, and (3) Problem solving actively in the group develop into active learners.

PACE model of learning activities involves computers both when learning in the laboratory and regular classroom. In next activity, students are grouped in small groups and placed on a computer. Computer usage involves one of dynamic geometry software, GeoGebra. By this software, students expected have better understanding the concepts in geometry. According Wees (Rahman, 2010) there are some considerations about the use of dynamic geometry software such as GeoGebra in mathematics, especially geometry, such as allowing students to be active in building understanding of geometry.

GeoGebra program allows a simple visualization of complex geometric concepts and help improve students' understanding of the concept. Putz (Rahman, 2010) added when students use GeoGebra they will always end up with a deeper understanding of material geometry. This may happen because the students are given a strong visual representation of object geometry, students engage in activities that lead to construct a deeper understanding of geometry so that students can doing good reasoning. By PACE model, reviewed the overall that statistical reasoning ability of students is better than using the conventional model (Dasari, 2009).

From the above description, the main problem in this study is "Does learning geometry through the PACE model of assisted GeoGebra can improve mathematical communication and reasoning ability in junior high school students?". Formulation of the problem can be broken down into the following research questions as follows: 1) How does the quality increase students' mathematical reasoning abilities that got learning geometry through the PACE model of assisted GeoGebra and students who received conventional learning? 2) What is the quality improvement of communication skills of students who got the mathematical learning of geometry through the PACE model of assisted GeoGebra and students who received conventional learning? 3) Is the increase in mathematical reasoning abilities of students who received learning geometry through the PACE model of assisted GeoGebra better than students who received conventional learning? 4) Is the increase in mathematical communication of students who received learning geometry through the PACE model of assisted GeoGebra better than students who received conventional learning? 5) Is there a relationship between reasoning and mathematical communication skills of students in learning geometry through the PACE model of assisted GeoGebra?

Mathematical Reasoning and Communication, and Learning through Pace Model Aided Geogebra

Shurter and Pierce (Dahan, 2004) explains reasoning is defined as the process of reaching logical conclusions based on the facts and relevant sources. In line with that Keraf (Sadiq, 2004) said that reasoning as a process of thinking that seeks connection between the facts or evidences are known that lead to a conclusion. Mathematical reasoning is required to determine whether a mathematical argument is right or wrong and also used to construct a mathematical argument. According to Mullis (Ulya, 2007) mathematical reasoning include the ability to discover conjecture, analysis, evaluation, generalization, connections, synthesis, problem solving is not routine, justification or evidence, and mathematical communication ability.

Students are said to be capable of reasoning when he/she able to use the pattern and attribute of reasoning, mathematical manipulation in making generalizations, compile evidence, or explain mathematical ideas and statements (Wardhani, 2008). In this regard the technical explanations charging indicator report cards described that students have the ability reasoning is: (a) asking the allegation; (b) mathematical manipulations (e) draw conclusions, compile evidence (proofing), giving reasons or evidence for the truth of the solution, (d) draw conclusions from the statement; (e) check the validity of an argument; (f) discovering patterns or mathematical nature of the symptoms to make generalizations.

Indicators of mathematical reasoning in this research is the ability to provide explanations by using pictures, facts, and relationships in solving problems; ability to solve mathematical problems by following logical arguments, as well as the ability to draw logical conclusions.
Another capability as more as important that must have by students is mathematical communication ability. In the learning process on classroom, teachers actually have to communicate with students. But communication ability that appropriate to the objectives of mathematics subjects not clearly. Mathematics is often conveyed in symbols, oral or written about mathematical ideas is not always recognized as an important part of mathematics education (Wahyuddin, 2008). Wahyuddin (2008) also revealed that communication is a way of sharing ideas and clarifying understanding.

Sumarman (2010a) describes the activities that belong to mathematical communication includes: (a) declare a situation, drawing, diagram, or tangible objects into the language, symbols, ideas, or mathematical models; (b) explain the idea, situation, and mathematical relationships orally or in writing; (c) listening, discussing, and writing about mathematics; (d) read with understanding written a mathematical representation; (e) Disclose back one paragraph description or math in their own language.

Related to mathematics courses at the school described in the content standards that students are said to be able to communicate mathematically when he/she is able to communicate ideas with symbols, tables, diagrams, or other media to clarify the situation or problem (Wardhani, 2008).

Indicator of mathematical communication in this study was measured by a written communication about achievement test that includes the ability to explain a problem in writing in the form of an image (drawing); ability expressed a concern in writing in the form of mathematical models (mathematical expression), as well as ability to explain an idea or situation of a given image with your own words in writing (writing).

PACE model was first published around 1990 by a professor of statistics from Central Michigan University named Carl Lee. This learning is based on constructivism, the basic principle are: (1) students learn better by constructing their own knowledge through a guided process, (2) training and feedback is an important element to understand new concepts, and (3) active problem solving in groups develop students into active (make active atmosphere).PACE model study intends to integrate new learning innovation and preserve the advantages of traditional learning approach. It provides a structure learning approach that blends project (project) and student activities (hands-on activities) are organized in groups or cooperative (cooperative) in a computer classroom. This model of learning puts students at the center, where the teacher as a facilitator who leads and guides the students to discover and understand new concepts, students also have ample opportunity to work as a team that will write and submit its report in a presentation to the class.

Each component of PACE itself is not new. PACE model provides a framework for integrating projects and worksheets are provided to the students are cooperative in a computer classroom. This model puts students at the center learning, teachers as facilitators who lead and guide the students to discover and understand new concepts. In addition, students also have ample opportunity to work as a team that will write and submit a report of its work in a presentation to the class (Lee, 2000). Compared to the conventional classroom learning environment, a learning environment PACE model provides many opportunities for students to develop their communication skills and their mathematical reasoning, to explore, search for solutions, communicate ideas, adapting resolution procedures, as well as working in a group. Here is an explanation of each component in the PACE model:

Projects:
Facilitate the students to have the opportunity to analyze the data, the report group work, and presentation of results. Project be structured with topic selection guidance of teachers or students themselves who choose it. It is intended to further arouse students’ motivation in learning geometry. The project was given after the essential concept behind the project is introduced first.

Activities:
Each activity is designed to introduce some new concepts and reviewing concepts already learned. Students are guided to work in a series of activities in the implementation of the activities of the group and present the results of their work to the class.

Cooperative learning:
There are many different types of cooperative, on the model of PACE, group work is implemented depends on the size of the class and the setting in the field. For class sizes of about 40 students and computer lab classes that have 15 to 20 units of computers, small group discussions around 2-4 people felt more appropriate. In learning geometry, the use of the software needed to facilitate student understanding and doing construction geometry, in this case GeoGebra as an option.

Exercises:
Exercise basically is an important element that will help the students to understand the concepts and skills of the material that has been studied previously. Students can use the computers to help with training provided.
PACE model is not sequential. That is, the placement of the project at the end of the lesson begins with giving worksheet (handout activity) is not a problem. However, the provision of early learning exercise should not be done, because students should already have the knowledge and concepts to given problem. In this study, the class begin with classifying learning activities (cooperative) students in a group that contains 2-4 people. Student-directed learning in the classroom to the computer lab working on a worksheet (handout activity) as a group. In working on the student handout activity using software GeoGebra to solve and investigate problems. At that time, teachers guide students through questions and answers, and brief explanation. The end, teacher give sheets to students as an exercises, students in these activities are no longer working in groups but do it by their self.

Once new concept (new knowledge) is given to students, teachers gave some projects that can be chosen by the students to be completed at future time, this activity also requires the implementation of group. After the agreed time arrived, teacher provides the opportunity for students to present their discussion in front of the class. Through these learning activities (PACE) students are expected to acquire knowledge of the concepts learned. In addition, the ability of students reasoning and communication will grow even much better.

Software that used in this study is GeoGebra. GeoGebra is a dynamic geometry software examples or interactive geometry software which is free used and obtained in www.geogebra.org. This tool was introduced by Markus Hohenwarter in 2001. During this initial appearance GeoGebra only consists of two views, which display algebra and graph display. GeoGebra latest version adds a third appearance on the GeoGebra: spreadsheet view.

2. RESEARCH METHOD

This research was quasi-experimental. Design research is a non-equivalent groups pretest-posttest design (McMillan & Schumacher, 2001). This design is chosen because the researchers assumed that the subject was not randomized, but the researchers received a sober state of the subject. In this study, there is also a pretest, a different treatment (treatment), and posttest. The following are non-equivalent groups design study pretest-posttest.

<table>
<thead>
<tr>
<th></th>
<th>Pretest</th>
<th>Treatment</th>
<th>Posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>O</td>
<td>X</td>
<td>O</td>
</tr>
</tbody>
</table>

Description:
O: pretest or posttest
X: Learning geometry through the PACE model of assisted GeoGebra

Learning in two classes are done by the researcher. This is done so that learning that has been planned by the researchers can be accomplished with maximum.

Population in this research were all students of class VII SMPIT in West Bandung Regency Academic Year 2010/2011. Selection of grade level, in this case the class VII, because researchers assume that the reasoning and communication ability of students at that level is still lacking. Of some existing SMPIT, SMPIT Fithrah Insani selected as study sites. SMPIT Fithrah Insani chosen because it has adequate facilities for research, such as the availability of notebooks on each student. The sample in this study consisted of two classes VII A and B, hereinafter referred to as the experimental and control group.

There are three major steps in the procedure that researchers do study, namely preparation, implementation, and data processing.

a. Preparation

At this stage there are some activities that are carried out by researchers, including identifying research problems, making the research proposal, the seminar proposal, and the improved proposal seminar, the activities included in the preliminary study. Furthermore, researchers compiled the lattice problem to manufacture research instruments were test mathematical communication and reasoning ability, the next step is making the rating scale student attitudes and statements related to the indicators that have been made. After research instruments examined by the supervisor, then conducted trials at two schools instruments equivalent to see legibility and empirical validity. The trial results were then analyzed. From the analysis of a few selected items of validity and reliability, then the instrument is ready to be used as a measuring tool. Preparation of learning tools for classroom experiments in the form of lesson plan and student activity sheets are also not spared from the preparatory steps that researchers do. After learning devices inspected by a supervisor the next step is to carry out research.
b. Implementation
Prior research conducted first step is to determine the population and sample will be used as research subjects, and then take the research permit at school. The next step is to determine the experimental and control groups based on consideration of the school, who continued with the pretest in each group there. The next activity is the provision of treatment in each group in the form of learning through the PACE model of GeoGebra aided in the experimental group and the control group of conventional learning. Materials provided on the triangle conducted over six sessions. After completion of learning activities, each group was given the posttest in order to see the results of student learning after being given treatment.

c. Data Processing
Based on the instrument used in this study, in the form of test data, the data were then processed through the following stages: Data such as the results of tests of mathematical reasoning and communication ability quantitatively analyzed using a statistical test. To determine the statistical test to be used, the data must first be tested for normality and homogeneity of variance. Before the test is performed should be determined in advance the average score and standard deviation for each group. For more details, the following is done in stages that researchers test data processing.

a. Scoring students' answers according to answer keys and scoring guidelines that have been made.
b. Calculate descriptive statistics pretest scores, posttest, and gain includes minimum score, maximum score, the average and standard deviation.
c. Calculate the increase in communication and mathematical reasoning abilities of students obtained from pretest and posttest scores using the normalized gain developed by Hake (1999) as follows:

\[ \text{Normalized Gain (g)} = \frac{\text{skorposttest} - \text{skorpretest}}{\text{skorideal} - \text{skorpretest}} \]

the gain index criteria:

<table>
<thead>
<tr>
<th>Criteria Score</th>
<th>Normalized Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain score</td>
<td>Interpretation</td>
</tr>
<tr>
<td>( g &gt; 0,70 )</td>
<td>High</td>
</tr>
<tr>
<td>( 0,30 &lt; g \leq 0,70 )</td>
<td>Middle</td>
</tr>
<tr>
<td>( g \leq 0,30 )</td>
<td>Low</td>
</tr>
</tbody>
</table>

d. Test normality of the data on any pretest scores and normalized gains for each group. Calculation through the Kolmogorov-Smirnov Test One Sample. According Rusefendi (1993) test is used instead of chi-square test for smaller sample sizes.

e. Variance test. Testing variance between experimental and control groups was conducted to determine whether the two groups of equal variance or different. The test is performed for the data normalized gain score reasoning and communication skills. Statistical test using Levene Test. To test the equality of two average pretest score on the data both experimental and control groups for each of reasoning and communication skills. Further to test the difference in the two mean gain score for the data is normalized in both groups.

f. If both the average scores are normally distributed and homogeneous then the statistical tests used were t-test. If the data is not normally distributed, the test statistic used is the non-parametric test, the Mann-Whitney test.

g. Further see the relationship between communication skills and mathematical reasoning in the experimental group. Correlation test was used to determine whether there is a relationship or association between two or more variables are observed. To see the correlation in both capabilities, used data from the experimental group posttest scores. Statistical test for data not normally distributed, non-parametric test was used Spearman's correlation.

3. RESULT AND DISCUSSION
Data analysis includes descriptive and inferential statistical analysis. Descriptive statistical analyzes performed to obtain a picture of students' ability before and after a given treatment, while the inferential statistics for withdrawal analysis conclusions on the differences increaseability of students achieved. Data
processing was performed using Microsoft Office applications Excel and SPSS 18 software. Here’s a description and discussion of research data.

**Combined Score Descriptive Statistics Mathematical Communication and Reasoning Ability**  
(Table 02)

<table>
<thead>
<tr>
<th>Test</th>
<th>Experiment</th>
<th>Control</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>$X_{\text{min}}$</td>
<td>$X_{\text{max}}$</td>
<td>$\bar{x}$</td>
<td>s</td>
<td>N</td>
<td>$X_{\text{min}}$</td>
<td>$X_{\text{max}}$</td>
<td>$\bar{x}$</td>
</tr>
<tr>
<td>Pretest</td>
<td>18</td>
<td>1</td>
<td>13</td>
<td>5.61</td>
<td>3.39</td>
<td>24</td>
<td>1</td>
<td>15</td>
<td>5.58</td>
</tr>
<tr>
<td>Posttest</td>
<td>18</td>
<td>8</td>
<td>33</td>
<td>16.8</td>
<td>6.57</td>
<td>24</td>
<td>4</td>
<td>29</td>
<td>13.12</td>
</tr>
<tr>
<td>g</td>
<td>18</td>
<td>0.17</td>
<td>0.96</td>
<td>0.40</td>
<td>0.21</td>
<td>24</td>
<td>0.03</td>
<td>0.74</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table (02) presents descriptive statistics on the combination of two abilities, mathematical reasoning, and communication from pretest, posttest, and normalized gain (g) by experimental and control group. Pretest and posttest scores are expressed in scores 0-34, while the normalized gain (g) is expressed in a score of 0-1. If staring the third at the table above shows that the average ability of experimental group is better than control group. To ascertain whether the increased ability of reasoning and communication experimental and control groups differed significantly or not, the researchers conducted tests on average, first tested for normality and homogeneity tests, with each test significance level of 0.05 or 95% in confidence level.

**The Similarity Test Average Pretest Score**  
(Table 03)

<table>
<thead>
<tr>
<th>Ability aspects</th>
<th>Group</th>
<th>Mann-Whitney</th>
<th>Asymp.Sig. (2-tailed)</th>
<th>Conclusion</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematics</td>
<td>Experiment</td>
<td>190,5</td>
<td>0,51</td>
<td>Accept H₀</td>
<td>There is no difference</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Communication</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematics</td>
<td>Experiment</td>
<td>169,5</td>
<td>0,22</td>
<td>Accept H₀</td>
<td>There is no difference</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table (03) above can be seen Asymp.Sig value is greater than the chosen significance level. This means an average of experimental and control groups for both abilities is no different. In other words, both groups had the same initial capabilities in reasoning and communication skills. Thus, before the experiment is performed both groups had equal ability in aspects of mathematical reasoning and communication. Thus, provided that the two groups should have the same initial capabilities are met.

Have known before that the equality test average pretest showed no significant difference in students’ mathematical reasoning to experimental and control group. Further, the analysis of improvement in mathematical reasoning abilities both groups. In general, as shown in Table (02) that the average score of students' mathematical reasoning ability in experimental group showed an increase of about 2.4 more than control group. For the spread of mathematical reasoning abilities after learning, the experimental group is more spread out than the control group and standard deviation of experimental group appear larger. However, to prove that the increase in students' mathematical reasoning ability experimental group is better than the control group required further statistical tests.

Type of statistical test used is known after tested normality of data distribution and homogeneity variance. If the data meet the requirements of normality and homogeneity, the mean difference test using t-test, whereas if the data is normal but not homogeneous then use the t-test, and for data that does not meet the requirements of normality, using the non-parametric test, ieUji Mann-Whitney.
Differences Test Average Normalized Gain Mathematical Reasoning And Communication

(Table 04)

<table>
<thead>
<tr>
<th>Ability aspect</th>
<th>Group</th>
<th>t-test</th>
<th>Asymp.Sig. (2-tailed)</th>
<th>Asymp.Sig. (1-tailed)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning mathematics</td>
<td>Experiment</td>
<td>2.53</td>
<td>0.02</td>
<td>0.01</td>
<td>Reject H0</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ability aspect</th>
<th>Group</th>
<th>Mann-Whitney</th>
<th>Asymp.Sig. (2-tailed)</th>
<th>Asymp.Sig. (1-tailed)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Communication</td>
<td>Experiment</td>
<td>1.03</td>
<td>0.07</td>
<td>0.04</td>
<td>Reject H0</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Having regard to the above test results, it can be concluded that the average normalized gain reasoning and communication ability better than the experimental group average normalized gain control group. It also shows that the PACE model of learning geometry through better GeoGebra aided in improving students’ mathematical reasoning ability compared with conventional learning.

Furthermore, an increase in the ability to see the quality of students’ mathematical reasoning and communication can be based on the criteria of normalized gain expressed by Hake (1999).

Normalized Gain Classification In Mathematical Communication And Reasoning Ability

(Table 05)

<table>
<thead>
<tr>
<th>Ability aspect</th>
<th>Improvement category</th>
<th>Experiment</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean</td>
<td>N</td>
</tr>
<tr>
<td>Reasoning mathematics</td>
<td>High</td>
<td>3</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>11</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>4</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>18</td>
<td>0.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ability aspect</th>
<th>Improvement category</th>
<th>Experiment</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean</td>
<td>N</td>
</tr>
<tr>
<td>Communication</td>
<td>High</td>
<td>1</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>6</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>11</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>18</td>
<td>0.30</td>
</tr>
</tbody>
</table>

In mathematical reasoning ability, seen majority of the students in experimental group achieved score normalized gain by medium category, there are only 3 people 4 people categorized as high and low. In the control group, a category seen an increase in normalized gain balance, i.e. 11 people with lower category and 12 people are being medium. However, when viewed as a whole the two groups reach the normalized gain in medium category. It can be said the quality of both groups improved reasoning abilities are not much different, which is located in the middle category.

For mathematical communication ability, seen majority of students in experimental group achieved score normalized gain by low category, there are only 6 people and 1 person categorized as being high. At control group, category increased normalized gain appears to be low, i.e. 19 people with low category and only 4 people in the middle. Both groups achieving overall gain normalized to low category. Can be said the quality of increasing in reasoning ability is not much different from the two groups, which are in the low category.
Although the results of statistical tests show the average normalized gain between the two groups differ, but the difference is not significant.

To determine whether there is a relationship in both studied mathematical ability, next step is to test the correlation. Such measures has been done before, to determine the test statistics that will be used, it must first be known whether the tested data comes from a normally distributed population or not. The data is used posttest score of the experimental group. Summary results of the calculation of SPSS output shown in the table below.

<table>
<thead>
<tr>
<th>Ability aspect</th>
<th>Mathematical communication</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spearman’s rho</td>
<td>Mathematical reasoning</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table (06) shows that the value of Sig. smaller than the significance level. It means that there is a correlation in both the ability, mathematical reasoning and communication. The magnitude of the correlation between the ability is 0.56 and significant at 0.02. This suggests there is a fundamental and direct correlation. In other words, if students' mathematical reasoning ability in experimental group is high, so does communication, and vice versa.

4. CONCLUSIONS

Based on research over teaching geometry through the PACE model of assisted GeoGebra following conclusions can be drawn:

1) Mathematical reasoning ability of students who are learning geometry through the PACE model of assisted GeoGebra better than students who learned through conventional geometry.
2) Mathematical communication skills of students who are learning geometry through the PACE model of assisted GeoGebra better than students who learned through conventional geometry.
3) There is a significant relationship between communication and mathematical reasoning abilities in students who learn geometry through the PACE model of assisted GeoGebra. Degree of correlation in both the ability to be in the middle category.

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THE ANALYSIS OF ALGEBRAIC THINKING SKILLS OF THE STUDENT IN SECONDARY SCHOOL

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ABSTRACT
Algebra is a branch of mathematics which is very important in building the mathematics character of the students, because the children are taught to think numerical algebra, critical, creative, reasoning and abstract thinking. Algebra is the gateway to further understanding of mathematics. One approach to explore the students’ understanding of algebra is the algebraic thinking. Algebraic thinking becomes a key for learning and teaching mathematics to prepare students succeed in math. This paper will discuss the analysis of algebraic thinking skills of the student in secondary school.

1. INTRODUCTION
Algebra is an area of study that must be mastered when students studying mathematics at secondary school (SMP) and high school (SMA) in the Kurikulum Tingkat Satuan Pendidikan (KTSP) [1]. This is in accordance with the National Council of Teachers of Mathematics [2] which has been determined that the expectations for middle and high school algebra are as follows. 1) In grades 6-8 all students should represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible, symbolic rules; 2) In grades 9-12 all students should use symbolic algebra to represent and explain mathematical relationships.

Further NCTM [3] states that developing algebra skills should start from kindergarten and became the focus of learning mathematics from kindergarten through 12th grade. This is in accordance with the Windsor [4] which states that algebra is the gateway to understanding further math. Knowing algebra opens doors and expands opportunities, instilling various mathematical ideas that are useful in many professions and careers.

Krieger [5] states that the algebraic thinking into a hold for the learning and teaching of mathematics to prepare students to succeed in math. This statement is in line with the phrase Math Panel [6] who say that one approach is to create a more integrated math curriculum is to develop algebraic thinking students at all grade levels.

Based on the above it can be said that the algebraic thinking skills should be owned by the student at the school so that they can succeed in mathematics that will be also their success in life. Buton the other hand still found a problem for students in algebraic thinking skills.

It has been identified that there is a decline in the participation of students studying advanced mathematics in junior high. In Australia as revealed by Windsor [7], only 12% of students enrolled in advanced mathematics courses. This could be due to declining student achievement in middle school mathematics. Decline in student mathematics achievement starts in junior high where they were introduced to algebra for the first time. In many cases, Algebra intimidates the students and influences their attitudes toward mathematics [8]. Students seem todo well in arithmetic, but have difficulty with the concept of algebra.

Decline in participation has a negative impact on the employment opportunities available to the community. Even more worrying is the fact that a limited understanding of mathematics can directly inhibit the effectiveness of a person to participate in a modern society where information, discussion and rhetoric in some cases more use of mathematics.

Based on the above, then this paper will discuss the analysis of algebraic thinking skill of junior high school students seen by the components of algebraic thinking, i.e. problem solving, representation, and reasoning in the field of algebra. The research objective was to determine the algebraic thinking skill of junior high school students and mistakes are made when students solve algebra.
The research method used is descriptive research as it relates to data collection to provide an overview of a phenomenon or event and the questions relating to the subject of the current study[9]. Symptoms or events in this study are the algebraic thinking skills students. Subjects were 8th grade junior high school students in three schools representing high school level (SMP A), medium level (SMP B), and low level (SMP C) in North Jakarta. Each school is selected one class to be the subject. The selection of schools and classes are conducted by random selection. The number of subjects entirely 98 people, with details of 25 people from the SMP A, 35 people from the SMP B, and 38 people from SMP C.

2. ALGEBRIC THINKING SKILLS

According to Kaput [10], one approach is to explore the students' understanding of algebra is algebraic thinking. Algebraic thinking is an essential and fundamental element from mathematical thinking and reasoning [4]. Algebraic thinking activities began in pattern recognition and common mathematical relationships among numbers, objects, and geometric shapes. This opinion more emphasis on reasoning to define algebraic thinking. According to McClure [6], algebraic thinking is thinking in certain ways, including analyzing the relationship between quantities, consider the structure, studying the change, generalization, problem solving, modeling, justifying, proving, and predicting. This statement also implies that the activity of algebraic thinking is reasoning activities, but expanded with problem-solving activities. Algebraic thinking is formed innumeracy and computing kills, geometric reasoning and skills related to the concept of measurement introduced and taught in elementary and secondary schools (Kaput) [4]. Extending the statement Kaput, Windsor [4] states that the algebraic thinking is more than just numbers perspective algebra, algebraic thinking into account a variety of student activities in doing Mathematics. Windsor opinion about algebraic thinking more emphasis on problem-solving activities.

According to Krieger [5], there are two major components in thinking of algebra, namely with respect to: 1) the development of mathematical thinking tools; and 2) the study of fundamental algebraic ideas.

1) Mathematical Thinking tools include:
   a. problem solving skills
      (1) Using a problem solving strategy
      (2) Exploring approaches / resolution multiplication
   b. representation skills
      (1) Showing relationships visually, symbolically, numerically, and verbally
      (2) Translating between different representations
      (3) Interpreting the information in the representation
   c. reasoning skills
      (1) Inductive reasoning
      (2) Deductive reasoning

2) Fundamental algebraic ideas include:
   a. algebra as generalized arithmetic
      (1) Conceptually based computational strategies
      (2) The ratio and proportion
      (3) Estimation
   b. algebra as the language of mathematics
      (1) Meaning of variables and variable expressions
      (2) Meaning of solution
      (3) Understanding and using properties of the number system
      (4) Reading, writing, manipulating numbers and symbols using algebraic conventions
      (5) Using the equivalent symbolic representation for manipulating formulas, expressions, equations, inequalities
   c. algebra as a tool to study the function and mathematical modeling
      (1) Seeking, expressing, generalizing patterns and rules in the context of real world
      (2) Representing mathematical ideas by using equations, tables, graphs, or words
      (3) Working with the pattern of input / output
      (4) Developing coordinate graphing skills

Examples of Algebraic Thinking

The following is an example of a problem that the solution describes the students in algebraic thinking.
A garden is surrounded by a single row of tile barrier as described below. (A garden with a length of 3 requires 12 tiles as a border).

The questions are:

a. How many tiles are required to park barrier with a length of 12?

b. How many tiles are required to park barrier of length "n"?

c. Explain to find the length if 52 tiles are used for barrier!

Several strategies can be used to solve the problem of the park, for example, create a table, looking for patterns, using models and diagrams, and work backwards. Another strategy used is numerically represents completion, symbols, graphs, and verbal. Thus to solve this park takes students' skills in problem solving, representing in various forms (tables, diagrams, symbols, graphs, verbal), and reasoning (looking for patterns, making generalizations).

Noting the definitions proposed by the experts on algebraic thinking, so the definition of algebraic thinking is the students' skills in problem solving, representation, and reasoning in the context of algebra. Thus, components of algebraic thinking are problem solving, representation, and reasoning.

3. RESULT AND ANALYSIS

Instruments used in the research is algebraic thinking essay test with the number of items was 13 items, which consists of 3 items test problem solving skills, 4 items test representation skills, and 6 items tests reasoning.

The average results of tests of algebraic thinking skills are illustrated in Figure 1 below.

Figure 1. The Average Value of Algebraic Thinking Skills

Figure 1 shows the average overall algebraic thinking skills in every school is still very low, which is still under 50 (with a scale of 100). The order of the average value of algebraic thinking skills in order of level of school. If seen each component algebraic thinking (problem solving, representation, and reasoning algebra), the average value for each component of each school is presented in Figure 2 below.

Figure 2. The Average Value for Each Algebraic Thinking Component
Proceeding
International Seminar on Mathematics, Science, and Computer Science Education

From the figure it is seen that in every school the lowest value among the three components of algebraic thinking is the value of problem-solving abilities. However, the highest score in each school varies. Highest value in the SMP A contained in reasoning abilities. SMP B and SMP C have the highest value on the ability of the representation abilities.

Based on Figure 1 has been shown that the order of algebraic thinking overall value according to order of level of school. Similarly, the value for each component of algebraic thinking, according to order of level of school. This is shown in Figure 3.

![Figure 3: The Average Value for Each Algebraic Thinking Component in Every School](image)

Figure 3. The Average Value for Each Algebraic Thinking Component in Every School

The value of algebraic thinking skills overall and the value of each component of algebraic thinking show that the order of values associated with school level. Based on these results can be justified that the school-level effect on the algebraic thinking skills. It can be caused by high-level school standards different from medium and low levels of school. With different entry standards can be assumed that they are also different initial ability. This is in line with the opinions Arends [11] who said that the students' ability to learn new ideas depend on their prior knowledge and previously existing cognitive structures.

Analysis of students' responses for each component of algebraic thinking is presented below.

**a. Algebraic Problem Solving**

Problem solving 1

a. Determine the range function.

b. Whether the range function is a set? Explain your opinion, what is the range?

c. Determine $f(7), f(9), f(11),$ and $f(13)$. What conclusions can you get?

Students' answers showed that the students' misconceptions in algebraic form. The students understand that $2x$ is not $2 \times x$, but these students understand that $2x$ as the number 23. So if $x = 11$, then $2x = 211$.

![Figure 4: The Answers Problem Solving 1](image)
Problem Solving 2

Curves sale and purchase of a fruit trader are represented by two curves in the figure beside. The questions are:

a. Find the equation of the curve sale and the curve purchase.

b. If the trader is buying 50 pieces, how many rupiahs of capital should be provided?

c. If the trader is selling 50 pieces, how many rupiahs earned money?

d. If many pieces are sold in 35 pieces, how much profit the trader?

e. When does occur the losing traders?

One of the students responded as shown in the side. The student does not have a strategy to resolve the problem, so he just made the estimate by using picture. And he made improper estimation.

b. **Algebraic Representation**

Representation Problem 1

Find the area of the shaded region in the image below in the simplest form.

![Figure 5. The Answer of Problem Solving 2](image)

Some the student’s answers to the following have not demonstrated the ability of students to translate mathematical problems of the image form to word/math symbols.
The students answer’s to the following indicate that they still do not understand operations on algebraic variables.

For example:

\[ 3s \times 2s = 6s^2 \]

should be

\[ 3 \times 2 = 6s \]

\[ 3^2 = 9 \]

should be

\[ 3 \times 9 = 27 \]

The following students did not understand the algebraic form. Students are directly using numbers to solve problems.

Representation Problem 2

A rectangular has area \( x^2 + 8x + 15 \) cm\(^2\) and width \( x + 3 \) cm. Determine the length and circumference of the rectangle in simplest form!

The following responses indicate that the students still do not understand the operation of algebraic forms, especially factoring and the division of the algebraic form.
In the division of $\frac{x^2 + 8x + 15}{x + 3}$, these students make mistakes when eliminating $x$ and divide 15 by 3.

The following students showed good ability in factoring of algebraic form and algebraic division, but this student made a mistake at the end of the answer. The students’ answers are correct, but in the end he reduces it to $x + 4$.

Likewise with this student, he already understands factoring and division of algebraic forms, but students make mistakes when adding up, namely: $x + 5 + x + 5 + x + 3 + x + 3 = x^2 + 10 + x^2 + 6 = x^4 + 16$.

Meanwhile, following the two students did not understand what to do with the matter.
Representation Problem 3
Given two sets of numbers $M = \{6, 7, 8, 9, 10\}$ and $N = \{8, 9, 10, 11, 12, 13\}$.

a. Draw an arrow diagram that fulfills relations “are two of the least” of the set $M$ to the set of $N$.
b. Express the relation as a set of sequential couples.
c. Express the relation with the Cartesian diagram.
d. Whether the relation is a function? Explain!

Answer the following two students indicated that they occurred misconceptions about the location of a point in Cartesian coordinates. They can present a problem to a diagram and set correctly, but they made a mistake when presenting problems in Cartesian coordinates.

![Image of Representation Problem 3 diagram]

Figure 14. The Answer of Representation Problem 3

c. **Algebraic reasoning**
Reasoning Problem 1

Note the graph of the function $f$ in Cartesian coordinates in the following.

a. Determine the range of the function $f$.
b. Determine the value of the function $f$ for $x = 0, x = 1, x = 2, x = 3$ and $x = 4$. What the pattern do you get?
c. Find a formula the function $f$ based on (b)?

![Image of Reasoning Problem 1 graph]

Students answer as follows.

![Image of Students' answers]

Figure 15. The Answer of Reasoning Problem 1
This answer shows that students understand that the formula of linear function is \( f(x) = ax + b \). However, these students do not understand the points that describe the function of the Cartesian coordinate plane.

Supposed to \( x = 1 \rightarrow f(1) = 3 \) so that \( a + b = 3 \)

**Reasoning Problem 2**

The table to the right shows the high germination (in mm) and the length of the growing season (in hours).

a. How tall sprouts at 9th?

b. How many hours have the sprouts height 10.5 mm?

The following is student response.

![Table image]

**Figure 16. The Answer of Reasoning Problem 2(a)**

To answer this problem, students do not make a pattern, but the list is expanded so that they find the answers. Although the student answers correctly, the pattern of thinking is not shown inductive thinking. Students will have trouble when the requested problem is high germination at the 30th.

Other students answer the following.

![Figure 17. The Answer of Reasoning Problem 2(b)]

Students who answered this already show the ability of reasoning.

4. **CONCLUSIONS**

   Based on the data presented in figure 1, figure 2, and figure 3, both overall and for each component of algebraic thinking skills in junior high school students in North Jakarta is still very low. Student mistakes in answering algebraic thinking problems of whom are students experiencing difficulty when working in a variable. As expressed by Kilpatrick, et al. [12] that for most students, learning algebra is a totally different experience from learning arithmetic, and they have difficulty in the transition from arithmetic to algebra. For example, in the arithmetic in elementary school, there is a relation between addition and subtraction operations. Form of the sum \( 35 + 42 = 77 \) is equivalent to \( 35 = 77 - 42 \). However, students who are just learning algebra in junior high will be difficult to discover the form \( x + 42 = 77 \) which is equivalent to \( x = 77 - 42 \).

   In line with the opinion Kilpatrick, et al. [12], Greer [10] also said that for most students, the experience of "manipulating symbols" generally give a negative impression. When students learn algebra, they are no longer just focus on counting, but they need to understand and represent the relationship through the use of letters and numbers. For students, the change in emphasis between elementary and secondary school can create a conceptual barrier to mathematics achievement. To remove this barrier, a new paradigm is evolving in math education—one that calls for teachers at all grade levels to help students develop “habits of mind that attend to the deeper underlying structure of mathematics” (Katz, 2007) [6].

   Of course, this problem can be used as material for a study to determine the causes of weakness in algebraic thinking skills students. The research could also be developed further is how learning strategies are used in order algebraic thinking skills students can increase.
REFERENCES

DEVELOPING INSTRUCTIONAL INSTRUMENTS OF MATHEMATICS USING THE GROUP INVESTIGATION MODELS TO ENHANCE MATHEMATICAL REASONING SKILLS OF PRESERVICE TEACHERS

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ABSTRACT

This study aimed to develop group investigation learning instrument to enhance mathematical reasoning skills. The term ‘development’ means to create a valid and effective learning instrument. A valid learning instrument means an instrument that get discretion and judgment from experts (validators), while an effective learning instrument is an instrument that can enhance students’ mathematical reasoning skills. This study is the development research by using model of Thiagarajan modification. The research was conducted in Unswagati Cirebon, by taking fourth term students of department of mathematics with two classes were randomly taken. The developed learning instrument are: (1) syllabus, (2) lesson plan, (3) students’ book, (4) work sheet of students activity, and (5) mathematical reasoning skills test. The result of this study indicate that development learning instrument of mathematics with group investigation is valid and its implementation is effective.

Keywords: Development of Instrument, The Group Investigation, Mathematical Reasoning

1. INTRODUCTION

Linear Algebra course is one of the subjects in the curriculum department or program of study of mathematics and mathematics education at all universities in Indonesia. Linear Algebra which includes a branch of mathematics, has some materials that require reasoning ability to understand and find solutions to the problem. In addition, Linear Algebra learning prerequisites that have to go through the Association mastery Mathematical Logic and certainly not out of the application of its use in everyday life, both concrete and abstract, so that linear algebra is identical with the use of logic and reasoning in solving problems and finding solutions.

Increase understanding of student thinking and logic arguments synonymous with improving critical reasoning. Likewise with Linear Algebra that can make students more in-depth reasoning. This is due to the frequent use of logical and systematic thinking in solving the problem, especially if linked to Linear Algebra solution which tends to have a definitive solution. But the fact is, that occurs in Linear Algebra is learning concepts in Linear Algebra highly abstract causes many examples of the concept can not be recognized by both the students and also many students who are not familiar with deductive proof, instead led to the low quality of students' understanding of linear algebra course.

Data were collected on mathematics education courses University of Swadaya Gunung Jati (UNSWAGATI) Cirebon show facts about the many student difficulties in understanding linear algebra. Difficulties experienced by students in general find it difficult to determine which is the first step to proving a theorem, properties, and statements of truth that must be investigated, so their reasoning process to be blocked, and there is a small percentage of students who did not able to prove a theorem, properties and investigate the truth of a statement.

In anticipation of such problems it is necessary to find a sustainable learning the right formula, so as to improve student mathematical reasoning ability. Learning that is not dominated by the teacher, the learning process will take place on the initiative of the students themselves. This could happen if the teacher gives
students the opportunity to put forward bold new ideas according to their interests and needs. In an atmosphere of learning such as motivation and student activities that can be cultivated. As a result, the selection and use of appropriate learning models is an important factor in an effort to develop the motivation and activities so students can take them on the activities of mathematical reasoning.

Murata [1] explains that learning could be expected to develop the ability to think is to implement the learning that is designed according to the view of constructivism. Hung [2] explains that learning should be motivated and mentored student teachers to construct ideas, concepts, and their own understanding of the material being studied based on prior knowledge that they already possess. Based on the above two opinions can be said that Epp [3] states that one of the best approach to develop the ability to think abstractly (mathematical reasoning) students is through meaningful involvement in constructing or solve the problems related to mathematical reasoning. Based on the opinion of the experts mentioned above, it is in developing mathematical reasoning skills students need a constructivism-based learning is capable of learning activities involve students in full and meaningful for learning. Kolawole [4] states that a cooperative approach can improve student achievement and retention, increase self-confidence and further develop intrinsic motivation and attitudes (activity) of the positive study skills and social skills.

One solution is deemed appropriate to achieve this goal is to develop an instructional instruments using Group Investigation model based constructivism as a learning strategy in an effort to condition and improve mathematical reasoning skills of students. Slavin [5] states, learning the type of Group Investigation (GI) in general makes the learners more active because of heterogeneity within the group, forming a positive attitude towards learning, mutual respect among fellow members of the group and also with other groups. This is because the type of GI is a learning model that combines the complexity of social and academic tasks in building academic and social learning situations.

Instructionals instruments in accordance with the demands of the curriculum and to consider the needs of teachers and students is needed in order to implement the learning GI. Learning tools must be developed in accordance with the characteristics and student social setting or environment, learning mathematical tools that can actively involve students intellectually, make students learn independently trained, guided and guided students in constructing a new understanding that is associated with the concept of understanding that already exists on the students so that their knowledge would be perceived as a science that are related to one another. Thus the effective learning of communication between teachers and students can be built.

According Winataputra [6], GI models have three main concepts, namely: research or inquiry, knowledge or knowledge, and group dynamics or the dynamic of the learning group. Research (inquiry) here is the dynamic process of the students to respond to the problem and solve the problem. Knowledge is the student learning experience gained either directly or indirectly, while the dynamics of the group showed atmosphere depicting a group of interacting involving various ideas and opinions as well as exchange of experience through the process of arguing with each other.

One of the underlying cognitive model of instructional learning is closely modeled on the GI models of Jerome Bruner, known as discovery learning (learning Discovery). According Dahar [7], Discovery learning is a teaching method that involves students in the process of mental activity (observe, digest, understand, make a guess, infer, and so on) through sharing ideas with discussions, seminars, reading alone, and try it your self, so that learners can learn to find their own, while the work of teachers as mentors and giver of instruction.

Sharan [8] sets out the steps in the application of learning models GI. These steps are: (1) Identify a topic and organize students into groups, (2) Planning for the task to be learned, (3) Conduct investigations, (4) Prepare final report, (5) Present the final report, and (6) Evaluation.

According Kusuma [9], the reasoning is reasoning that the translation of the word is defined as a conclusion in an argument. Reasoning (some refer to it as proof), are often interpreted as a way of thinking, an explanation as an attempt shows the relationship between two things or more based on the properties or certain laws that have recognized the truth with certain steps that ends with a conclusion. This is in line with the statement Shufer and Pierce [10] which defines the process of achieving penalaransebagai logical conclusion based on the facts and the relevant sources. It can be concluded that the reasoning is the high-level mathematical thinking stage that includes the ability to think logically and mathematics based on the facts and sources that mendukung. Berdasarkan description above, mathematical reasoning skills of students in this study focused on several indicators, namely: (1) draw a logical conclusion, (2) examine the validity of the argument, (3) provide explanations using models, facts, properties and relationships in solving non-routine problems.

Hiebert [11] asserts that effective mathematics teaching requires understanding that students know and need to learn and then challenging and supporting hem to learn it well. To achieve high-quality mathematics teachers should (1) understand deeply thema thematic they teach, (2) understand how students learn mathematics, including mathematical know the progress of individual learners, and (3) select the duties and strategies that will improve the quality of the teaching process.
Application of learning with GI model provide many opportunities for students to develop their mathematical skills, to explore, try, adapt, and change there solution procedures, including verifying the solution, which is in accordance with the new situation. If students are familiar with the conventional classroom exercises, theorems, and equations, which limited its implementation in an unknown situation, students in GI learning environments generally have more opportunities to learn mathematical processes related to communication, connections, representation, reasoning, and modeling.

Based on the description that has served researchers interested in studying "the development of mathematics learning with GI models to improve student mathematical reasoning ability". The main question of this study is "How is the development and implementation of GI model study on the topic of Vector Space?"

From the primary research question, formulated some subresearch questions are: (a) how the results of the development of the GI model study on the topic of Vector Spaces, and (b) how the results of the implementation of the development of the GI model study of mathematical reasoning skills of students?

2. RESEARCH METHOD
This research is the development of instruments development instructional using model of learning GI on material vector space. Development of learning instruments is focused on developing learning tools that can create effective learning communication between teachers and students. Software development aims to improve student mathematical reasoning ability is more evenly on the material vector space that meets the criteria valid and effective.

Instrument developed and tested for its effectiveness in this study is the vector space of mathematics learning materials with model GI. Developed learning tools includes Syllabus, Lectures Events Unit(SAP), Textbook Student, Student Worksheet(LKM), and Mathematical Reasoning Ability Test(TKPM).

Learning instruments development model used is a modification of the model developed by Thiagarajan, Semmeland Semmel namely 4-D Model which is composed of fours tages of development, namely Define, Design, Develop, and Disseminate or adapted into Model 4-D. Modifications made are simplified models of the four stage into three stages, namely Defining, Designing, and Development.

Data analysis techniques used in this research is descriptive analysis. Data analysis was performed on each instruments validation sheet Space Vector learning material, write validators assessment of each instruments comprising: Syllabus, SAP, Student Textbook, LKM and TKPM. Assessment consists of 5 categories, which is not good (score 1), poor (2), fairly good (3), good (4), very good (5). Assessment data experts for each of the instruments were analyzed by mean score. Description of the mean scores are presented in Table 1. Learning instruments said to be good, if each instruments is in the category of "good enough", "good", or "very good".

<table>
<thead>
<tr>
<th>Table 1. The Determination Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1.00 ≤ x ≤ 1.80</td>
</tr>
<tr>
<td>1.80 ≤ x ≤ 2.60</td>
</tr>
<tr>
<td>2.60 ≤ x ≤ 3.40</td>
</tr>
<tr>
<td>3.40 ≤ x ≤ 4.20</td>
</tr>
<tr>
<td>4.20 ≤ x ≤ 5.00</td>
</tr>
</tbody>
</table>

3. RESULT AND ANALYSIS
3.1. Define
At this stage of the analysis focused on the study of college studentstoexaminate the characteristics of UNSWAGATI mathematics education student in accordance with the design and development of leaning tools. These characteristics include cognitive development, academic background, background knowledge. To determine the characteristics of the students mentioned above, the researchers conducted a literature review of cognitive development of students and provide Test of Logical Thinking (TolT) which was first developed by Tobin and Capie 1980 to examine the ability of reasoning, assessing students' academic background through background data collection regarding the origin of the school as well as assess the ability of students to Linear Algebra prerequisites material covering materials Mathematical Logic and set to know the background knowledge of students.
3.2. Design

At this stage, several activities including the preparation of mathematical reasoning of TKPM, the first begins with the preparation of the lattice about TKPM. Grating (test blue print or table of specification) a description of the competence and the material to be tested. Lattice test twich is based on learning objectives in it is a map of the spread of the questions that have been prepared in such a way that the grain of that question can be determined by a proper level of preservice teachers of mathematical reasoning.

The next activity format adapted to the learning tools developed learning format. The selection of the format associated with the learning objectives to be achieved, the learning model used, and the development of specific indicators GI learning model. Format the selected learning is used to design the content, instructional strategy selection, and source instrument. Design learning content developed in this research include the design of general and special design. General design refers to the general rule learning development tools, and special design is the fulfillment of the criteria GI models.

The next activity is the design of the initial design study that describes the activity of teaching and learning with students in GI learning model. The initial design of learning tools that are designed, called, called the Draft I. This design can not be used for field trials because it must be validated by the experts.

3.3. Develop

Development stage is the stage of learning to produce the draft has been revised based on the input of experts, namely validation and testing experts.

Validation experts include content validation and construct validation of all instruments that have been developed at the planning stage. Validation is done by 5 people competent to assess the feasibility of the study. Revisions were made based on suggestions/ hints of validator then produce a second draft. Results of the evaluation of the experts (Draft II), who later tested on learning. Instruments of instructional developed in this study include Syllabus, SAP, Student Textbook, LKM, and TKPM.

Table 2. Recapitulation of Expert Validation Results

<table>
<thead>
<tr>
<th>No</th>
<th>Validator</th>
<th>Mean Syllabus</th>
<th>SAP</th>
<th>Student Textbook</th>
<th>LKM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>4.8</td>
<td>4.7</td>
<td>4.8</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>3.8</td>
<td>4.6</td>
<td>3.8</td>
<td>3.8</td>
</tr>
<tr>
<td>3</td>
<td>III</td>
<td>4.8</td>
<td>3.8</td>
<td>3.8</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>IV</td>
<td>4.9</td>
<td>3.8</td>
<td>3.8</td>
<td>5.0</td>
</tr>
<tr>
<td>5</td>
<td>V</td>
<td>4.8</td>
<td>4.1</td>
<td>3.8</td>
<td>3.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Validation Criteria</th>
<th>Excellent</th>
<th>Excellent</th>
<th>Good</th>
<th>Excellent</th>
</tr>
</thead>
</table>

| Category | V / MR | V / MR | V / MR | V / MR |

Keterangan :
V : Valid
MR : Minor Revisions

Based on the data in Table 2, the results of the assessment with the assessment obtained validator category "Good" and "Excellent", and multiple entries with a little revision it was concluded that all the instruments produced in a valid category. Results valid instruments shows that the instruments meet the standards validation, i.e., content validity and face validity.

After all the learning tools validated and declared eligible to be tested, then performed testing on classroom learning instruments testing instruments, while specifically TKPM tested in the pilot class and control class. During this trial, carried out the data collection process includes student activity observed data. Furthermore at the end of the trial process, conducted TKPM to measure mathematical reasoning ability in the trial class and control class.

The data were used to determine the level of learning effectiveness by using the results of the development. Level of effectiveness measured through three test statistics, namely: (1) trial completeness learning outcomes, (2) test the effect, and (3) test the differences.
Based on Table 4 obtained the proportion of students passing grade on an individual basis is 75%, meaning that students who scored above the established criteria as much as 75%. For the average mastery of mathematical reasoning skills in the class overall trial. The average value (mean), the mean value of the average mastery of mathematical reasoning ability test class more than 65 instruments. Analysis of the influence of activity on mathematical reasoning tested by linear regression and obtained results are \( F = 241.738 \) with \( \text{sig} = 0.000 \), which means that \( H_0 \) is rejected, meaning that the linear regression equation. The amount of influence the activity of mathematical reasoning can be seen from the R square = 0.910 which means that 91.0% of students are influenced mathematical reasoning student activities, and by 9% influenced by other factors. While the form of the regression equation is \( \hat{Y} = −38.377 + 1.439X \).

Comparative analysis of the test data in this study using the Independent Sample Test and the obtained results are \( F = 2.154 \) and \( \text{sig} = 0.148 \) (more than 5%), meaning that the two samples have the same variance. Afterwards, Equal variance assumed, acquired \( \text{sig} = 0.004 \) (less than 5%). This means that \( H_0 \) is rejected, which means that both samples have an average value of different completeness. To determine which classes have an average value higher use Statistical analysis showed the TKPM Mean values for each class. Group 1 (trials) had a higher mean value, that is equal to 71.9615 compared with group 2 (control) which only has a mean value of 64.4231.

4. CONCLUSIONS

Has obtained a valid development of learning tools, including: (1) Syllabus, (2) SAP, (3) Student Textbook, (4) studentwork sheets, and (5) tests of mathematical reasoning skills. Results of the implementation of the development of learning with models investigation group generate and effective instruments

REFERENCES


RECONTEXTUALISING DIDACTICAL SITUATIONS IN PRIMARY MATHEMATICS INSTRUCTION

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ABSTRACT

Improving mathematics instruction requires substantial preparation in terms of designing teaching and learning materials and processes. In this paper we argue that such endeavor should pay attention to the way teachers develop a didactical situation. By using ‘didactical design research’ (Suryadi, 2013) as thinking tool, we explicate the notion of learning obstacle on the volume lesson delivered by a primary teacher in Bandung. From lesson observation (February 2013), we analyse the feature of didactical situations based on the interactive function of mathematical representation and its corresponding students’ responses. Accordingly, we hypothesise alternative didactical situations of such lesson as we consider it would promote a higher mathematical thinking for students. Finally, we recontextualise Brousseau’s idea of action-formulation-validation situations based on our empirical findings.

Keywords: didactical design research, didactical situation, learning obstacle, primary mathematics

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1. INTRODUCTION

For many, teaching and professional learning such as Lesson Study are both simple and yet difficult. Although Lesson Study has spread out across the country, its direct impact on teaching and learning is questionable (Suratno, 2012). Indeed, it needs a robust and fruitful framework for designing and analysing teaching-learning.

Generally, the very nature of teaching-learning processes involves three important aspects: teacher, content-students. This key feature of classroom practice engages rich, complex and interdependent interaction and relationship. At least, there are two important relations: teacher-student(s) or pedagogical relations and teacher-student(s) mediated by teaching materials or didactical relations. However, in most part of the lesson, didactical relations play key role in classroom communication and knowledge construction. Brousseau (1997) called it didactical situations and the way the situation is constructed constitutes the quality of such relationships. Suryadi (2010; 2013) called it metapedadidactic.

It is well known that the problem of teaching lies in the preparation phase. In most parts, teachers tended to teach by following or simply transmitting content as appeared in the text book without further analysis. In fact, scientists produced scientific knowledge in an uneasy ways. They involved and engaged themselves in the knowledge construction with pay off, facing with challenges, obstacles and other supporting and inhibiting factors. In this regard, they experienced recontextualisation and repersonalisation. In order to present knowledge to broader audiences, they simplified its representation through the process of depersonalisation and decontextualisation. Accordingly, if teachers only follow what is said in the textbook then they will not grasp the complexity of the topics to be taught. In this case, a rich and fruitful lesson design is desirable.

This paper is based on assumption that most teachers do not fully consider the importance of projecting what will happen in the classroom. At least, teachers should think of students’ responses as logical consequences of presenting instruction or question. We call such classroom situations as didactical situations which are complex and uncertain. Such assumption is based on our practical engagement with Lesson Study which approaches professional learning through observation and interpretation of current teaching practice. In addition, we studied didactics tradition of continental Europe to frame our work to such analysis. We thought
that there are complex interplay between Lesson Study as practical approach and didactics as theoretical stance of teaching-learning. The relation is centered on teacher thinking in regard to the study of teaching materials. In this paper we introduce the notion of didactical situations, metapedadidactic and Didactical Design Research (DDR) as thinking tools and methodological approach in studying teaching-learning. By drawing on specific case of volume lesson, we explicate how to analyse teaching-learning from perspectives of didactical situations, metapedadidactic and DDR.

2. THEORETICAL PERSPECTIVES

In his book “Theory of didactical situations in mathematics”, Guy Brousseau (1997) identifies three situations that many considered as key features of problem-solving and constructivist mathematics teaching-learning: 1) non-didactical situation; 2) didactical situation; and 3) adidactical situation. Non-didactical situation is one that is not explicitly organized to allow the learning of presented knowledge. Didactical situation is deliberately designed to introduce the presented knowledge in which teacher intended to teach it through tasks that should be undertaken by students. Adidactical situation is conditions that allow students to establish relationship with the presented knowledge without the help of teacher. Students are at the process of devolution of responsibility in action in which their responses (answer, argument) to the presented knowledge (or problem) were related each other, including its problem solving approaches and the difficulty that may be encountered during students interaction with the situation of presented knowledge/problem.

For Brousseau, teaching situations, that is didactical situations, constitute the exchange between students, teachers and the milieu (what students act on or what act on students in term of mathematical content such as problem solving). Such interactive system establishes what so called as didactical contract in which the role of teacher and student is put into play reciprocally in dealing with the way the knowledge is presented (milieu). This relationship involves the concept of didactical contract with its key features of didactical situations is indeed complex with many things to be considered by a teacher in terms of psychological development. Didactical obstacle is originated from the ways teachers and others deal with educating children. Epistemological obstacle is located in the nature of knowledge itself and the history and philosophy of knowledge would vividly show the formative role of knowledge. With regard to dialectical pattern of didactical situations and obstacles, Brousseau maintains that overcoming such obstacles involve “a complete restructuring of models of action, language and proof-system. But the didactician can precipitate these breakdowns by favoring the multiplication and alternation of specific dialectics.” Thus, it is the very nature of reciprocal relationship between obstacles and problems, that characterises the most part of milieu.

Suryadi (2010; 2013) develops the idea of metapedadidactic as to engage educators not only to understand the interrelationship between didactics theory and Lesson Study, but also to conceptualise a universal categories of what is good quality of teaching-learning. He proposes three key indicators: unity (of situations), coherence (logical sequence of situations) and flexibility (dealing with didactical obstacles). To analyse such criteria, Suryadi proposes new genre of research called Didactical Design Research (DDR).

DDR constitutes the way teachers think about the lesson, i.e didactical situations, from preparation to implementation and evaluation. Hence, the object of the study in DDR is teacher thinking before, during and after the lesson; 1) analysis hypothetical didactical design by means of prospecting Didactical-Pedagogical Anticipation (DPA); 2) analysis of metapedadidaktic; dan 3) retrospective analysis by comparing hypothetical design vs. metapedadidactic. Through such kind of analysis we can have one possible Empirical Didactical Design than can be improved through DDR cycles.

However, thinking of didactical situations is challenging; there are many things to be considered by a teacher before he/she makes instructional decisions. Therefore, we would argue that it is important for teacher to make robust preparation of didactical design. One of such efforts is by DPA analysis in terms of predicting and anticipating students’ responses to the presented knowledge or milieu in general. However, to understand the whole context and content of teaching materials, teachers should consider epistemological obstacle, i.e.
logical structure of knowledge. Therefore, it is important for teachers to do recontextualisation and repersonalisation. By doing so, teachers can consider possible learning obstacles that should be predicted and anticipated.

Such processes demand teachers’ deep thinking about preparation and design of didactical situations. It is through such habits of mind a teacher can do innovative works regarding designing and analyzing teaching-learning. In this regards, DDR may play as thinking tools not only for improving teaching-learning but also teachers’ intellectual capital as the core of their key competencies and capacities.

3. THE CONTEXT OF THE STUDY

This study is part of collaborative inquiry between a group of UPI’s faculty members and a private primary school that has been conducted since September 2012. The school has implemented a self-directed Lesson Study practice since 2008. Until 2011, the development of Lesson Study in that school was at stake and at the same time the development of Lesson Study at UPI was in the critical time. Hence, when UPI’s faculty members and school leaders met at the first time, we strategised series of workshops centering on developing teacher thinking. Firstly, the workshops focused on three important questions: What is math? How we learn math? Then how we teach it? It aimed to extend knowledge of math teachers about mathematics teaching-learning as to improve their ways of thinking of pre-lesson planning (repersonalisation and recontextualisation). Secondly, the activity system for DDR was developed as to recontextualise the plan-do-see cycle of Lesson Study. Compared to original Lesson Study practice which tended to focus on classroom observation and immediate post-class discussion (Hendayana et al., 2007), our approach is to focus on preparation phase and employs delayed post-class reflection.

One of key vignettes is the math lesson in which students were expected to formulate the volume of rectangular and cube. The teacher started the lesson by asking students to compare which one can contain more book as illustrated by the following figure. The students could easily answer this question.

![Figure 1. Comparison between two containers](image)

Then the teacher asked similar question, which one contain more as illustrated by the following figure.

![Figure 2. Comparison between two balls.](image)

In this stage, teacher tried to introduce the concept of contain as the first step before introducing the concept of volume. By presenting two comparisons of containers in the concrete ways, most students could grasp the differences of spaces for containing books or other things. Then teacher asked students to work in small group to fill in bricks into two other designed containers as illustrated by the following figure.

![Figure 3. Comparison of containers by filling in the bricks.](image)

In the following steps, teacher asked students in group to count how many bricks in two different sizes of rectangular as represented by pictures. The teacher tried to move on from concrete representations into
more abstract one. Next steps were: 1) determining how many bricks within a cube of 5x5x5; and 2) determining how many bricks within several rectangles with different sizes. Again, basically all groups could make correct counting. It was assumed that once students could solve counting how many bricks contained in particular shape and size of rectangular then students could formulate the volume of rectangular.

These stages aimed at pointing out children to the idea of volume. Our observation revealed that at these stages, students could do all tasks, including counting, comparing and determining how many bricks within a rectangle. They seemed enjoying these sessions and actively used manipulatives. However, when it arrived to the phase in which students were asked to make formulation, our observation revealed that most students were not concentrate anymore. Most students were confused on how to formulate the volume of a rectangle. This is the key finding of learning obstacle. It was also perceived by teachers. According to their reflection, the lesson run well and equipped with rich teaching materials. Children were considered as having joyful and active learning. However, model teacher sensed that children had difficulty and lost focus when asked to formulate the volume of rectangle.

Since it was started, the lesson tended to engage students with comparative thinking between two or more different cubes or rectangular. We analysed that students were dealing with didactical obstacles as represented by the ways teacher develop teaching sequences and materials. In addition, we identified that the lesson only applied action situation which clearly explained why students were in difficulty to make mathematical formulation. However, most teachers and other observers viewed the lesson positively, as opposite to our interpretation. We then share our finding as well as our hypothetical didactical situations of the lesson.

4. HYPOTHESIZING DIDACTICAL DESIGN

One of possible improvement to such lesson is by hypothesizing didactical design according to identified learning obstacles and the nature of didactical situations of action, formulation and validation. Based on the observed lesson, children faced didactical obstacle of which teacher design the lesson that could inhibit mental action of students as it was expected. We call it the problem of unity in regard to the idea of metapedadidactic: Logical arrangement of didactical situations of action, formulation and possibly validation could not fully be structured.

By doing so, our hypothetical design can focus on what critical feature of teaching volume. In our view, rather than focusing on comparison of two volumes, the lesson should focus more on the nature of volume, that is, multiplying three variables as volume formula is. The following figures depict our Hypothetical Didactical Design.

To start the lesson, teacher should develop action situations by inviting all students to think of the presented problems. At this stage, the aim is to raise the sense of necessary to think in part of students. Therefore, we assume that by presenting more difficult one followed by easier one, students could sense the hop-step-jump guidance from teacher and their peers. Figure 4 shows the logical structure of action situations.

Figure 4. Hypothesizing action situations.

Based on Figure 4, teacher can develop dialectical relation of action situations through following interaction: 1) There are 64 bricks arranged as such. Explain how you find out way of counting them?; 2) Then, how many bricks arranged as such? Explain how you find out way of counting them!; 3) Basically, the picture contains bricks as represented by number (5, 6, 10). How many bricks in total are they?; and 4) This picture also contains bricks as represented by number (5, 8, 12). How many bricks are they in total? It is expected that by counting concrete object of cube and rectangular, students can engage in developing
strategies for counting it by multiplying three variables. It is opening the door to formulation situations in which students discuss their strategies to formulate the volume of rectangular by presenting more abstract representation as illustrated by the following figure.

According to Figure 5, the teacher can develop the dialectical relation of formulation situation: 1) This picture contains bricks as represented by \(a, b\) and \(c\). How many bricks are they in total?; and 2) This picture contains bricks as represented by \(p, l\) and \(t\). Explain how to formulate so that you can explain how many bricks inside. These situations will engage students to focus on their thinking to multiplying three variables represented by symbols. It is expected that students can speak out the formula of volume of rectangular. Once this phase was achieved, it may lead to validation situations by developing the following dialectics: 1) How many bricks are they? Try to count them without multiplication. Then find out whether the result similar with using multiplication!; 2) A and B are part of rectangular with \(5 \times 6 \times 4\) in size. How many bricks are in A and in B? Count A and B without multiplication and prove it with multiplication. Do you find similar results? Figure 6 presents the overall hypothetical didactical design from action to formulation to validation situations (red one).

5. CONCLUDING REMARKS

One of key factors to improve teaching-learning is through reflective practice that focuses on analyzing the overall relationship between designed and enacted classroom practice. However, it is important to consider how substantial reflective practice can be achieved fruitfully. Our case of volume lesson gives lessons learned about how teachers think about interconnected didactical situations, learning obstacles and learning trajectory of students.

The problem held by such lesson according to theory of didactical situation and metapedadidactic was the problem of unity. The following is the possible explanations of such problem. Consider the objective of the lesson: was it comparing or formulating the volume? The teacher started the lesson by comparing containers. Then teachers provided materials that focused on comparing how many bricks inside cube and rectangle with different sizes. Hence, the lesson consistently focused on comparing the volume until finally asked children to formulate volume. At the end, children felt confuse and lost focus. It raises the question whether students were engaged in joyful and active learning. At beginning, it was appropriate to present the idea of contain. However, next steps kept comparison as ways of thinking. Children felt confuse because they spent time to perform tasks of comparing. Children had clear image that the aim of the lesson was comparing volume, not formulating volume. Overall, the lesson was designed to make comparison rather than abstraction (formulating). It means that the lesson spent too much on action situation that resulted in confusion among children due to incoherent focus of what should be achieved.
From such kind of analysis, we constructed hypothetical design based on reflection session of the lesson. However, it needs to be tested again, and again in order to have Empirical Didactical Design leading to didactical innovation. In this regard, DDR, particularly the notion didactical situations and metadidactic is promising as intellectual tools to analyse deeply the lesson: Unity, Coherence, Flexibility. Such construct is represented by focusing on multiplication of three variables (p x l x t) as the very nature of the concept of volume. In addition, a hop-step-jump arrangement is created to promote collaborative learning by considering possible learning obstacles and trajectories (particularly for slow learners). Therefore, our case suggests that DDR provides methodological approach of teacher research as to both hypothesize and revise the lesson.

REFERENCES


CRITICAL THINKING ABILITY AND EMOTIONAL INTELLIGENCE IN MATHEMATICS LEARNING

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ABSTRACT
Critical thinking is not only involves cognitive, but it is also a feeling or emotion. Critical thinking has a relationship with emotional intelligence. Is it true? Then how mathematical learning model can improve critical thinking ability and emotional intelligence students? This paper will discuss the relationship between the critical mathematical thinking and emotional intelligence and how the learning model can improve critical thinking ability and emotional intelligence.

INRODUCTION
Critical thinking skills is one of the skills that learners have achieved as listed in the Competency Standards in Indonesia Curriculum. The Indonesia curriculum states that mathematics needs to be given to all students ranging from elementary school to equip students with the ability to think logically, analytical, systematic, critical, and creative as well as the ability to cooperate. Critical thinking skills need to be achieved by students so they can study the problems in a systematic, organized way to face challenges, formulate innovative questions, and design are considered relatively new settlement (Johnson, in Ibrahim, 2010). Furthermore, in this regard can not be denied that there cent very rapid flow of information, and in between there is indeed in formation that needs to be consume are should not be consumed. For that, of course, the necessary critical thinking skills that can be in choosing a filter, process, and receive information. Wijaya in Muhammad (2002) suggested that the critical thinking skills that are part of the thinking skills need to be owned by every member of the community (students).

Developing critical thinking skills is very possible to be developed through the study of mathematics, but in general critical thinking processes are rarely trained in mathematics learning in school. Learning math is focused on memorization and find answers to questions that are routine or procedural.

Successful mathematics learning should use the students potential optimally. Mean while, to create a learning process with the optimal use of the potential of students, according to Shapiro (Ibrahim, 2010) states that the emotional intelligence of the students needs to be a concern. Emotional considerations in mathematics learning in particular may be a little more help in receiving math, amid assumptions that are still believed by most students, the math is a difficult subject. Thus, the presence of emotional intelligence can be viewed as aspects to be considered, it can even be used as the basis to follow the process of learning math. This article discusses the relationship between the critical mathematical thinking and emotional intelligence in learning mathematics.

MATHMATICS CRITICAL THINKING SKILLS
In mathematics learning, critical thinking skills are skills that must be achieved by students. In this regard, there is some sense of critical thinking presented some experts. Ennis (1996) states that critical thinking is a process that aims to make sensible decisions about what we believe and what we do. Furthermore, John Chaffee (Ibrahim, 2010) defines critical thinking as the thinking used to investigate systematically from one thought process of using evidence and logic in the thinking process. While according Sumamro (2010), critical thinking is not equivalent to high-level thinking skills. In critical thinking contained all components of high-level thinking, but includes a disposition that is not contained in the high-level thinking. John Dewey (Fisher, A.,
EMOTIONAL INTELLIGENCE

Emotional intelligence or known as Emotional Intelligence (EI) is the ability to understand and control emotions. Including the ability to build relationships with other people around. EI is not collide with IQ because it has regions' powers are different. IQ is generally associated with the ability to think critically and analytically, and to be associated with the left brain. Meanwhile, the more EI associated with feelings and emotions (right brain). If you want to get intelligent behavior it must also be honed emotional skills. Because to be able to relate to others well we need the ability to understand and control the emotions of self and others as well. This is where the function of emotional intelligence.

EI is not a talent, but the emotional aspect within us which can be developed and trained. So every one has been end owed by Godemotional intelligence. Stay extent of its development, it depends on our own accord. One is for sure, we'll EI formed well when trained and developed intensively with the means, methods and the right time.

1) Five Basic Ability in the Theory of Emotional Intelligence

There are five basic skills in the Theory of Emotional Intelligence by Daniel Goleman (Enditiara, 2011), namely:

a. Recognizing Self Emotions

Recognizing emotions themselves is an ability to recognize feelings when those feelings happen. This ability is the basis of emotional intelligence, the consciousness of one's own emotions going. Self-awareness makes us more alert to the moods and houghts of mood, when the individual becomes less vigilant soluble emotions flow and controlled by emotion. Self-awareness does not guarantee mastery of emotion, but it is one important prerequisite for controlling emotions so that people easily master the emotions.

b. Managing Emotions

Managing emotions is an individual's ability to deal with feelings that can be revealed precisely, in order to reach a balance within the individual. Keeping the troubling emotions under control is the key to emotional well-being. Excessive emotion, of which increases within tensity for too long will taurusstability. This capability includes the ability to entertain yourself, let go of anxiety, moodiness or offense and the consequences there of, and the ability to rise from the feelings that push.

c. Motivate Yourself

Achievement must be passed with its motivation within the individual, which means it has the perseverance to resist the gratification and impulse control, as well as having a positive sense of motivation, namely antusiasisme, passion, optimism and self-confidence.

d. Recognizing Emotions of Others

The ability to recognize emotions in others is called empathy. According to Goleman (Harris, 2011), person's ability to recognize others or care, demonstrate the ability of one's empathy. Individuals who have the ability to be able to capture more empathy social signals hidden anything that suggests it takes every one else so he is able to accept the other person's perspective, sensitive to the feelings of others and be able to listen to others.

e. Fostering Relationships

Ability to build relationships is a skill that supports the popularity, leadership and success among fellow. Skills in communicating a basic ability in successful relationships. Sometimes if it is difficult for men to get what she wants and it is also hard to understand the desires and wishes of others.

2) Factors Affecting Emotional Intelligence

Walgito (Enditiara, 2011) divides the factors that affect emotional intelligence into two factors, namely:
1. Internal Factors

The internal factor is what is in the individual that affect their emotional intelligence. The internal factors have two sources, namely in terms of physical and psychological terms. In terms of the physical factors and individual health, and physical health if a person can be subject to possible influence the process of emotional intelligence. Psychological reasons include there in the experience, feeling, thinking ability and motivation.

2. External Factors

External factor is the environment in which the stimulus and ongoing emotional intelligence. External factors include: 1) the stimulus alone, stimulus saturation is one of the factors that affect one’s success in treating emotional intelligence without distorts; and 2) environment or in particular circumstances underlying the process of emotional intelligence. Object underlying environment is very difficult to separate roundness.

4. RELATIONSHIP CRITICAL THINKING WITH EMOTIONAL INTELLIGENCE

Critical thinking involves some emotional and intellectual development of distance between us and the ideas. Key characteristics required for critical thinking according to Cline, A.(2011) are:

1) Open-Minded

A person who thinks critically about some thing must be open-minded. Open to the possibility that some one else is not only true, but also that you are wrong. Similarly, even if a person holds true, we cannot take it for granted that view, we need to think about the truth and analysis. And we also can refuse if their views are wrong.

2) Distinguish Emotions and Reason

If we have a logical and empirical reasons obvious to accept an idea, may be it is also influenced by emotional reasons and psychological that we do not realize. To think critically, we must separate the two things. Emotional reasons to believe something that may be quite understandable, but if the logic behind the beliefs wrong, then ultimately we should not consider our rational beliefs. If we are really going to approach our faith in a way, skeptics fair, then we must be willing to set as ide our emotions and evaluate the logic and reasoning—may even reject our belief, sit hey fail to live up to logical criteria.

3) The Opinion of Knowledge, Instead of Ignorance

Sometimes a person maintain the idea even though he did not know much about it. A person who thinks critically, should try to avoid the assumption that hea lready knew every thing he needed to know.

4) Probability is Uncertainty

There are ideas that may be true, and the ideas that it is true, but it is a good idea indeed. When one thinks critically, they remember that just because they can demonstrate this conclusion may be true, it does not mean they have demonstrated or can demonstrate that it is true. Certain truth requires faith, but the truth may only require a tentative belief—that said, we have to trust them with the same power a possible evidence and reason.

5) Avoid Misunderstandings Language

Language is a communication tool to convey a variety of ideas, including new ideas, but in the delivery of these ideas sometimes misunderstanding, ambiguity, and obscurity. Often what we think and then we communicate, may be other people do not accept it as what we think, and vice versa we did not catch what the other person intends to communicate. Critical thinking some ones should try to eliminate these misunderstandings factors, for example, by trying to get the key words when ideas are communicated so as to avoid misunderstandings.

6) Avoid Common Mistakes

Common errors are used commonly performed by the general public with out realizing that it was a mistake. Errors in reasoning errors that creep into arguments and debates all the time; practice critical thinking will help people avoid involving themselves and assist in identifying their views in the arguments of others. An argument that one cannot give a good reason to accept the conclusion, therefore, as long as the error is being done, the argument is not very productive.

7) Do Not Jump to Conclusions

People often immediately draw conclusions when faced with a dilemma, but the conclusions drawn are not always correct. Therefore it is better to avoid problems than to try to get out of trouble, critical thinking also emphasizes careful thinking—this means that we do not jump to conclusions if we can avoid it. Go ahead and admit the existence of a clear conclusion as it may be true, but not really adopted until other options have been considered.

Emotional intelligence is understood as a measure of the extent to which a person succeed (or fail) using a logical assessment and reasoning to situations in the process of determining the emotional or feeling responses to situations. This brings intelligence (cognitive) for emotional support. Both positive and negative emotions. It will be a measure of the extent to which affective responses to basic reasoning. A person with high emotional
intelligence will be the one that responds to the situation by stating the feeling that "good sense," given what happened in the situations. The resulting flavor serves as a motivation to pursue reasonable behavior or action. Naturally emerge from emotion "rational" to desire "rational" behavior and "rational".

According to Ennis (1996), critical thinking emotional support. Emotions and feelings are very involved within faith and our actions. For example, if I feel fear, it is because I believe that I was being threatened. Therefore I tend to attack or escape.

Based on the above descriptions, it can be a relationship between critical thinking and emotional intelligence. Critical thinking involves not only our cognitive abilities, but also our feelings or emotions. Critical thinking has a relationship with "emotional intelligence". Critical thinking is the only sensible way that can lead to emotional intelligence. Critical thinking that allows us to take an active command not only the mind, but also feelings, emotions, and desires. Critical thinking provides us with the mental tools needed to explicitly understand how the reasoning works, and how they can be used to take over command of what we think, feel, want, and do.

Through critical thinking, we obtain the means to assess and improve our ability to assess properly. It provides way for us to learn from new experiences through a process of continuous self-assessment. Critical thinking, allowing us to form a belief logic and judgment, and in doing so, gives us the basis for the emotional life of a "rational and reasonable".

A person who thinks critically able to remain objective, even in the most emotional situations. They are able to eliminate their emotional investment, anxiety, sensitivity and even frightened to stay focused and on task (Harris, B., 2011).

5. EMOTIONAL INTELLIGENCE AND CRITICAL THINKING SKILLS IN MATHEMATICS LEARNING.

Given (Hasrattudin, 2010) proposed a 21st century learning is learning with the natural functioning of the brain by combining components of emotional, social, cognitive, and reflection. According to Izard, C.E. (Hasrattudin, 2010) which says that the critical thinking skills and emotional intelligence needs to be developed in the schools through problem solving, especially in shaping morality better learners, in addition to helping them understand the problems and conflicts in the learning or around student life.

Covey (Hasrattudin, 2010) also explains that the learning patterns are able to develop emotional intelligence and thought the child is learning the nuances of social patterns, ie patterns of learning that involves interactive learning community. While Oleinik (Harris, 2011) said that the process of learning that can improve critical thinking skills and emotional intelligence of students is centered on student learning (student centered) and takes place in a social context.

Principles of active learning students, referring to the notion of learning as something that is done by the students, and not something that is done to students. The statement constructivism adopts the view that students as individual who actively build the knowledge and not just recipients of information that is so. In the constructivist view of learning is a process, situation, and efforts are designed so that teachers makes students learn in accordance with the principle of learning how to learn. In other words, the teacher acts as a facilitator of learning, motivator, and manager of learning for their students. Teacher's task is to choose the information/job/new problems related to students' prior knowledge, and creating a learning environment (role as facilitator) to enable the interaction between new information with prior knowledge (is not in balance). Then the teacher helps the students that through accommodation and a new equilibrium occurs asosiation (role as motivator) to form new knowledge to the students. Activities teachers select information (task) new, creating an environment, and motivate students as a whole describe the role of the teacher as a learning manager.

There are various models of learning that applies the philosophy of constructivism (Sumarno, 2010), namely:
1. Indirect and direct learning. Both approaches are similar to the present case that begins or contextual issues then gradually guided students find meaningful concepts, followed by a more complex problem solving.
2. Problem-based learning, discovery, and investigation. Both of these approaches are also quite similar to the direct and indirect approach that begins with the presentation of the issues covered and open-ended.
3. Metacognitive approach and discursive approach. In this approach the students asked a number of questions not just memorization but that encourages students to provide answers along with reasons.
4. Various cooperative learning strategies. In this strategy students are encouraged to work together on a common task and they have to coordinate its efforts to complete the tasks set by the teacher. Aim of the group in a cooperative learning strategy is to provide opportunities for all students to be actively involved in the process of thinking and learning activities.
5. APOS theory based learning. This learning cycle following the ADL (Action, Discussion, and exercise) were packed using programming languages and modification.
6. Abductive-deductive strategy. This approach is designed to develop the ability to prove that begins with understanding the evidence first, then proceed to the proof that is not too formal and gradually guided students understand and can carry out a formal proof.
Ibrahim (2010) and Hasrattudin (2010) suggest that within their mathematical learning model to enhance critical thinking skills and emotional intelligence of students is a model of problem-based learning approach and realistic mathematics. Here is a summary of the results of the research papers or model of learning that can improve critical thinking skills and emotional intelligence of students.

1. Teaching Learning Process that bring emotional intelligence to seek provision of a sufficient portion of the task (a mathematical problem) that can only be resolved through the help of the teacher or the students' collaborations with other students who are more skilled. This resulted in interaction with a good emotional state and control. In a good emotional state and then trigger a lucid controlled to provide in-depth reasons and considerations in creating, evaluating, taking, and reinforce a decision or conclusion about a situation or mathematical problems that it faces. Moreover, in a good emotional state and controlled the trigger lucid smoothly and to reason properly, can solve the problem, finding original ideas, and to communicate clearly and well detailed. Ties and cooperation between the emotional and mind this raises the complementarities between the two. It gives extraordinary power to students in their region emotions affect the functioning of the centers of critical thinking mathematically (Ibrahim, 2010).

2. Learning mathematics with a realistic approach can be implemented in an effort to improve vecritical thinking skills and emotional intelligence of students with no school should not distinguish rank and gender. In addition, there was no significant correlation between critical thinking skills and emotional intelligence of students, therefore in an effort to improve students' critical thinking skills in the learning of mathematics by using a realistic mathematical approach does not rely on emotional intelligence, and in improving emotional intelligence does not depend on critical thinking skills (Hasrattudin, 2010).

Emotion management as an important element in the learning of mathematics, because the emotions are managed well, students will not experience many problems in interacting with the math.

6. CONCLUSIONS

Critical thinking skills is one of the skills that students must achieve in math. Successful mathematics learning students should use the potential optimally. Meanwhile, to create a learning process with the optimal use of the potential of the student, then the emotional intelligence of the students needs to be a concern. Learning model that is able to develop emotional intelligence and thought the child was a nuanced social learning model, which involves a model of learning in an interactive learning community. Learning process that can improve critical thinking skills and emotional intelligence of students is centered on student learning (student centered) and takes place in a social context.

Some experts say that there is a relationship between critical thinking and emotional intelligence. Critical thinking involves not only your cognitive abilities, but also our feelings or emotions. Critical thinking has a relationship with "emotional intelligence". Critical thinking is the only sensible way that can lead to emotional intelligence (Cline, 2011). Haris said that a critical thinking are able to remain objective, even in the most emotional situations. They are able to eliminate their emotional investment, anxiety, sensitivity and even frightened to stay focused and on task. However, the research results of Hasrattudin (2010), say that there is no significant correlation between critical thinking skills and emotional intelligence of students.

References

CONTRIBUTIONS THE MATHEMATICAL THINKING OF STUDENTS ABILITY TO WRITE ARGUMENTS ON PGSD

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ABSTRACT

This article contains the results of the exposure study conducted on the basis of mathematical thinking that contributed to the writing ability argument. But there has been no scientific information about mathematical thinking skills that can contribute to the ability of writing argument. The research was carried out using the method of correlation. Data collection techniques include tests/exams are written. The instruments used are a test/exam math that can measure the cognitive stage, and tests/exams of Indonesia that can measure the ability writing argument. The sample is determined by purposive, i.e. student in semester VI PGSD UPI campus of Tasikmalaya. The Data was successfully collected processed using SPSS 20.0 to help further analyzed. The processing and analysis of data, give the conclusion that: a) the mathematical thinking ability do not contribute significantly to the ability of writing arguments. b) low level of mathematical thinking ability contribute significantly to the ability of writing argument. c) high level of mathematical thinking ability contribute significantly to the ability of writing argument. d) the low of mathematical thinking ability greater contributions than the high level of mathematical thinking ability.

Keywords: Mathematical thinking ability, The ability to write arguments.

1. INTRODUCTION

Individuals having the faculty of thought mathematical, moreover individuals formal educational. The faculty of thought mathematical is the process of dynamic who demands of their diverse complex idea so increase understanding. There are two levels activity in thinking mathematical, namely: 1) the activity think a low level, and 2) activity think a high degree. Activity think high levels is activity think that uses capacity knowledge taller. That activity aims to be on a level higher than the information, so as to be making evaluation, having cognizance metakognitif, and have the ability problem-solving. Think high levels other is a critical thinking creative, and constructive.

Ruseffendi (1994) says, the activity of higher-order thinking in three areas, namely: according to Bloom cognitive aspects of analysis, synthesis, and evaluation. This aspect of the analysis is the cognitive domain which gives facilities to develop/demonstrate the ability to separate the material into sections, looking for connections between the parts, see the components, how the components relate and are organized, as well as the ability to solve problems that are not routine. Aspects of the synthesis is the cognitive domain which gives facilities to develop skills in working with parts, elements for structured into a new unit like the patterns and structures. The last aspect is the evaluation of a thinking activity that facilities develop the ability to measure using the previous aspects.

As for the low level of thinking activity in mathematics learning process demonstrated by activities in resolving things/questions routinely, related to the use of direct in accordance with the rules; While the activity of higher-order thinking begins with observation data, extracting ideas, compiled a conjecture, analog as well as generalizations, besides doing the reasoning, connections, communication, and problem solving. Therefore, the atmosphere of mathematical thinking activity can show the process as well as the work of one simple and complex issues in the context of mathematics and outside of mathematics.

Mathematical thinking activity which is used for the context outside of mathematics among others thought to declare/communicate through writing skills in the language of the discipline. Writing skills are integral part of the activities of thinking. Without the activity of thinking there will be no work lettering. Therefore, the activity of writing make active thinking when constructing the individual words into sentences, stringing sentences into paragraphs.
There are several kinds of paragraph in writing, include a paragraph of argumentation. Paragraph of argumentation is a paragraph that explains the information and opinion with reasons. Argumentative paragraph intended to convince the reader, developed with development patterns as a result. Causal relationships at first from an event that is considered as the cause is known, then move forward toward a conclusion as an effect or result. The effect that appears can be either single-and multiple-effect effect (together).

Any individual/human thinking, either in high or low level. The results that made each individual/humanity is like what she thinks. Each individual is different in his ability of human/, tergantun his experiences. IV semester student logically already has enough math learning experience, so having the ability of State sentences a good argument. To know the contribution of mathematical thinking ability in writing ability argument needs to be done to research carefully. Therefore examined the contribution of mathematical thinking of students ’ writing ability in the argument. The student in question is prospective elementary school teacher (elementary school).

The formulation of the issues raised is: “What contribution to the mathematical thinking ability of students to write arguments on prospective elementary school teachers (PGSD)?” The details revealed on several questions as follows, 1) How low-level contribution to the mathematical thinking skills in students write arguments PGSD? 2) How high-level mathematical thinking contribute to the ability of students to write arguments in PGSD?

MATHMATICAL THINKING AND WRITING ARGUMENT

Thinking to solve math problems in algorithmic mathematical, show mathematical thinking activity (Sumarmo, U.: 2003). Mathematical thinking activities form two different levels, namely level higher-order thinking and level lower-order thinking.

Activity / cognitive process is broken down into tangible steps that are then used to guide thinking in solving problems / issues called thinking skills. For example, to teach thinking skills draw conclusions, firstly infer cognitive processes should be broken into steps: (a) identify the focus of the question or the conclusions to be made, (b) identify the known facts, (c) identify the knowledge that relevant previously known, and (d) make the formulation of the final outcome prediction (Sutrisno, J.: 2010).

There are three terms related to thinking skills, which is actually quite different, namely higher level thinking, complex thinking, and critical thinking. Higher-order thinking is the cognitive operations that are needed in the thinking processes that occur in the short-term memory. Thinking is a complex cognitive process that involves many stages or parts. Critical thinking is one type of convergent thinking, which is leading to a point. Opponents of critical thinking is creative thinking, the kind of divergent thinking, which is spread from a point.

Experience completing Higher Level Questions (rich questions), the question is asked to infer, hypothesize, analyze, apply, synthesize, evaluate, compare, contrast or imagined, showing a high level answers will develop high-level thinking skills.

Critical thinking is the ability to think critically. According Desmita, Lau and Chan (Abdul Kholis, 2007:16) argues that critical thinking is an intellectually disciplined process of actively and are met by the ability to conceptualize, use, analyze, synthesize, or evaluate information obtained, or generalize, observe, experiment, refleksi, thinking, or communication as a hint of any that do.

Critical thinking can foster creativity in finding the best solution in a problem. Gerhand (Abdul Kholis, 2007:16) states critical thinking as a complex process that involves acceptance and reinforcement of data, data analysis ask ek evaluation by developing qualitative and quantitative, and make decisions based on the evaluation.

Tarwin (2005: 9) says that in education, critical thinking is defined as the formation of aspects such as the ability to argue logic, syllogisms, and proportional reasoning. The statement was supported by Tapilouw M. (2009) which says that critical thinking is a disciplined way of thinking and controlled by consciousness. This way follow the logical flow of thought and signs in accordance with the known facts or theories.

Based on earlier exposure, the ability of critical berpikir not mean gathering information, but also must have the ability to make or draw conclusions from all the information that he knew, he was able to figure out how to use the information he has to solve a problem, and find the resources relevant to help resolve a problem.

Critical thinking skills every person is different or not necessarily the same. Necessary to distinguish an indicator so that we can assess a person’s level of critical thinking. Bayer (Abdul Kholis, 2007:19) specifies 12 indicators of critical thinking skills, which include:
1. To know what the problem is
2. Compare the similarities and differences
3. Determine relevant information
4. Formulate the right questions
5. Distinguish between evidence and opinion reasoned opinion
6. Correcting precision argument
7. Knowing the assumptions set
8. Recognizing the allusion or imitation
9. Recognize bias, factors, emotional, propaganda and the lack of proper sense of the word
10. Recognize differences in value orientation and outlook
11. Recognizes the adequacy of the data
12. Foresee the possible consequences

According to Ennis (Abdul Kholis, 2007:19) indicators of critical thinking skills are divided into five groups, namely:

1. Provide a simple explanation
2. Build basic skills
3. Make inference
4. Provide further explanation
5. Adjust strategies and tactics

The fifth group of indicators is described in more detail in the following table.

### Table: Indicators of Critical Thinking Skills

<table>
<thead>
<tr>
<th>Critical Thinking Skills</th>
<th>Sub Critical Thinking Ability</th>
<th>Explanation</th>
</tr>
</thead>
</table>
| 1. Provide a simple explanation | 1. Focusing question | a. Identify or formulate questions  
b. Identify criteria-criteria to consider possible answer  
c. Maintain the state of mind |
| 2. Analyze arguments | a. Identify conclusions  
b. Identify the reason (cause) stated (explicit)  
c. Identify the reason (cause) is not expressed (implicit)  
d. Identify irrelevance and relevance of  
e. Look for similarities and differences  
f. Looking for the structure of an argument  
g. Embrace |
| 3. Ask and answer questions that require explanation | a. Why  
b. What is the point, what does it mean  
c. What for example, what is not an example  
d. How to apply in such cases  
e. What causes the difference  
f. Are you stating that more than |
| 2. Build basic skills | 1. Consider the credibility of the criteria of a source | a. expert  
b. No conflicts of interest  
c. Inter source agreement  
d. reputation  
e. Using existing procedures  
f. Knowing the risks  
g. Board member's ability  
h. Careful habits |
| 2. Observe and consider the observation | a. Participate in summing  
b. Reported by the observers themselves  
c. Record things desired  
d. Reinforcement and strengthening possibilities  
e. Access to good condition  
f. Competent use of technology  
g. Observer satisfaction of creditworthiness criteria |
| 3. Make conclusions | 1. Conduct and consider the deduction | a. Logical group  
b. Logical conditions  
c. interpretation statements |
| 2. Conduct and consider the induction | a. Make generalizations  
b. Making inferences and hypotheses |
| 3. Decision making and consider the value of | a. The background facts  
b. consequence  
c. Application of the principles  
d. think of alternative  
e. Balancing, decided |
4. Make further explanation
   1. Define the term and consider the value of the decision
      a. Form: synonyms, classification, range, same expression, operational, examples and non examples
      b. Strategy definition (equation identifies actions)
      c. Content (the content)
   2. Identify terms and definitions consider
      a. Reasoning implicitly
      b. Assumptions are necessary, reconstruction argument
   5. Adjust strategies and tactics
      1. Decide to act
         a. Defining the problem
         b. Select criteria makes the solution
         c. Formulate alternative memungkinkan
         d. Decided that things would be done tentative
         e. Reviewing
         f. Monitor the implementation of

2. Interacting with others


Glazer (Abdul Kholis, 2007: 20) says that the process of critical thinking in mathematics must include: 1). Unfamiliar situations where individuals are not able to quickly understand how to determine a solution to the problem; 2). Using prior knowledge, mathematical reasoning, and cognitive strategies; 3). Generalization, verification, or evaluation; 4). Reflective thinking that involves communicating solutions with full consideration, making meaning with an answer or a reasonable argument, specify an alternative to explain concepts or solve problems or raise and extension for continued study.

Therefore critical thinking activities include: 1) Pay attention to the details thoroughly. 2) Identify trends and patterns, such as mapping information, identifying similarities and inequality. 3) Repeating pengembanan to make sure nothing is missed. 4) Looking at the information obtained from different viewpoints. 5) Selecting the preferred solution objectively. 6) Consider the impact and long-term consequences of the chosen solution.

Creative thinking is a way of thinking a person (people) in trying to find new relationships to acquire new answer of a problem. In creative thinking, are required to obtain more than one correct answer of a question. To obtain a correct answer, not acquired suddenly (soon). Required several stages of the process of thinking as follows.

1. Preparation Phase. In the stage of formulating the problem is a process of thinking with gathering all the facts and data necessary to solve the problem.
2. Incubation stage. Phase shifting of attention to the problems being faced by the new experience gained.
3. Stage Illumination. Think the solution search phase (reflection) based on the acquisition of experience to find a way of solving the problem.
4. Evaluation Phase. Phase assessing the results of solving the problem is to re-examine the obtained solution.
5. Revision Stage. The advanced stage of examining the results back, which is reviewing measures to produce a better solution.

Of exposure of critical and creative thinking, we get an explanation that someone making capabilities and determine problem resolution / answers needed capabilities / skills to think critically and creatively.

In contrast to the low-level thinking activities, which is the development of mathematical thinking skills early stage (first), perform simple arithmetic operations performed by applying mathematical formulas directly, follow the procedure (algorithm) standard. This has the ability to think mathematically functions include guiding mindset and attitude formation (Ruseffendi: 1994). Shafer and Foster (1997) stated that the ability to think mathematically low level or rate of reproduction involves the ability to know the basic facts, apply the standard algorithm, developed technical skills.

Low-level thinking skills is the ability to demonstrate an understanding of mathematical concepts which include mastery of arithmetic oprasi simple, direct application of mathematical formulas, and can work on mathematical tasks in accordance with applicable procedures. Ruseffendi line with the opinion, the mindset and attitudes of individuals who are familiar with the low-level math problems, will have the mindset and attitude that low anyway.

Associated with writing, which has the form of an idea or subject matter of mind. While the idea / thought to be the basis for the development of writing. And ideas / thoughts on a paper can vary, depending on the wishes of the author. We know that the author is the originator / thinker, because it's there in the writing ability / skill thinking writer. Thus the mathematical thinking ability will affect one's writing.

Posts that require interaction opponents understand what the necessary mastery of patterned speech argument. Write sentences exposure or argument is stated / write penyentesian reasons and opinions to build a conclusion. Posts argument is made with the intent to provide an excuse to reinforce or reject an opinion, understanding or idea. As said Supamo, Yunus M (2002), have led to the writing arguments or reasons and convincing evidence, that the reader or listener is affected and justify ideas, opinions, attitudes, and beliefs.

On any scientific work, the argument is usually used to pay attention to or encouraged the opinions, ideas or concepts of a problem to the reader based on the data, phenomena, or the facts presented. The argument
always describes a relationship between two statements that sort. The first statement is a statement of the reasons for both. For example: "Usually after a cloudy day at 12.00 and rain, so we had to bring a raincoat or an umbrella if you will be traveling". Further arguments are not just paper aims to convince the reader, but also can be: a) denial or against a proposal or statement without trying to convince or influence the reader to take sides, this argument merely aim to convey a view, b) a statement of the reason or rebuttal which affects the reader to agree, c) exposure to a problem-solving, d) a discussion of the problem without the need to reach a settlement.

To achieve the above purposes, the authors argue are required to have some requirements. Requirements write arguments are: a) must be able to think critically and logically and be able to accept other people's opinions into account, b) must have knowledge and a broad view of the context of the material / issues discussed, c) present the results of creative thinking / ideas so exposure can understand other people.

Step preparation of sentence / statement argument is as follows.

a) Determine the theme / topic of the argument
b) The purpose is to convince the reader argues
c) Develop outline based on the theme / topic and purpose
d) Looking for facts, data, information, and evidence in accordance with the framework of argumentation
e) Conducting research and assessment fact, the collected data that support the topic and purpose arguments
f) Develop a framework of argumentation argumentation posts

Authorship arguments often developed from exposure to things that are special to achieve a generalization, and sometimes constructed from common exposure to the specific. Therefore, there are two arguments development techniques that can be used, namely: a) inductive techniques, and b) deductive techniques.

Development of the technique of inductive argument is an argument that the preparation is done by arguing first that the evidence relating to the topic. Based on the evidence then drawn a general conclusion. Evidence can be presented examples, facts, experiences, reports, statistical data, and so on.

There are two things to note in gathering and using evidence to support the general conclusions. First, the evidence collected should be relevant to the topic and purpose of writing. Thus, the general conclusion is not deviating argument writing. Second, the evidence used to support the general conclusions should be plenty. How big is the amount of evidence that, depending on: (1) the importance of the issues discussed, (2) the extent of the range of issues, and (3) the difficulty for the reader to be convinced (Syafi’ie in Suparno: 2002).

Development of the technique of deductive argument, begins with a general conclusion then followed a description of the specific things. As the development of inductive techniques, development of the technique of deductive arguments are also necessary evidence to support the description presented. Reasons or evidence that strengthens or supports the conclusion in a deductive argument is called a premise. Argument with deductive technique is none other than mathematical logic. example:

Premise: All speak Sundanese Sundanese
Sule Sundanese people.

Conclusion: Sule speak Sundanese

Deductive reasoning systems with array elements as examples (two premises and a conclusion) is called syllogism (syllogism). The first premise as a major premise to make a general statement about the object, idea, or situation. Major premise contains a proposition is true for all members of a particular class. The second premise as the minor premise that contain more about a term in the major premise. The minor premise is a proposition which identifies a specific event as a member of the class.

2. RESEARCH METHOD

The method used in this study are correlational study two variables. The first variable of the mathematical thinking ability and other variables Indonesian skills in writing argumentative sentence. Mathematical thinking skills and the ability to write the argument derived from the test / final exam given to students at 37 PGSD the fourth semester students. With the use of correlational methods, will know the extent of the relationship between mathematical thinking skills with the ability to write argumentative.

Form this research paradigm is the paradigm that consists of one independent and dependent variables as shown below.

![Picture 1. Paradigm Research](image)

Description: X = Thinking Mathematically (independent)
Y = Writing Argument (dependent)
In data collection, the instrument required a research essay questions given at the end of semester exams. There are two components of the instrument, namely:

1) Instrument to measure the ability to think mathematically.
   Questions used is about tracking low-level thinking skills and high as follows:
   a. What is a .............
   b. How can the concept be given to elementary students? Explain to each concept is in question no. a.
   c. What are the reasons you explain the concept as stated in answer no. b.

2) Instrument to measure the ability to write argumentative.
   Questions used are as follows.
   In learning Indonesian in lower class are terms relating to the implementation of learning, namely:
   1. The types of communication
   2. Child language development
   3. The role of teachers as mentors
   4. The role of the teacher as a model
   5. Teacher's role as administrator
   Explain the terms are based on the answers that includes (1) What is .... (2) Why is it necessary / important ... (3) How to practice

3. RESULT AND ANALYSIS
   Data is the ability to think mathematically (X) and the ability to write the argument (Y) be reviewed to the highest, the lowest, average and standard deviation can be found in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Score Ideal</th>
<th>Highest Score</th>
<th>Lower Score</th>
<th>Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking Mathematically (X)</td>
<td>4.00</td>
<td>4.00</td>
<td>1.17</td>
<td>2.35</td>
<td>0.61</td>
</tr>
<tr>
<td>Argument Writing Ability (Y)</td>
<td>3.53</td>
<td>1.33</td>
<td>2.69</td>
<td></td>
<td>0.51</td>
</tr>
</tbody>
</table>

Ability to think mathematically and write arguments specifically in terms of indicator ability, can be seen in the average ability of each indicator in Table 2. following.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Subject</th>
<th>Score Ideal</th>
<th>Average score Capabilities thinking Mathematically</th>
<th>Average score Ability Writing Argument</th>
<th>Percentage Score Every Ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge and Comprehension questions (What)</td>
<td>37</td>
<td>4.00</td>
<td>2.45</td>
<td>2.51</td>
<td>61.25</td>
</tr>
<tr>
<td>Questions and Synthesis Applications (How)</td>
<td></td>
<td></td>
<td>2.79</td>
<td>2.74</td>
<td>69.75</td>
</tr>
<tr>
<td>Question Analysis and Evaluation (Why)</td>
<td></td>
<td></td>
<td>1.81</td>
<td>2.82</td>
<td>45.25</td>
</tr>
</tbody>
</table>

Based on Table 2, mathematical thinking skills possessed by students synthesize dominant indicator. On the indicator synthesizing ability in math is quite good (69.75%), and the knowledge and comprehension ability is quite (61.25%), while the evaluation of mathematical ability is low (45.25%).
In language skills, specifically the ability to write the argument quite well (70.50%), while the ability of knowledge and understanding of the language is quite (62.75%), and the ability to synthesize quite good (68.50%). In addition, the acquisition of knowledge and the ability to synthesize and is quite good enough. However, to evaluate the ability of existing mathematical level is less, while the evaluation of language ability is on good level.

Results of test calculations of data linear mathematical thinking skills and writing arguments by using SPSS 20.0 shows the relationship between the ability to write an argument with the ability of students to think mathematically PGSD very low (r = 0.19 to 0.13 significance). And the contribution of mathematical thinking to write the argument is 3.6% (R² = 0.036), or 96.4% there are other factors that contribute to the ability of students to write arguments. Price F = 1.303 with a significant level of 0.261 confirms that there is no significant relationship between mathematical thinking skills with the ability to write an argument. Therefore not significant mathematical thinking skills contribute writing argument (t test). Analysis of the regression coefficients are as follows, Writing Ability Argument = 2.318 + 0.158 Mathematical Thinking. This shows that the ability to write an argument already owned enough students (of 2,138) before it affected the ability of mathematical thinking, mathematical thinking skills while contributing only 0.158.

In terms of low-level mathematical thinking skills, writing skills contribute sebesar18 arguments, 1%, amounting to 81.9% contributed by other than low-level mathematical thinking. ANOVA test results stating no significant influence of low-level mathematical thinking skills with the ability to write an argument. However, from the results of the t test showed significantly lower levels of mathematical thinking affects the ability to write an argument. Great influence can be seen from the following regression equation, \( Y = 1.802 + 0.362 X \) (Writing ability Arguments = 1.802 + 0.362 Mathematical Thinking Low Level). In other words, the ability of written arguments already owned by 1.802 students before the affected low-level mathematical thinking skills. Constants that are less relevant to the category. While the contribution of low-level mathematical thinking skills is 0.362; greater than the contribution of the ability to think mathematically as a whole.

Judging from the high-level mathematical thinking skills, writing skills contribute sebesar18 arguments, 1%, amounting to 81.9% contributed by other than high-level mathematical thinking. ANOVA test results stating no significant influence of high-level mathematical thinking skills with the ability to write an argument. Through t-test, high-level mathematical thinking significantly affect the ability to write arguments. Learn obtained the following regression equation, \( Y = 2.655 + 0.018 X \) (Capability Writing Arguments = 2.655 + 0.018 Higher Level Mathematical Thinking Skills). It was argued that without a high level of mathematical thinking, already has enough ability to write arguments (2,655). While the contribution of high-level mathematical thinking skills is 0.081; least lower than the contribution of the ability to think mathematically as a whole let alone the ability of low-level mathematical thinking.

4. CONCLUSIONS

Based on the analysis of research data, the ability to think mathematically have no significant contribution to the ability to write an argument. Value of mathematical thinking very small contribution to the ability to write the argument (just 0.158), and previous students have had sufficient ability to write argument (2.138).

At a low level of mathematical thinking skills revealed that there is a significant contribution to the ability to write an argument, although worth only 0.362; students with initial conditions that already have the ability to write an argument at a low level (1.802). For high-level mathematical thinking skills gained significant contribution also to the ability to write an argument, although its value is very low at 0.018, and previous students already has the ability to write an argument on the level enough (2.655). From the analysis of the test data, the researcher found that the contribution of information to think mathematically greater than the low-level contribution to the high-level mathematical thinking ability of students to write arguments.

Based on these results, it should be a concern that some of the other factors that contribute to the ability to write an argument, especially in the context of mathematical thinking should be a concern for anticipated and acted upon. Take on high-level mathematical thinking skills. High-level mathematical thinking skills need to be carefully addressed in order of ability to think this gives a greater contribution in the realization of the ability to write an argument.

REFERENCES


EFFECT OF REACT MATHEMATICS LEARNING STRATEGY ON THE IMPROVEMENT OF JUNIOR HIGH SCHOOL STUDENTS MATHEMATICAL UNDERSTANDING, REASONING, AND COMMUNICATION SKILLS

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ABSTRACT

The main problem of this study is that “Teachers tend to inform, but they do not provide opportunity for their students to construct their knowledge, so they are unable to increase the quality of teaching”. Hence, the study aims at identifying differences in the improvement of mathematical understanding/comprehending, reasoning, and communication skills of junior high school students who were learning mathematics in REACT Mathematics Learning Strategy (RMLS) and who were learning mathematics in conventional strategy (conventional learning strategy/CLS). A pretest-posttest control group design in which experiment group was treated in RMLS and control group was treated in CLS has been adopted. The study has involved four seventh grade classes, of two good and medium quality schools. Two classes, experiment and control classes, represented each school. Data were collected by tests consisting of two mathematics tests constructed on the basis of learning materials, and this instrument has qualified surface and content validity and reliability. Based on statistical analysis, it can be concluded that: (1) on the overall mathematical ability and learning approach, a) there are different abilities, b) in KMA, there is a difference among medium and low KMA students, c) in school category there are different abilities in good and medium quality schools, d) school category, approach, and KMA, there is a difference among medium and low KMA students; (2) on mathematical understanding, reasoning, and communication skill, a) student in RMLS and CLS are different in their mathematical skills, b) in KMA, high KMA student are only difference in their understanding, whereas low KMA students are different in reasoning and communication skills, c) in school category, good quality school indicates a difference in understanding and reasoning, and medium category school is different in those three skills, d) school category and KMA students, there is a difference in understanding among high KMA students of good quality school, medium KMA students are different in reasoning and communication, low KMA students are different in reasoning only, and the medium KMA students of medium quality school are different in understanding and communication skill, but low KMA students are only different in reasoning and communication skills.

Keywords : React strategy, Comprehending, Reasoning, Communication skills

A. Introduction

Mathematics is an essential basic science to be studied, because the mindset of mathematics can help students to think of logical, analytical, systematic, critical, and creative, and able to work in groups. Through mathematical mindset of the students are unable to deal with various changes in life that is always evolving, and can provide convenience in studying various sciences.

Mathematics has many uses, but the math is deductive and abstract, making it difficult for many people to understand, including the students. Therefore it is necessary that efforts be made to help students in learning mathematics. Efforts to improve students' understanding of mathematics such as by improving the quality of teachers in the understanding of the material, and the quality of learning mathematics. This increase is necessary because of the quality of mathematics education in our country is still not increased significantly (Hudoyo, 2003; Marpaung, 2003; Sabandar, 2001; Zulkardi, 2001).

One concrete effort that needs to be done is to increase the quality of mathematics learning, because learning mathematics, especially in middle school still felt to have a problem. One such problem is the result of students' mathematics learning is low, when compared with the other subjects in the UAN. This is consistent with the results of interviews between researchers with junior high math teachers, there are still many students...
who have difficulties in learning mathematics, especially when solving problems in the form of stories (problems) and student learning outcomes in general was lower when compared with the value exams obtained when the end of school in elementary school.

The low student achievement in school mathematics is caused by several factors, as said Ruseffendi (2006: 7) that in the process of learning mathematics there are ten factors that influence the success of the children studied, one of which is the presentation of the material models.

Model presentation of the material in the learning of mathematics is an interesting factor to be assessed and investigated, as it turns out in the field in general presentation of the material is still mostly in the form of information, fewer question and answer, and giving the task to the questions that have been changed in the language and symbols mathematics is far from the reality of everyday life, as well as not providing opportunities for students to develop the power of reason. When the teacher in the classroom usually active themselves while students are passive. This research, learning like this called the conventional learning (PMKv).

To overcome the situation seems to actions that need to be done especially in the presentation of mathematical material which is much more focused on contextual issues, the issues that presents a real environment in everyday life of students. Learning should be student-centered and teacher acts as a resource is not dominant.

Based on the description of the problem context as stated above, according to Crawford (2001) the application of mathematical learning approach that uses contextual issues and is expected to improve the ability of understanding of mathematical concepts, and other mathematical skills, and can improve the performance of the students are learning using strategies REACT (Relating, Experiencing, Applying, Cooperating, and Transferring). The further learning of mathematics as we call REACT Strategy Learning Mathematics with the abbreviated PMSR. Through this study are expected skills of understanding, reasoning, and mathematical communication students can increase significantly.

B. Research Problems

The main thing that want to look for the answer as follows:

1) Are there differences in enhancing of mathematical skills overall (combined) based on (a) approach PMKv and PMSR?, (b) instructional approaches and students' KMA? (c) instructional approaches and school level?, and (d) instructional approaches, school level, and KMA students?
2) Are there differences in enhancing understanding, reasoning, and mathematical communication skills of students is reviewed at each based on (a) instructional approach? (b) instructional approaches and students' KMA?, (c) instructional approach and school level; (d) instructional approaches, school level, and students' KMA?

C. Research Purposes

In general, this study aims to examine and reveal the influence of mathematics learning using REACT strategy to increase understanding, reasoning, and mathematical communication junior high school students, and the difference when compared to Conventional Learning Mathematics. Both in terms of overall mathematical skills as well as each, based on KMA, School Level, and the instructional approach.

D. Hypothesis

Based on the background of the problem, research of the problem, and the purpose of the research, then drafted the hypothesis to be tested, are:

1) There are differences in enhancing of mathematical skills overall (combined) based on (a) approach PMKv and PMSR, (b) instructional approaches and students' KMA; (c) instructional approaches and school level, and (d) instructional approaches, school level, and KMA students;
2) There are differences in enhancing understanding, reasoning, and mathematical communication skills of students is reviewed at each based on (a) instructional approach; (b) instructional approaches and students' KMA; (c) instructional approach and school level; (d) instructional approaches, school level, and students' KMA.

E. Literature Studies

1. Contextual Learning Approach to Strategy REACT

The purpose given mathematics in school is for students to use mathematics as a way of reasoning on any state, such as critical thinking, logical thinking, systematic thinking, objective, honest, disciplined, in looking at and solving a problem, being able to use mathematics as a tool of communication, and has solve mathematical problems skills as well as issues related to real life.

One approach to learning that is expected to achieve these objectives are based on the context of learning, ie learning by Contextual Approach. Contextual approach is the concept of learning that helps teachers relate the
content to be studied by students real-world situations, and encourages students to make connections between the knowledge he has with the application in their lives as members of families and communities (Team Director General of Primary and Secondary Education in Sukestiyarno, 2003). It is within the meaning of Contextual Teaching and Learning (CTL) as stated Johnson (Setiawan and Sitompul, 2007: 67) that:

CTL is a learning approach that aims to help students see meaning in the material they are learning by plugging in the context of their daily lives, ie the context of his personal environment, social, and cultural. To achieve these objectives, CTL will guide students through the eight major components, namely: making meaningful linkages, doing meaningful work, organize their own learning, collaboration, critical and creative thinking, helping students to grow and thrive, achieve high standards, and uses authentic assessment.

Based on the understanding of CTL as noted above, the contextual learning of mathematics there are five strategies that can be used, namely: Relating, experiencing, applying, cooperating, and transferring. Learning the five strategies is called Learning Mathematics with REACT Strategy (PMSR). How the application of the five strategies in learning mathematics, Crawford (2001: 3-14) gives a fairly detailed explanation as follows.

(1) Relating

Relating is learning that begins by linking new concepts that will be studied with the concepts that have been taught (Crawford, 2001: 3). For more details see the following examples, for example, students will learn about the surface area of the beam, then the teacher can provide the questions on the following activities:

Questions on the first activity:
1. What shape our classrooms?
2. There how many pieces of the wall?
3. What shape the wall of our class up?
4. What shape the floor of our class and the roof of our classroom?
5. How many pairs of the same in our classrooms?
6. How the area of walls, roof, and floor?
7. How the area entirely?

Questions on the second activity:
1. How wide carton paper needed to make a block that you created?
2. If unit length of the timber is p, q unit width, and height r units, how the formula to find the surface area of the timber?

(2) Experiencing

Students in building a new concept, will be based on the experiences that occur in the classroom. As said by Crawford (2001: 5),

Experiencing strategy can help students to build a new concept by concentrating the experiences that occur in the classroom through exploration, search, and discovery. This experience can include the use of manipulation, problem solving, and activity in the laboratory.

Manipulation, can help students build abstract concepts clearly. Here is an example of how the role of manipulation in building an abstract concept.

Board or cartons fraction simple fraction or demonstrate the meaning of addition and multiplication of fractions. This is considered as manipulation because it can help students visualize and explore concepts and can make students see answers quickly to the question "How about?"

Problem Solving, by providing a form of problem issues can teach students problem-solving skills, analytical thinking, communicating and interacting with the group.

Laboratory activities, through this activity students can work in groups to obtain data by measuring, analyzing data, making predictions and inferences. Example: Students take measurements of their height. They compared their data with other groups in a diagram. Then they made a plot coordinate system (drawing data) in pairs.

(3) Applying

Application or the application is fairly important aspect in the study of mathematics, as someone who has been able to apply a mathematical concept means he was able to understand the concepts in depth. As stated by Crawford (2001: 8),

Applying of strategy, a strategy of learning by the use of the concept. Students can use the concept when they engage in problem solving activities or other mathematical activities. Teachers can also provide motivation for understanding the concept of providing realistic and relevant tasks

Applying strategies to clarify understanding in mathematics learning we see the following example:

We want to paint all the walls and ceiling of a classroom with white. The chamber length 6 m, width 5 m, and 4 m high. When a tin can paint an area of 8 m². How many cans are needed for all of the walls and roof of the room can be painted?

In general, students would prefer to work on word problems, rather than practicing counting something
unrealistic, because through word problems students may engage in more meaningful in learning mathematics.

(4) Cooperating

Cooperation among students is important issue in learning mathematics, because through the cooperation of students will be able to discuss, share, and respond to their neighbour. As said by Crawford (2001: 11), "Learning strategies are cooperating in the context of sharing, responding, and communicating with fellow friends. In this study them more freely and not feel embarrassed to ask, they will also be better prepared to express their understanding of the concept of his friend. Together with his friend they learn and revise their own understanding memformula. Learning with this strategy will be more successful if students have the opportunity to express ideas and get feedback from his friends."

(5) Transferring

Teacher's role in contextual learning, they are creating a learning experience that focuses on understanding rather than recall. Students who learn with understanding can also learn to transfer knowledge. According to Crawford (2001: 13), "Transferring a teaching strategy, which is described as the use knowledge in new contexts or situations, where a person has not been done in the classroom".

For example, to provoke students' curiosity, Crawford (2001: 4) give examples of problems in the form of problem solving.

Write a book about the thickness of two miles (one mile is 1000 inches), this book If you fold it into 4 miles thick. And if it is thick again folded to 8 miles. If you fold it 50 times. Which question below which shows the thickness.

a. Less than of 10 times.
b. More than 10 times but less than ten-Story tinggi building.
c. More than of ten high-Story building but less than Mount Everest.
d. More than of Mount Everest but less than the distance to the moon.
e. More than of the distance to the moon.

2. The Basic Steps of Mathematical Learning Using REACT of Strategy

1) Relating, meaning after bringing students in a learning situation further conditioning the student teacher to be able to relate the new concepts that will be studied with the concepts he had learned by way of providing what is appropriate to the material being studied.

2) Experiencing, learning takes place when teachers have to create a situation that can help students to build a new concept by way of concentrating the experiences that occur in the classroom through exploration, search, and discovery. This experience can include the use of manipulation, solving problems, and laboratory activities.

3) Applying, to check whether the students have understood very well about the concepts being taught, the teacher can give problems that require students to be able to use the concepts they have learned is a learning strategy and how to use the concept. Teachers can also provide motivation to deepen understanding of the concept by providing realistic and relevant tasks.

4) Cooperating, to improve the quality of learning in activity Relating, Experiencing, Applying, and Transferring can be implemented through cooperation among students, because through the cooperation of the students will be able to discuss, share, and respond to his other friends.

5) Transferring, at this stage students should be able to use their newly acquired knowledge in the face of a new context or situation given by the teacher.

3. Mathematical Understanding, Reasoning, and Communication Skills

1) Mathematical Understanding

Understanding is a translation of the understanding that the intention is the absorption of the meaning of the subject matter being studied. Understanding according to Hewson and Thorley (Kusuma, 2005: 3) is a conception that can be digested or understood by students so that students understand what is meant, is able to find a way to express the concept, as well as to explore the possibilities related.

Understanding of the concept, according to Skemp (Sumarno, 1987 : 24) there are two kinds of understanding, the understanding of instrumental and relational understanding. Instrumental understanding is defined as understanding (mastery) are still mutually exclusive concepts between one concept with other concepts and be able to apply new concepts / principles in the calculation routine / simple, or do something as algorithmic. While relational understanding, is the ability to link multiple interrelated concepts. This notion implies that in addition to one to understand a number of concepts that have been learned, it can also understand the relationships between the concepts are interrelated.

Knowledge and students 'understanding of mathematical concepts can be seen from the students' ability, 1. Define of concepts verbally and in writing.
2. Define and make an example and not an example.
3. Use models, diagrams and symbols to represent a concept.
4. Change a representation to other forms of representation.
5. Know various meanings and interpretations of the concept.
6. Identify properties of a concept and know the terms that define a concept.

The following problem is an example that is used to reveal the capability of understanding the instrumental and relational understanding of a student.

Wake square PQRS below (with a length of one side of the unit) is divided into several areas, each area shows the plan the amount of money that will be issued by Miller in the coming months.

![Image of Wake square PQRS](image-url)

a) Calculate the area A, B, D, and J!
b) If the area A shows the amount of pocket expenses for Rp.75,000.00, how much spending it all on each of these plans?

2) Mathematical Reasoning

Reasoning is a translation of the reasoning which is defined as the process of thinking that is done in a way to draw conclusions. General conclusions can be drawn from the cases is individual. But it can also be the opposite, of things that are common to the individual cases (Suherman and Winataputra, 1993: 222). The second type of reasoning is called inductive reasoning and deductive reasoning. Inductive reasoning by Keedy (1963: 67), "When a person makes observations and on the basis of his observations Reaches Conclusions, ... " while deductive reasoning, " ... proceeds from assumptions, rather than experience".

Mathematical reasoning abilities that were examined in this study is inductive reasoning, with emphasis on aspects of the assessment of logical reasoning, and inductive analogy. Logical reasoning according to Kennedy (Hudoyo, 1990) is the ability to identify or add logical arguments necessary to resolve a matter, whereas logical reasoning referred to in this study is the ability of students to give a logical reason necessary for the completion of a matter. Shurter and Pierce (Sumarmo, 1987) states that analogy reasoning is reasoning from a specific case to another similar case later concluded.

The following problem is an example of logical reasoning ability,

How many different fractions can be formed from the numbers 1, 2, 3, and 4, with the denominator and the numerator, each consisting of one digit and the numerator smaller than the denominator. Write down and sort the fragments from the smallest value to the largest value! Why did you answer like that, please explain! What can you conclude about the order of fraction.

The following issues are examples that reveal about the ability of analogy reasoning.

Two square picture below that the width of each of the unit area, showing the plan the amount of money that will be spent by Astri at one month to come.

![Image of Two square pictures](image-url)

When the cost of public transportation that is needed in a month is Rp 90,000.00, Which plan should be taken by Astri? Why? Explain!

3) Mathematical Communication

Communication according to Indonesian dictionary (Surayin, 2003: 250) means the sending and receiving of news or messages between two or more people. This means that the interaction in communication both written and oral messages between the giver and the recipient. The interaction can take place unidirectional, bidirectional or many directions. Unidirectional communication occurs in many conventional learning where teachers are more dominant, while the two-way communication is used in many learning towards a greater emphasis on student activity, including communication in PMSR.
At the time of teaching and learning take place usually students obtain information about facts/definitions/concepts/theorems of mathematics or reading teacher, so that when it happens the transformation of information from sources to the poor students. Students would respond based on his interpretation of the information. Problems will arise if the response does not match what students are expected by the teacher. To cope with this kind of thing students need to learn to communicate the results of their study habit or ideas both orally and in writing.

In order for communication skills in mathematics in students can be formed, it needs to make the learning objectives. Objectivity must necessarily be tailored to the student and the student's ability in the classroom how. NCTM (1989: 78) recommend the learning objectives for mathematical communication grade 5-8, is for students to:

a) Using a model of the situation by way of oral, written, concrete, images, graphics, and algebraically;

b) Reflect on the situation and clearly mathematical ideas;

c) Develop understanding of mathematical ideas, including the definition;

d) Using the skills of reading, hearing, and seeing to interpret and evaluate mathematical ideas;

e) Discuss mathematical ideas, make conjectures and convincing arguments;

f) Understand the meaning of mathematical notation and rules in developing mathematical ideas.

Baroody (1993) states that there are at least two important reasons that make communication in mathematics learning should be the focus of attention, they are: (1) mathematics as a language: mathematics not only as a tool for thinking, tools for finding patterns, or solve the problem, but mathematics as well as a good tool for communicating various ideas so clear, precise, and concise, and (2) the learning of mathematics as a social activity: interaction between student, teacher communication with students in learning mathematics is a fairly important part to develop the potential of students.

Through communication students can organize and consolidate mathematical thinking (NCTM, 2000), and also students can explore ideas in mathematics. Communication in learning mathematics developed in this study is a mathematical communication that is convergent and has the following indicators:

a) Declare a daily occurrence in the language or mathematical symbols;

b) Declare a picture/diagram to the idea of math or other forms;

The following issues are examples of questions that reveal about the ability of communication expressed everyday events into mathematical symbols.

A truck can carry a charge no more than 2,115 kg. Weight of the driver and kennentinya 150 kg. He will transport several boxes of items, each box weighs 50 kg. Write the mathematical model for the problem!

The following problem is an example of the problem of mathematical communication skills expressed in the form of an image to another.

The following figure shows the floor plan in a place that looks a combination of several flat geometry up to form a square ABCD with the size of the sides is a unit of length. Based on the picture below, show with images that AGPCPQ heptagon is a combination of some geometry! Mention anything up!

**F. Research Method**

This study is an experimental study, the design of the pretest-posttest control group of Ruseffendi (2003a: 45)

A O X O

A O O

Information:

A = Selection of subjects (sample) research is purposive based class.

X= Treatment strategy that is learning with REACT.

O = Pretest = Postest of mathematical skills.

Subjects of the study were students of class VII SMP purposively selected four classes. Students study sampled for each class is 34 people, two classes of schools on level well and two more classes from school on moderate level.

The instrument used in this study consisted of two sets of questions, which for the first test and the second
G. Data Analysis
To answer the hypothesis and the formulation of the problem, using the Independent Samples Test, ANOVA, QQ plots, and other necessary present in SPSS 17.

H. Results and Discussion
1. Prior Knowledge of Students Mathematics (KMA)
   Determination KMA scores obtained from an average of three times a math test, and consideration of teachers who teach in classroom research. KMA students are classified into three categories, are KMA students who were high (score of 75-100), moderate (score 55-75), and low categories (scores 0-54).

2. Mathematical Skills Students In Overall
   a. Based Learning Approach, the results of t-test analysis showed a mean gain score differences in mathematical skills gain scores overall between students learning via PMKv with PMSR at a significance level of 5%. When viewed from the difference in mean score gain, then the students PMSR better than students PMKv.
   b. Based Student Learning Approach and KMA, the results of the student t test analysis PMSR and PMKv with KMA moderate and low, there are differences in the mean scores showed gains in overall mathematical skills. While the analysis of the students' high KMA there is no difference between the mean gain scores of students PMSR and students PMKv.
   c. Based Learning Approach and School Level, the results of t-test analysis at both the school level and are being, showed no overall differences in mathematical ability between students learning via PMSR with students PMKv. When seen from the mean scores, students are at the school level of moderate and the level of well, students PMSR higher than students PMKv.
   d. Based on Level School, Learning Approach, and KMA of students, the results of t-test analysis of student data KMA moderate and low groups at both school level and are showing no difference in gain scores on the 5% significance level. Meanwhile, a group of student data at the school level high KMA either reflected no differences in gain scores, and student data at the school level KMA are high can not be determined because the students in the group there is only one data.

3. Skills of Mathematical Understanding, Reasoning, and Communication of Students
   a. Based Learning Approach, the t test results indicate a difference in score gains in understanding, reasoning, and mathematical communication between students PMSR and students PMKv at a significance level of 5%.
      When based on the average score gain of each, the skills of understanding, reasoning, and mathematical communication students PMSR higher than students PMKv.
   b. Based Learning Approach and Student KMA: (i) The results of the t test mathematical reasoning ability and communication at high KMA students, showed no differences in gain scores, whereas no difference in the ability of understanding between the students PMSR and the students' learning with PMKv at a significance level 5%; (ii) the results of the t test understanding, reasoning and mathematical communication in the group of students KMA of moderate, there are differences in the mean scores gain between students PMKv and students PMSR. When viewed from the mean score gain, suggesting that the ability of understanding, reasoning, and mathematical communication students PMSR higher than students PMKv; (iii) The results of the t test mathematical reasoning ability and communication is no difference in the gain scores of students in the group with low KMA, while the ability understanding scores showed no difference in the ability to gain mathematical understanding, between students PMSR and students PMKv.
   c. Based Learning Approach and School Level: (i) The results of the t test for the mean understanding and mathematical reasoning skills at school level both showed no difference in the mean gain scores between students PMKv and students PMSR, whereas in the no communication skills mean difference score gain; (ii) the results of the t test for the mean understanding, mathematical reasoning and communication at the school level is showing the difference in mean scores gain between students PMKv and students PMSR. In general, the skills of understanding, reasoning, and mathematical communication students PMSR higher than students PMKv.
   d. Based on Level Schools, Learning Approaches, and Student KMA Group: (i) t-test results mean similarity mathematical reasoning ability and communication at the high school level students KMA either showed no difference in the mean gain scores between students and students PMKv, whereas in there are differences in the ability of understanding the significance level of 5%, (ii) at the school level students are well KMA, shows there is a difference score gain understanding and mathematical reasoning abilities, between students and students PMSR PMKv, whereas in mathematical communication skills are not there differences in mean score gain (iii) the results of the t test on low KMA students at both school levels, the
ability of reasoning indicate a difference in gain scores on the 5% significance level between the student PMKv and the student PMSR, while the ability of understanding and communication there is no difference.

I. Conclusion

Based on the analysis of the results of the study as described in the previous section, obtained some conclusions as follows:

1. Understanding, reasoning, and communication skills between student mathematical learning through PMSR with students learning through PMKv be reviewed as a whole:
   a) there are significant differences in mathematical ability, which is the average score gain of students PMSR better than students PMKv.
   b) and KMA students, there are significant differences in the students' mathematical skills KMA medium, and low, whereas at high KMA students there are differences in ability. Nevertheless, the students' mean score gainnya PMSR stay better.
   c) and the school level, there are differences in mathematical abilities in school children and moderate with a good level. This difference would be based on gain scores, students' mathematical skills PMSR better than students PMKv. While the students PMSR, students at the school level are better than good level school students.
   d) school level, and KMA, there is a difference in students' mathematical ability and low KMA are available at the school level and are good. While the high KMA students, both students at the school level as well as being good and there were no differences in mathematical ability.

2. On the skills of understanding, reasoning, mathematical communication, respectively are reviewed:
   a) Based on the learning approaches, there are significant differences in the ability of understanding, reasoning, and mathematical communication respectively. When viewed gainnya mean score, then students are learning with PMSR higher than students PMKv.
   b) Based on the approach to learning and KMA,(i) At high KMA student group is not there a significant difference in reasoning ability, and mathematical communication between students PMKv and students PMSR, except for the ability of understanding there is a difference; (ii) In the group of students moderate KMA, there are differences in mathematical ability of third respectively, among students PMSR with student PMKv; (iii) Students low group, there are differences in reasoning ability and communication between students PMSR with PMKv students, while the ability of understanding there is no difference in mathematical ability. This difference when viewed gainnya score, then each group of students the mathematical knowledge at the KMA high, medium, and low on student learning PMSR better than students with PMKv;
   c) If any mathematical ability level visits by school and learning approaches, then: (i) At either level school students, there is a difference score gains in understanding and reasoning skills among students PMKv with student PMSR, while the communication skills there is no difference; (ii) At the school level students are, there are differences in mathematical reasoning ability and communication between students PMSR with students PMKv. This difference when viewed by the score gainnya, both students at the school level and learning through PMSR, the ability of understanding, reasoning, and mathematical communication better than students with PMKv;
   d) If the ability of understanding, reasoning, and mathematical communication, respectively, seen by school level, learning approaches, and KMA students, then: (i) At the level of a good school, for students KMA high, there was no difference in mathematical reasoning ability and communication, between students PMKv and students PMSR, except in the mathematical understanding of the capabilities there is a difference. In the group of students KMA moderate, there are differences in the ability of understanding and mathematical reasoning, the students PMSR with PMKv students, whereas in mathematical communication skills no difference. At KMA low student group, only the differences in mathematical reasoning ability, while the communication skills and mathematical understanding there is no difference between students' mathematical skills PMSR with students PMKv.

In general, when viewed by the score gain, the ability of understanding, reasoning and mathematical...
communication PMSR students better than students PMKv. (ii) At the school level for students KMA are high, the difference can not be determined; because there is only one data is data on student PMSR, while data on student PMKv. At KMA students are, there are differences in the ability of mathematical understanding and communication between student PMKv with students, while the mathematical reasoning ability there is no difference. At low KMA students, there are differences in reasoning ability, and mathematical communication, between students PMSR and students PMKv. But in the mathematical understanding there is no difference. When viewed by the score gain understanding, reasoning, and mathematical communication, PMSR students better than students PMKv.

J. Suggestions
1. Based on conclusion of the study, suggests that the approach to learning with a REACT strategy can significantly improve the skills of understanding, reasoning, and certain mathematical communication better than using the conventional approach to learning, both in terms based on prior knowledge, and school level. This means learning approach with REACT strategy is very likely to be implemented in the mathematical learning.

2. Suggests for teacher, if the teacher will be teaching with the aim to improve the understanding, reasoning, and mathematical communication in a comprehensive unity, then study the effect of learning with REACT strategy against the skills capabilities can be used as an alternative in learning. In order implement this learning approach can achieve satisfactory results, the implementation of teachers need to pay attention to: (a) teaching materials that students should be taught in relevant and appropriate, further teaching materials should be designed specifically to suit the objectives to be achieved, so as to learning occurs in students who are able to optimize the increase understanding, reasoning, and mathematical communication, (b) at the time the student is trying to solve the problems it faces, teachers do not be too quick to give an answer, but try to give direction to subsequent questions that lead to resolution of the problem, (c) the implications of the results of this study can be used as the main base, and (d) adequate mastery of both theoretical and implemented, on approach to learning with the REACT strategy.

3. In giving attention to learning process conditions implementation on the ground is still a lot of learning activities using student-centered, it is possible that teachers face difficulties in applying REACT strategy learning approach with the results of this study. However, due to the prevailing curriculum in schools today require changes in the learning more student-centered, the teacher would not want to want to start trying to change those habits. For example, the teacher would try to apply this approach to learning the results of the study.

4. Suggests for school, because the current curriculum is learning to prioritize activities to the students, the school leaders should always try to condition that the teacher is always motivated in implementing learning more student-centered activities such as REACT strategy learning approach with the results of this study. Because it is consistent with the view that mathematics is seen as something to be constructed by the students.

5. This research using only certain materials, and take place in the seventh grade, it is recommended that further research conducted on the ability of understanding, reasoning, and mathematical communication on the scope of the other materials that may be, the broader sample, the class higher, and at other levels of education such as in schools at the SD, SMP, or SMA.

6. This research is about use of PMSR in an effort to improve the skills of understanding, reasoning, and mathematical communication is more use of Student Activity Sheet (LKS), it is recommended that further research conducted using other media in accordance with the characteristics PMSR.
REFERENCES


CLASSROOM REFORMATION ON MATHEMATICS LEARNING
(Learning from Misconception through Lesson Study)

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ABSTRACT
This discussion material based on an assumption that to build teacher professionalism as “learning expert” can’t be separated from reality of learning practice in classroom. A Professional teacher not only has to have standard qualification of education and standard competency but also has to have standard performance which appropriate with law. In line with knowledge, technology, and culture placed education as a strategies process with role of teacher as the most important thing. Reformation of education and reformation of school need teachers as “learning expert”. So that teachers should learn each other continuously in mathematics and pedagogy. Based on the problem, classroom reformation is needed, because classroom is a place for teachers to teach and learn as responsibility to their profession. On the other hand, the existence of lesson study can be viewed as model of conventional training to reform professionalism of teacher. To enhance teachers’ competences in classroom it should be related with learning practice problem. After that, teachers have to study the learning process based on principle of lesson study, namely: colaboratif, colegiality, and mutual learning to reform learning community. Further, through naturalistic inquiry study, empirical data of characteristic of learning misconception of school mathematics is founded. Misconception characteristic profile which is founded in steps of lesson study (Plan-Do-See-Redesign) namely: (1) weakness of pre requisite, (2) false on algorithm process, and (3) lack of understanding written language.

Keywords: Classroom reformation, Misconception, Lesson study

1. INTRODUCTION
Build the professionalism of teachers can not be separated from issues related to the learning process in the classroom. Real quality of education take place in the classroom with a major role is the teacher. The process of learning in classroom is a real curriculum needs to be assessed continuously by the teacher as an expert learning.

In the global atmosphere which chaos, complex, turbulences, full of change and uncertainty, requires teachers to continually add to their knowledge and insights. More so at the present time with the implementation of Education Reform and School Reform, of course, requires a Class Reform. The atmosphere was laid teacher positions as professional workers.

Professional teachers always able to provide a challenge, satisfaction, and always meet the needs of their students and their communities, including himself. As a learning specialist teachers should have educational qualification standards, competency standards, and performance standards as well. The presence of the teacher in accordance with the standardization of regulations, namely Law, 14 Year 2005 on Teachers and Lecturers, Government Regulation no. 19 Year 2005 on National Education Standard, Regulation of Manaster of Education No. 16 of 2007 on the Guru , and Decision of minister for Administrative and Birocracy Reforms No . 16 of 2009 on the Functional Teacher and credit figures.

The existence of class reform is inseparable from the idea of education reform. Expressed by Jalal and Supriyadi (2001 : v - vi ) that education reform should be left out of the education problems in the past and now and then get ready for a change. The purpose is to build a better education system by empowering all potential optimally.
In line with the education reform appeared an idea of school reform that has been rolled out in Japan. Sato (2013:1) "Reform of the school with learning community has spread outside of Japan since 2000". School reform has been introduced in America, Europe, Australia, and Asia, including Indonesia.

Inspired by the idea of education reform in the macro level, and school reform in the meso level, it is raised in this paper the term reforms in the micro level class. This term is deemed strategic and allow it to be realized. In particular we will discuss issues about classroom reform related to the teaching misconceptions of school mathematics in the context of lesson study (LS).

Misconception of school mathematics learning is the fact of the practice field as the expression of LS start of the discussion in planning (plan), researching learning in the classroom (do), post-lesson discussion (see), and re-design to the next plan. This paper is just a recommendation as a form of participation in building a culture of quality awareness in educational environment, especially in mathematics education.

2. CONTENT OF STUDY

1. Philosophy of Classroom Reform in Learning Mathematics

Talking about the class nature of the reform in the context of mathematics learning will relate to discussion about the nature of the teacher, students, learning, and the nature of mathematics itself. The following discussion largely abstracted from several posts, including Hudojo (1979 and 1990), Tambunan (1987), and his own writings (Karso, 2008 and 2010) that are short are as follows.

In general, the learning process is basically a dynamic interaction between students, teachers, and materials. In fact, in the classroom reform requires dialogical collaborative interactions between students and students, students and teachers, teachers with teachers, even teachers with principals and supervisors, and teachers with other parties concerned with education in schools.

Learning process expect the students to master the knowledge, skills, and attitudes, as well as to be relevant to their cognitive structure. Learning should provide an opportunity for the student to seek, to ask, to guess, to build, and even argue for dialogue and collaboration.

Philosophical view of constructivism is one of the basic classes used as the basis of classroom reform in the teaching of mathematics in schools. Processes of mathematical learning necessary emphasis on the role of students in shaping concepts as knowledge, while teachers act more as facilitators. Students are encouraged to actively learn, digest, and proceed in understanding the concept. Mathematical concepts as knowledge is not something ready-made and can be transferred from a teacher to his students. Classroom atmosphere should be able to proceed to enable the students who want to know to evolve continuously build knowledge.

In the early years in primary school (SD / MI) and the first-year class at Junior high school (SMP / MTs) and senior high school (SMA / MA / SMK (vocational school)) is a critical year for students who study mathematics. First year as an initial gain experience in learning mathematics. Hence, it need for more serious attention in learning mathematics.

Mathematics related to abstract ideas of the use of language symbols arranged in hierarchical with deductive reasoning which require relatively high mental activity. The existence of learning theory is a strategy for understanding mathematics. Mathematics should learn fairly by students in accordance with their mental development. The student with different abilities must be actively engage in learning mathematics.

Learning activities including learning of mathematics in schools is a very complex process and comprehensively. This causes education program of mathematics teacher not usually warrant graduates become "outstanding teacher" that a lot of people's expectations. Learning process in the classroom involve many interacting variables, so resulting in the number of teaching and learning interaction situations. Coupled with the large number of variables humanity. Therefore there is no method or approach to teaching which is generic for all situations.

Although measurement has been developed and qualified test in validity and reliability, but it is still difficult to design a test for mental or psychological processes. In mathematics learning is still difficult to develop test and measurements which are valid and reliable to measure precisely and accurately about changing attitudes, perception, intuition, motivation, and emotion. The complexity of the learning process, various of teachers, students, showed that teaching, learning, and learning how to teach is a very personal activity and individuals.

Teacher education programs, books, journals, media are helper to be a competent teacher of mathematics, if we have provided ourselves for that profession. In short, teaching is not an exact and definitive activities, but it is a process that is difficult and full of challenges that require continuous improvement.

Understanding the nature of the teacher, the nature of the child, the nature of learning, and the nature of mathematics, is a part of the discussions of classroom reform in the context of LS. All of them helped us as a liability to the teaching profession, as well as the meaning of worship to him.
2. Classroom Reform in the Context of Lesson Study

Discuss about classroom as a place of learning to all what happened is part of a learning community practices in the LS. Discuss about LS as a community of learning can not be separated by the presence of school reform and reform as well as the class which is seen as a follow-up. LS concept in the context of classroom in line with the learning organization concept of corporate practices as presented by Mc Gill and Garvin in Ismawan (2007) which is basically the need for ongoing assessment of the experience of the organization in order to improve organizational success.

LS existence certainly not new for us, especially for the academic community of Indonesia University of Education and society who care about education. LS should be viewed as an alternative reform model of in-service training. LS is an effort to overcome the conventional training models. LS is not a method of teaching and it's not a technique or approach to learning. According Hendayana, et al. (2006 : 10) "Lesson study is a model of professional development of educators through collaborative and sustainable learning assessment based on the principles of collegiality and mutual learning to build a learning community".

From the expression above, it appears that LS can be seen as a strategy to improve the quality of learning through a process of continuous learning assessment. Further disclosed by Masaaki (2012 : 4) "lesson study comprises the Plan - Do – See - Redesign”. This stage is certainly familiar to those who have practiced LS. However there are some records submitted by Masaaki (2012) and Sato (2013) that needs attention in understanding the principles of learning community practices through LS to build professional teachers who are in short as follows.

a. Learning Plan

Given the learning process is a complex and comprehensive activity, then there are a few notes about the learning plan (plan) :

- a. Before conducting learning activity, lesson plan should be drawn up in writing, such as lesson plans, lesson design chapter design, worksheets, etc.. This draft should estimate the thoughts and reactions of students in learning.

- b. Preparation of the lesson plan for teachers is a process performed collaboratively to discuss and explore each subject matter, each ferreting out various media, each revealing the problems of learning, mutually enriching learning ideas.

b. Implementation of Learning

Stages do is implement stage design have made. A teacher model teaching while others observe, collect data on student activity. Some important notes on this second stage include :

- a. This stage is a process for teachers discover and reveal the problems that occur in learning activities.

- b. Observations focus not only on the way teachers teach, but also pay attention to the students, how student learning or student confusion, and how teachers can help students in learning process in order to get more qualified students. Class should guarantee the right of every child to learn to improve the quality of learning. Learning from ongoing learning not to judge teachers.

c. Reflection

The third stage in the LS is a reflection activities (see) is performed directly after the completion of learning activities. Reflection together to form a learning community. Every member of the community involved potential to develop themselves with other members. There are also some notes on the reflection stage to perform the next redesign, including :

- a. After the observer convey the fact of learning in the classroom, then the next step be mutual learning, mutual well honed skills in their field of study and pedagogy. The principle of collegiality, and mutual learning, applied in collaboration not only in the see stage but since the plan and do.

- b. Forum reflection is not a forum to judge any shortcomings and mistakes made by the teacher models. Learning activities are not seen as good or bad, success or failure, not an evaluation or advice, but rather be closely monitored as a fact that said in this forum. In which part of learning already established? In which parts are still having trouble learning? In which learning is still possible? Thanks the teacher models and show expression of respect, it means collegiality developing. Facilitators or mentors are not advisory, but who can learn together.

Through the classroom reform as part of school reform in the context of LS we hope that every teacher turned into a professional teacher. Hence in LS every teacher should learn each other continuously through observation how the others teach repeatedly. Based on the knowledge and insights gained, the teachers are expected to improve learning on an ongoing basis, so that the students could see the look on his face is cheerful.

3. Misconceptions of School Mathematics Learning

This study is more naturalistic inquiry. The data analyzed is empirical data from misconceptions cases occurred in mathematics learning in school. These data revealed through exploration ranging from planning, implementation, reflection, up to redesign for subsequent cycles learning. Field data revealed described and
organized into units according to their characteristic concepts. Then the data analyzed for comment among its relations with reference to the essential concepts underlying. The results are used as are commendation for further development. But in this limited opportunity can only presented a part of the analysis of the findings.

1) Set

On the elementary topic often happens misconceptions in mathematics learning in school. For example on the topics set in the early discussion about the concept of a set and the membership of a set, and concept of subsets. In both concepts ever happened misconceptions in LS, when problems developed slightly to the questions that are not routinely required in the learning of mathematics in schools to develop creative skills. For example, in resolving following simple problems.

(a) Given \( A = \{\{a, b, c\}, \{d\}, \{e\}\} \).  
then 1) \( \{a, b, c\} \subseteq A \) and 2) \( a \in A \)  
Both of these statements are false. Where is the misconception? Remember the concept of subset and the membership concept of a set, so the correct answer adala1) \( \{a, b, c\} \subseteq A \) and 2) \( a \notin A \)  

(b) If \( A = \{a, b, c, d, e\} \),  
then 1) \( \{a, b, c\} \notin A \) and \( \{a, b, c\} \subset A \) and 2) \( a \in A \) but \( a \notin \{a, b, c\} \)  
Such misconceptions can certainly be avoided if there is a discussion forum that discusses the fact of learning in the classroom as we did in the LS.

2) Equation

Solve the following equation.  
\[
\frac{x}{x-3} = \frac{3}{x-3} + 2 \quad \text{.......................... (1)}
\]

First try to do this equation and check your answer, whether it meets the original equation, ie equation 1 or not? Usually to obtain an equation which is independent of the fractional equation, than equation (1) above multiplied by \( x - 3 \) for both sides so we get:

\[
(x - 3) \cdot \frac{x}{x-3} = (x - 3) \cdot \frac{3}{x-3} + (x - 3) \cdot 2
\]

\[
x = 3 + 2x - 6 \quad \text{.......................... (2)}
\]

So, 3 is the answer of the equation (2). However, in equation (1), \( x \) is replaced by 3 then obtained:  
\[
\frac{3}{0} = \frac{3}{0} + 2
\]

which is not defined shape. **Where is the misconception?**

To obtain equation (2), equation (1) has been multiplied by \( x - 3 \), but for \( x = 3 \) it turns out to \( x - 3 \) is zero so the theorem prohibit multiplying by zero. Thus, equation (2) is not equivalent to equation (1) for \( x = 3 \), then equation (1) does not have the resolution or \( HP = \emptyset \). Equations that do not have a settlement called false equation. Falsehood equation (1) will appear anyway if the equation (1) completed, as follows:

\[
\frac{x}{x-3} = \frac{3}{x-3} + 2 \quad \text{.......................... (1)}
\]

\[
\Leftrightarrow \frac{x}{x-3} - \frac{3}{x-3} - 2 = 0
\]

\[
\Leftrightarrow \frac{x - 3 - 2(x - 3)}{x-3} = 0
\]

\[
\Leftrightarrow \frac{-x + 3}{x-3} = 0
\]

\[
\Leftrightarrow \frac{- (x-3)}{x-3} = 0
\]

\[
\Leftrightarrow -1 = 0
\]

Of equation (1) can already be seen that does not apply to \( x = 3 \). However, for \( x \neq 3 \), equation (1) yields the statement -1 = 0. Such misconceptions common in learning discussions facilitated by LS.

3) Inequality
Need to be careful in solving following inequalities because in LS discussion it is the common misconception. For example, determining set completion (HP) follows:

(a) \( \frac{1}{x-2} > 1 \)
(b) \( \frac{1}{x-2} > 0 \)
(c) \( \frac{x}{x-3} \geq \frac{3}{x-3} + 2 \)
(d) \( \frac{0}{x-2} > 0 \)
(e) \( \frac{1}{x} > 0 \)
(f) \( \frac{a}{x} > 0 \)

Please first try all the questions done. For (a) HP = \( \{x / 2 < x < 3\} \), \( x \in \mathbb{R} \), for (b) HP = \( \{x / x > 2\} \), \( x \in \mathbb{R} \), for (c) HP = \( \{\} \), \( x \neq 3 \), for (d) HP = \( \{\} \), \( x \neq 2 \), for (e) HP = \( \{x / x > 0\} \), \( x \neq 0 \), to (f) please try with \( a \in \mathbb{R} \). In HP is not obtained in accordance with the above HP, then misconception happened. Where is the misconception? Remember the prerequisite knowledge of the concept of fractions and the properties of inequality, and algorithm accuracy in the process.

4) Powers and Withdrawal of Roots

Are often happened elementary misconceptions made by the students in the learning of powers and withdrawal of roots, including:
(a). For example: \( 3^2 = 3 \times 2 \) and \( 3^3 = 3 \times 3 \times 3 \), \( 4^2 = 4 \times 4 \).
(b). \( 2^1 \times 2^2 = 2^1 \times 2^2 \) and \( 4^2 \times 4^3 = 4^2 \times 4^3 \).
(c). \( 3^5 \times 3^6 = 3^{11} \) and \( 5^4 \times 5^4 = 5^8 \).
(d). \( 3^3 : 3^2 = 3^1 = -9 \) and \( 2^2 : 2^1 = 2^1 = 2^1 = -8 \).

(e). \( \sqrt{81} = \sqrt{3^4} = \sqrt{3^2} = 3^2 \) and \( \sqrt{64} = \sqrt{2^6} = \sqrt{2^3} = 2^2 \), although the result is true, where is the misconception?

(f). It’s still common misconceptions: \( \sqrt{9} = \pm 3 \), \( \sqrt{25} = \pm 5 \). They reasoned, \( \sqrt{25} = 5 \) because \( 25 = 5^2 \) and \( \sqrt{25} = -5 \), since \( 25 = (-5)^2 \). So, \( \sqrt{25} = \pm 5 \). Where is the misconception? Square root of a positive number is a positive number.

(g). What if \( x^2 = 25 \), then \( x = \pm \sqrt{25} \). What is \( \sqrt{x^2} = \ldots \).

Some misconceptions of essential concepts as above common and need to be discussed in LS because these concepts are prerequisite concepts.

5) Function Composition

There are some records that need to be associated with the concept of a function, for example in the discussion of the function of the composition. Let \( f(x) = \frac{1}{x} \) and \( g(x) = x + 1 \), then \( D_f = \{x / x \in \mathbb{R}, x \neq 0\} = \mathbb{R} \setminus \{0\} \) and \( D_g = \mathbb{R} \). How to do with \( D_{fg} \) and \( D_{gf} \)? In this case we will get something interesting in mathematics learning in school, since there are often misconceptions when discussion at LS. For more details please discussed as a forum for mutual learning, as facilitated in the LS.

In addition to the examples above, of course there are many more concepts associated with misconceptions in mathematics learning in school is often encountered in the assessment of learning through practice LS. In a nutshell, the main characteristic profile of misconceptions were revealed today include, (1) lack of prerequisite knowledge, (2) an error of algorithms processing, (3) lack of understanding of the written language. It will more interesting if the misconception problems of school mathematics learning is devoted to pedagogic problems, or adapted to level of education.

3. CONCLUSIONS

In the era of globalization with a complex full of change and uncertainty, has demanded the existence of qualified human resources and are able to compete side by side with other countries. This condition has put education in a strategic position, so impact on the commencement of education reform and school reform. Reforms in the macro and the meso level has demanded the existence of a classroom reform can be viewed as the micro level by positioning the teacher as a professional worker. Therefore, classroom reform with existence of professional teachers has been a requirement and a necessity.

Class should be viewed as a public space, guaranteeing the right to learn every student, must be a place of establishment of collaborative dialogue for mutual learning. All these conditions were facilitated in the LS as
a model teacher professional development through learning assessment on an ongoing basis in order to improve the quality of learning. It is still associated with LS the empirical data about some misconceptions in mathematics learning in school also revealed. Through the study of analysis and commentary on misconceptions of school mathematics learning can be viewed as a process of mutual learning and making friendship (berasilatuhim). This strategic conditions can also be viewed as an attempt to improve themselves in providing quality education as a form of worship to Him sincerity.

Expected impact on individual teachers, especially mathematics teacher turned into a learning expert who can change the paradigm of learning mathematics which is viewed as a negative object. The students who learn math in school should feel challenged, happy, spirit, enjoy, and be able to smile with a cheerful countenance. Wallahualam bissawab.

REFERENCES
IMPROVING THE SIXTH GRADERS’ COMPREHENSION ON PERCENTS CONCEPT USING PERCENTAGE BAR

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ABSTRACT

Three methods in making comparison i.e. fractions, decimals, and percentages are taught in the fifth and sixth grade. Amongst them, percents seem more flexible than fractions and decimals since it is possible to do arithmetic flexibly. According to sixth-grade mathematics teacher, pupils are too much trained to do arithmetical calculation when working with percentages. However, a number of pupils still show incompetency in mastering the procedure. By conducting self-reflection, the teacher came to a finding that such incompetency was caused by their formal understanding without any contextual and conceptual comprehension. This practical problem then was shared to the researcher while thinking of the teaching improvement could be made. Giving the pupils many contextual problems became a good solution to promote the presence of a model functioning as a bridge between the informal and formal percents. It was percentage bar, the most important model for percentages having a powerful role in making the pupils understand the concept of percents. By using Realistic Mathematics Education as an approach and Classroom Action Research as the research design, this study was aimed to improve the pupils’ comprehension on the percents concept by using a percentage bar model. The data were collected from observation in the form of pupils’ written works and video recording. The data obtained then was analyzed descriptively. From the reflective step, it was found that some pupils made a remarkable shift by using the percentage bar as calculation model, even thought model, instead of context-connected representation.

Keywords : Concept of percents, Percentage bar, Realistic mathematics education, Classroom action research.

INTRODUCTION

In daily live, proportions are broadly used in many aspects of live. Considering the importance of the daily-used proportions, mathematics plays its role by involving proportions as one of the important topics taught in primary schools. In certain sense, proportion is a more general concept, which can be reflected in a more specific way as fractions, decimals, or percents [1]. Substantially, ratio, or in simpler term, division [2] or partitioning [3] serves as the most important concept underlying these three specific proportions. Although, each of them has its own functions, rules, and procedures, percents seem more flexible than fractions and decimals since it is possible to do arithmetic flexibly in percents as a standardized way of describing proportions [1]. Percents figure out relationship based on a one-hundred-part whole [4] and are numbers on a fixed scale that runs from 0 to 100 [1].

Understanding percents inside and outside school are necessary for the learner. There can be no question on the social necessity of having an understanding of percent although it is often misused and misunderstood when applied in the real world [5]. In Indonesian curriculum, percents are taught in the fifth and sixth grade. In the fifth grade, the discussion was concentrated on introducing percents as a related topic with fractions and decimals and how to convert the percents into fractions or vice versa, while in the sixth grade, the pupils are trained to find a part or percent of a number, find a percent when a part and a whole are known, and find a whole when a certain part or percent of that number is known. These types of percent topics are not much more different with what was outlined by Ashlock, Johnson, Wilson and Jones [5] 30 years ago and illustrated by Frans [1] portraying learning-teaching trajectories for and Marja [6] listing the key goals of primary school mathematics in Dutch curriculum. It implicitly indicates that the ranges of percents topics are similar, past and present, abroad and in Indonesia. However, percents in Indonesian primary school mathematics are, still, mostly too syntactic rather than semantic, too instrumental rather than relational, and too computational rather than conceptual. Therefore, the development regarding the stressed mathematical contents that are taught as...
well as the enhancement of the teacher competencies in percents is required.

In general, there are three domains of mathematical knowledge that are particularly and should relevant to teachers’ instructional practices. They are mathematics content, mathematics specific pedagogy, and professional identity [7]. In particular, the mathematics contents of percents are the central strands and concept of percents, the mathematics specific pedagogies are the uses of percents tasks and orchestration of classroom discourse and the professional identity is the teacher’s conceptions of being a teacher. However in practice, many teachers still need guidelines for conducting science, or in particular mathematics, teaching process [8]. They have limited understanding on the mathematics contents causing them couldn’t design the lessons scenario. The only references they use are mathematics school books. This fact causes Indonesia lack of teachers having mastered science approach [8], include mathematics. Most teachers use conventional approach and are very rare to use hands on and practical work activities [8]. Therefore, a guideline seems required by these teachers to set their learning-teaching scenario.

Mathematics teaching is intended to promote the learning of mathematics [9] and understanding mathematics is the main goal of mathematics teaching. Understanding mathematics has been related to mathematical concept and procedure. Understanding mathematics requires proficiency in arithmetical calculation but arithmetical proficiency alone is no guarantee of having mathematical understanding [5]. It was deterioration when the mathematics teachers only focus on the arithmetical proficiency in the meaning of procedure. However, in fact, most of the teachers believed that doing mathematics means finding correct answers, quickly, using the one-correct standard procedure, and that learning mathematics means mastering these procedures [7]. These teachers don’t recognize that what they stressed is not the core of mathematics. It is only about calculations, procedures, and not about the mathematical concept that should be much more emphasized in teaching and learning mathematics. This trend is also happen in Indonesian primary school mathematics practice although many researches on mathematics education have been widely conducted. It implicitly shows that there is discrepancy between what was happening in the classroom, or at least what was reported to be happening in the classroom and the ideas about teaching methods on paper, the teaching theory so to speak [10]. Therefore, involving and collaborating with the practitioners, the teachers, in a study is very important to make them learn how to set and organize a mathematics class while considering the mathematical content.

The improvement of mathematics learning in classrooms is fundamentally related to development in teaching and that teaching develops through a learning process in which teachers and others grow into the practices in which they engage [9]. Further Barbara [9] argued that such practice can involve addressing mathematical tasks in classrooms, developing approaches to mathematics teaching or finding ways of working with teachers to promote teaching development. It was Realistic Mathematics Education (RME) that recently is well known as an alternative approach in mathematics teaching. The RME theory has been strongly influenced by Freudenthal’s view and concept on mathematics [10, 11]. As a theory, RME incorporates views on what mathematics is, how pupils learn mathematics, and how mathematics should be taught [11]. Mathematics must be connected to reality, actually not in truly ‘real’ but include ‘can be imagined’ [10, 12], stay close to children’s experience and be relevant to society [10]. The lessons should guide the pupils towards using opportunities to discover and reinvent mathematics by doing it themselves [11]. This means that in mathematics education, the focus point should not be on mathematics as a closed system but on the activity, on the process of mathematization [10].

It was Treffers who explicitly formulated the idea of two types of mathematization in an educational context [10]. He made a distinction between ‘horizontal’ and ‘vertical’ mathematization [10, 13, 14]. In broad terms, horizontal mathematization means converting contextual problem into a mathematical problem [13] or happens when the mathematical tools which can help pupils to organize and solve a problem set in a real life situation come up [10]. Vertical mathematization can be induced by setting problems which offer solutions on different mathematical level [13]. It also happens when, for instance, the pupils were finding shortcuts and discovering connections between concepts and strategies and then applying these discoveries [10].

To promote mathematization, the teachers have to engage pupils in rich and meaningful tasks as part of a coherent curriculum [7]. Dominic [7] also argued that the pupils’ thinking shared orally and in writing, should be used by teachers to guide the classroom community’s exploration of important mathematical ideas. In RME, the use of model is very important because it, together with the rich and meaningful tasks, can elicit mathematical ideas [15]. There is no mathematics without mathematizing and talk about mathematizing without talking about modeling because the mathematical models are mental maps of relationship that can be used as tools when solving problems [4]. In building an understanding of multiple and equivalent forms and the relationship among fractions, decimals and percents, a visual model is a critical point [16]. Particularly, understanding percents then require appropriate mental model to accommodate the various notion of percent as well as the procedures solving percent problem [5]. Therefore, in this study, the researcher would investigate how percentage bar model can be used to develop the pupils’ comprehension on the concept of percents.
The percentage bar model is the most important model for percentages [1]. Its area makes the pupils easier to figure out parts of a whole. The percentage bar is also appropriate to support a flexible arithmetic, later becomes mental arithmetic [1]. The nice context to develop this model is, for example, the bar shown when copying or downloading a large file on the computer [4]. By using percentage bar as a model to teach percents, it is possible for the pupils shift their understanding from context-connected representation, abstract representation, estimation model, calculation model, and thought model. Therefore, by considering the powerful function and impact of percentage bar in the pupils’ comprehension on the concepts of percents, this study was conducted.

The aforementioned literature reviews were used as a grounded theory for the background of the present study raised through in-depth discussion during reconnaissance process. Through the reconnaissance process based on the teacher’s perspective, it was known that the pupils’ performance on percents problems was too procedural rather than contextual. What mostly written in the mathematics school books also plays a role in the pupils’ and teacher’s perspective on how the learning-teaching process on percents was conducted. The collaborative action research projects set in the mathematics classroom have not been reported in large numbers [17]. However, in recent decades there are many bachelor students took action research in mathematics area as their final assignments. Most of them were concentrated on the teaching strategy reform, not on the mathematical content. Hence, the present study focused on the pupils’ lack of understanding on the concepts of percents that was considered as the area of focus in the present study.

2. RESEARCH METHOD

This study belongs to classroom action research with one cycle involving four phases namely planning, acting/implementing, observing, and reflecting [18, 19]. Before conducting the cycle, the reconnaissance process served as the initial step [19] to identify the classroom problems related to learning process on the percents. During the reconnaissance process, the intensive discussion between the teacher and the researcher was held. It was found that the sixth grade pupils’ comprehension on percents, they learned in the previous grade, was too procedural causing their ignorance on what the percent means, except a calculation involving something per hundred. Therefore, in the planning phase, the proposed alternative instructional theory, using Realistic Mathematics Education approach, was translated to more conceptual teaching scenario and lesson plan. The mathematical content was fully considered. Reintroducing the percents in a more make sense way has to do to provide the pupils more insight on what the percents and its applications are. The discount and double discount were given as an initial situational activity. Modeling the situation has been supported by providing download process context. A little bit improvement has been made by working with the model using mineral volume. This teaching scenario then was outlined in more detail lesson plan before implemented.

This study was a collaborative research between the researcher and the teacher as the practitioner. During the planning phase, both the researcher and the teacher were sitting together to find the core problems related to the teacher’s practice and to design the learning scenario and elaborate the lesson plan. It was the teacher who taught in the acting phase, in total three meetings or 8 times 35 minutes, while the researcher served as an observer. There were 26 sixth-grade pupils involved that were divided into four groups of five and one group of six. During the lesson, the researcher observed and videotaped the teaching and learning process to get all the data needed. The instruments used in this study were problems of discount and double discount, figuring percentages, download process, and mineral water volume, even 6-percent-problem test. Those problems should be discussed in group before being shared in class. The discount and the double discount were used to investigate whether the pupils could find the correct and different meaning of those two discounts. In addition, to check how far they could see percents not in too procedural way, formal calculation. Still to investigate the pupils’ comprehension, the problem of figuring percentages was intended to check pupils’ comprehension on the relationship between percents and fractions. After the pupils recognized that relationship, the context of download process was provided to bridge the emergence of percentage bar model. There were three units used in this context, namely download progress in percent, time in minute and second, and file capacity in Megabytes (MB). By working with this context, it was expected the pupils could see the proportional comparison of percent in the download bar. This idea underlies the additive and multiplicative properties in which the pupils should perform on the last context, mineral water volume. A 6-percent-problem test was given in the end to investigate in what degree the pupils could work with percentage bar model.

All the data, in the form of pupils’ written text and video recording, were then analyzed descriptively. The description was based on the portraits taken from the pupils’ written works and video recording that reflected to what degree the pupils’ understanding on percents and strategies in solving percents problems was influenced by the bar model. This process fell under the fourth phase, reflective. This analysis process was aimed to address the research question and to meet the criteria of success. Not only the way to analyze, but also the process in gaining the data that was to refer to the criteria of success determined in the planning phase. The criteria of success for the present study were the pupils’ ability in using percentage bar as a calculation, estimation, or even thought model. Finally, this analysis process has determined that the criteria of success were...
addressed implying there is no need to have the second cycle.

### 3. RESULT AND ANALYSIS

#### 3.1. Discount and double discount

![Figure 1. Discount and double discount problem](image)

Having learnt percents in the fifth grade, doesn’t guarantee the sixth grade pupils comprehend the fundamental idea of percents. Moreover, their applications that sometimes need more contextual understanding rather than only arithmetical proficiency. The context of discount and double discount were intended to reintroduce the percents in different way as well as to investigate the pupils’ memory on percents. It was retelling activity what pupils ought to do with these discounts problems. They were asked to re-explain the information, not the result, they have got from the pictures. The first picture was about the end year promo. There were 20% off for the minimum purchase of 100000 rupiahs. As conjectured before, some pupils would tend to perform direct computation, by calculating 100000 rupiahs minus 20000 that equal to 20% or by stating 80000 is the amount should be paid since it was same with 80%. Realizing his pupils were hard to capture the contextual meaning, the teacher asked the pupils to re-read the text thoroughly. It was proved as an effective instruction due to some pupils found the important keyword which is “minimum purchase”. They knew that “minimum purchase” has the same meaning as “purchase of at least”. Being involved in the discussion the pupils then totally understood that the discount was only for the buyer purchasing more than and equal to 100000 rupiahs.

The second picture was about double discount (50% plus 20%). Without providing the price, the pupils were asked to explain what it meant. It was a common mistake, as the other pupils knowing such context for the first time, when they firstly assumed that it was not different from 70%. There was a big shift when the pupils finally could grasp that it was about part-whole relationship. What is whole of the part has to compare with, in the pupils’ language after the first discount was performed to the initial price the second discount was performed to another whole, the new price after being reduced by 50%. It was remarkable moment when the discussion focused on whether the double discount, 50% plus 20%, is same with discount 70% off. The pupils said for sure that those discounts were different. Again a big shift was occurred when, with quick responses and without calculation, some pupils could recognize that 50% of certain price makes the price become a half and 20% of a certain price means one-fifth of that price. This overview of the pupils’ struggle in understanding the contextual problems and the teacher’s attempt to facilitate the class discussion gave portrait how the contextual discount problems could work and serve as a rich open-up activity of percents learning-teaching process.

#### 3.2. Figuring percents

![Figure 2. The figuring percents’ problems](image)

Including the discussion about the relationship between fractions and percents, especially for the common percentages namely 50% and 20% as a half and one-fifth have functioned as a valuable entrance for this second activity. Here, the pupils were asked to figure out 25%, 50%, and 40% in a circle and a
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rectangle. This activity would re-lead them to the idea of partitioning a whole into many parts, two, four, five or ten equal size. Further, for introducing pupils the proportional comparison in the sense of a half, twice, less than, and more than.

Figuring 25% and 50%

During the group discussion, four out of five groups almost find no difficulties in partitioning both the circle and rectangle for 25% and 50% (figure 3(a)). They used a strategy of halving and repeated halving to show 50% and 25%. This strategy might be much influenced by their familiarity with the equivalence of 50% and a half, and also the fact that 25% must be a half of 50% (figure 3(a)). On the other sides, another one group seemed do the same shading as the four groups did to show 25% and 50% in circle. But, the interesting thing was when they figured it out in a rectangle shape. Take for example, to show 25%, they divided the rectangle into 5 parts, vertically. This idea was not easy but great for the learners. It was not an easy strategy to divide the rectangle into many equal parts, vertically or horizontally. Moreover, although it was written that they divided the rectangle into five parts, but actually they didn’t (figure 3(b)). There was a remaining part they didn’t count as a part. In mathematical point of view, they couldn’t completely figure out the correct whole partition. Another interesting thing was about how much they did shading. They have lost in track, in figuring 25%. It should be one and one-fourth part instead of two parts and a half shaded. However, their strategy in showing 50% in a rectangle seemed good, although there was still a mistake in performing partitioning of a whole into five equal parts.

![Figure 3. Some pupils’ written works figuring 25% and 50%](image)

It was different with 25% and 50%, 40% gave pupils a little more challenge since mostly they were accustomed with easy fractions or percents. They were challenged to show how much it is 40% out of a whole, 100%. Among five groups, no one group could solve this problem. For that reason the class discussion was conducted to discuss and share many possibilities in figuring 25% and 50% both in circle and rectangle. This class discussion was also intended to emphasize the part-whole relationship and the relation of 25% and 50%, namely, 25% is a half of 50% or vice versa. This understanding bridged, one of the uses, the idea of 40% as twice as 20% and 20% is one-fifth of a whole.

The struggle in figuring 40%

![Figure 4. Some pupils’ written work figuring 40%](image)

During the class discussion, the teacher deliberately raised and stressed the idea of proportional comparison by taking an example 25% and 50%. This property also applies for all cases in percentages.
including 40%. It was expected the pupils could work in group again to show how much be shaded for 40% without any difficulties. Though, their answers were various and in this condition the teacher was bright and skilful enough to find the best method in conducting class discussion so that all the pupils could grasp. The discussion was set and started from the improper representation (figure 4(a)) then continued to the more accurate shading (figure 4(b)). The discussion was rich and interesting since the pupils’ senses of estimating and partitioning have been brought from the rough and without-basic guess to more-or-less estimation and later to more structured idea that is ten-equal partitioning. The three groups (figure 4(a)) seemed to understand the qualitative meaning of 40% that is less than 50% or less than a half. It was good but much better if they also can visualize it with the correct representation. Therefore, the pupils’ end point of this activity was their ability in figuring 40% by using ten-equal partitioning and shading four out of ten parts.

Also, by using five-equal partitioning and shading two out of five parts

3.3. Download context and problems

Understanding the context

This context was about download process that needed 2 minutes and 30 seconds to completely download a file with 50 MB capacity. When the download progress was on 38%, the downloaded capacities were 19 MB from the total 50 MB and the remained time was 93 seconds. By that information, the pupils were asked to understand deeper any information implicitly stated. This context was designed to promote pupils the emergence of the percentage bar model. The download bar was deliberately designed to give pupils opportunities both working informally and formally.

As conjectured, when confronted to this context, the pupils confused what should they do. For them, the information was all given clearly and they were questioning each other what kind of information was still needed. Again, they preferred problems using calculation rather than word or contextual problems. Realizing the conjectured situation happened, the teacher asked the pupils to match the written information and the download bar. As a hint, the teacher probed whether the pupils could capture the idea of “completely downloaded” as a hundred percent. For some pupils, this hint worked but for some others the context was still confusing. However, in writing up what information they got from the context, the pupils still tended to re-write what clearly stated in the text, of course, in their own sentences. Only one out of five groups that tried to expand the information such as mentioning how many MB and minutes during 38% and how many percents, MB, and minutes left when the download process showed 38%. Similar as the previous activity, a rich class discussion was needed to facilitate the confused pupils and also to ensure that all or most the pupils have understood. Having understood the context and being able to relate the written information with the download bar, the pupils were given tasks to solve two problems. One problem was about how many percents when a number as a part of a whole is known and another one asked for certain number as a part when a percentage of the whole is known. These two problems were still related to download context.

Finding a percentage when a part of a whole is known

“When the download running time is 2 minutes, what percentage it is the download process” was the question the pupils should solve. It was not easy for them to really understand what they had to do to solve this problem. By moving from one to another group, the teacher kept asking the pupils to back to the context before starting to solve the problem. How many minutes to complete the download, how many percents it means when the download completed, and how to figure it out are types of questions posed by the teacher. Such questions were expected to help the pupils realized that there were two units, percents and minutes/seconds, proportionally related.
To find the answer, most of the pupils changed the time unit into seconds or minutes (150 seconds or 2.5 minutes) and wrote it in a bar. It was a long journey to bring the pupils to the percentage bar. However, the pupils' shift on the percentage bar use was quite fast. They have already used it as an estimation (figure 6(a)) and calculation model (figure 6(b) and (c)), not a contextual representation anymore. In figure 6(a) the pupils have divided the bar into five equal parts. They knew that each part represents 20%. However, they haven’t come to an understanding that each part also equals to 30 seconds. This was the reason why they couldn’t give the exact representation of how many percents it is 2 minutes out of 150 seconds. Without giving a clear reason, this group only estimated that 2 minutes equal to 70%. The strategy they located 70% was clear since it was on 3.5 out of 5 parts. Another group as in figure 6(b) divided the bar into ten equal parts and shaded eight parts. It was quite clear the strategy they have applied. It seemed they have known that each part equals to 10% and 15 seconds. Therefore they shaded eight parts since it represented 120 seconds (2 minutes) and thus 80%. A remarkable strategy was performed by another group (figure 6(c)). The pupils in this group have divided the bar into five equal parts. The reason behind this partitioning was the idea of 30-second-based partitioning that would lead them to the position of 2 minutes or 120 seconds and when the time showed 120 seconds the percents showed 80%.

Finding a part when a certain percent of a whole is known

When the download process showed 38%, there was 19 MB of 50 MB have been downloaded. Then “how many MB would be when the download reached 22%” was the second question the pupils had to solve. The figures above showed the pupils’ two different strategies. The first strategy indicated that the pupils could see the proportional relation of two units, percents and MB. “If 100% is 50 MB, it means a half” was the first sentences they wrote. Then they continued writing “if 22%, it means a half that is 11 MB, 22:2:11”. Further, this strategy was confronted to another strategy as shown in figure 7(b). To show 22%, the pupils in this group firstly divided the bar into ten equal parts. The fact that 22% is between 20% and 30%, gave meaning that the downloaded capacity is should be between 10MB and 15MB. Finally their high understanding on partitions has leaded them to re-divide that part into five equally. As a consequence, the first partition would show 22% and 11MB. The combinations of both strategies have made the pupils saw deeper how the percentage bar works and the proportional relation between the units. However, the pupils need to see how the additive and multiplicative properties can be applied.
3.4. Mineral water context

The use of percentage bar as a calculation model was the aim of this context. In this context the pupils were confronted to a situation later on bringing them to 1% as a benchmark. The situation was easy enough to be understood. It was about sixth-grade sport time and the pupils in this class were distributed into 6 sport groups. One 380 ml bottle of mineral water was provided for each group. The pupils could fill and refill the empty bottle from the 19 liters gallon of mineral water. There were three questions, one about how many percents it is one bottle volume compared with one gallon volume and followed by the same question for three bottles. In the last problem, the pupils were challenged to prove that 11% of the gallon volume is 2090 ml.

In working with the first problem, some groups seemed being confused since they did ten partitions or started with 50%. However, it was easy for all pupils to change the volume unit become 19000 milliliters. Although the group that did ten partitioning has predicted one bottle volume must be less than 10%, but they couldn’t find the exact percentages for it. Through an explorative activity and rich discussion, the pupils invented that 380 ml equals to 2% because 1% of 19000 ml is 190 ml. This strategy influenced the way they solve the second problem. There were two quite similar strategies emerged at this time. Both strategies involved 2% as benchmark. Some groups came to an understanding if the volume of one bottle equals to 2% then three bottles equal to 6% and not important to know what the volume of those three bottles were. The other groups tried to find the volume of three bottles first, 1140 ml, and using it as reference volume. Started from 380 ml they walked through the bar with 380-based counting. When they reached 1140 ml, the corresponding percents showed 6%. Thus they concluded that three 380-milliliter bottles equal to 6% of 19 liters gallon.

Proving 11% of 19 liters is 2090 milliliters

In the process of proving 11% of 19 liters is 2090 milliliters, there was a remarkable shift in the pupils’ understanding especially in the pupils from two groups. The way they made use percentage bar can be categorized as in between of calculation and thought model. However, their ideas have been shared in a class discussion in order that all the pupils could grasp the process of proving. The first proof was using multiplicative properties (figure 9(a)). This group used 1% as benchmark. From the bar, it was clear that 11% is eleven times 1%, and this was what the pupils in this group saw and the reason behind their strategy. Then, to find how many milliliters it is 11% of 19000 ml can be calculated by 11 times 190 that equal to 2090 ml. Different with the first group, the second group was using additive properties in which 11% can be
gained by adding 1% to 10%. For that reason, they divided the bar into ten partitions, from this they got 1900 ml as the volume represented by 10%. To make 11%, they need 1% more. The way they got 1% was interesting since they didn’t recognize it directly as 190 ml but as a half of 2%. If 2% represented 380 ml, then 1% must be 190 ml. This idea came from the first task they have solved; 380 ml is 2% of 19000 ml. Without finding any difficulties, what they had to do was finding the sum of 1900 ml and 190 ml, which is 2090 ml.

3.5. Ending point

To make sure in what degree the pupils have made use of the percentage bar, they were given a task consisting of 6 percent problems. The problems were various in types involving three routine problems, one word problem, one situational problem, and one open problem. They were given 70 minutes to solve those six problems individually. Only four problems will be discussed to see how the percentage bar influenced the pupils’ strategy. Peculiarly, the situational problem was intended to investigate how the pupils could explain the correct meaning of the percents in that situation. The first problem was “what is your strategy to find the solution of 13% of 1300 kg” while the second problem was “how many is 39\% \times 1300”. The third problem was word problem and here was the story “there are 35 pupils in the class, if 14 of them are male then how many percents is it the female one?”. The fourth problem was a situational problem and in this problem the pupils were asked to explain the information they got from the picture.

**Some answers for 13% of 1300 kg and 39\% \times 1300**

Both problems above actually have multiplicative relation that was expected addressed by the pupils. However, there was no pupil who came up with that idea. For the brief overview, some answers will be analyzed.

![Figure 10. Milk problem in the test](image)

4. Tulis dan Jelaskan apa yang kalian pahami dari gambar di bawah ini!

(a)  

(b)  

(c)  

Figure 11. Some pupils’ written work on 13% of 1300 kg
The strategies shown in figure 11 (a) and figure (b) only showed the position of 13% not the result of 13% of 1300. The strategies they did partitioning were different. One used five partitions and then the first partition was divided into two to show 10%. Then, they were very clever to position the 13% as you can see in the figure 11 (a). A quite similar strategy was performed by another pupil namely using ten partitions. To show 13%, they used repeated ten-based partitioning that was clear enough (figure 11(b)). The two strategies mentioned above indicated that the pupils have been using percentage bar as a calculation model.

A very outstanding strategy was performed by one of the pupils (figure 11(c)). This pupil did ten-based partition to identify 10% that he also stated as 130 kg. Realizing the 13% was between 10% and 20%, he has tried to zoom that partition in and divide it into ten. What an amazing strategy was the way he counted on based on 13 in which represented 1%. This counting has brought him to a conclusion that 13% of 1300 kg was 169 kg. It can be concluded that this pupil has been using percentage bar as a calculation model, of course, in high level.

The first idea coming up when trying to understand the pupils’ strategies above was arithmetical calculation. The deeper it was examined, the clearer the strategies were. In fact, only two of them used arithmetical procedure. The estimated reason behind this strategy was the problem form. They have worked with such problems in the previous grade. Another one strategy (figure 12(c)) was percentage-bar-based calculation. To make 39%, he subtracted 1% to 40%. It was beautiful strategy since there was no specific discussion on subtractive properties. Later on, this idea has underlain his calculation although there were little typos. “1300:10 = 130” means 130 was 10% since the bar has been divided by ten. “130=520” indicated 40% equals to 520. Because of 1% of 1300 is 13 then 39% can be found by subtracting 13 to 520 that implied subtracting 1% to 40%. At last, he got 507 as the answer of the given problem. This pupil has shown how the percentage bar can be used as thought model. The bar and the partition was enough on imagination, but the written calculation has clearly expressed the strategy.

If 14 of 35 pupils are male then how many percents it is the female one?

There was something interesting when examining the pupils’ written work. It also applied in the strategies pupils used to find the solution of this problem. In the first strategy (figure 13 (a)), the pupil has
performed the correct partitioning. However, she made a mistake in labeling the partition. It must be related to the percentages instead of the pupils. The fateful mistake she did was when she tried to show 14 pupils using partitions that must be used for 14%. Another strategy was quite well and clear (figure 13 (b)). What needs to be explained was his first step underlying the partition. He first determined the numbers of female pupils in that class that were 21 female pupils. By using 21 as reference point he found it as 60%. This way of thinking then can be categorized in the high level using of percentage bar as calculation model.

The pupils’ struggle in explaining the situational problem

![Figure 14. Some pupils’ written work on situational problem on test](image)

The correct meaning of this situational problem was the volume of the milk in the package was 200 ml. It came from 160 ml plus 40 ml. 40 ml was the 25% of 160 ml which is the volume of the usual package. There was no need to use percentage bar here. However, not all pupils could catch this idea. The figure above was examples of the pupils’ answers. The three pupils have answered correctly with an appropriate reasoning (figure 14). These answers indicated that the pupils were getting used to situational problem that they seldom met before.

4. CONCLUSIONS

A learning trajectory using some contextual problems should be designed to propose the percentage bar model building in explorative activities. From the analysis process, it can be seen how struggle the pupils were in discussing and solving the problems using percentage bar. Their shifts in understanding the percents concept have been influenced by the way they were using the percentage bar, or vice versa. A big shift happened when the pupils were able to use the percentage bar as a calculation model, even thought model, although their level of understanding were different. The teachers’ belief on the pupils’ mathematical understanding and the teachers’ perspective on the mathematical content have also determined the pupils’ level of understanding.

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DIDACTICAL DESIGN OF KITE AREA DIMENSIONAL CONCEPT ON MATHEMATICS LEARNING IN THE FIFTH GRADE OF ELEMENTARY SCHOOL

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ABSTRACT

The background of this research was learning obstacle of students in understanding the kite area dimensional concept. It was identified by analysis result of introduction learning that showed that students did not understand yet about kite area dimensional concept completely. They also did not understand yet about its formula and element, for example diagonal concept, relation between the shape, and the practice in story question form. The purpose of this research is to arrange, and develop a didactical design to minimize or overcome the students’ learning obstacle. This research was done at three elementary schools in Tasikmalaya. They were Galunggung, Dadaha 1, and Citapen elementary school. This research used Didactical Design Research (DDR). The steps of the research were: (1) didactical situation analysis before learning process, (2) metapedadidactical analysis, (3) retrospective analysis. The data collection technique used was triangulation. The main research instrument was the researcher completed by introduction learning question and student work sheet. The data had been got was analyzed qualitatively. The research of this research was arranged a didactical design could be used as an alternative in mathematics learning in elementary school. The implementation result showed that by using this didactical design could minimize students’ learning obstacle in understanding the kite area dimensional.

Keywords: Didactical design, Learning obstacle, Kite, Mathematics learning

1. INTRODUCTION

The kite area dimensional is one of topics on geometry. The reality at elementary school showed that many students do not master about kite area dimensional concept. As suggested by Chairani (2012: 173) in her research report, that:

…(1) researched students experience difficulty in mentioning the width formula of kite and rhombus correctly, because they memorize it, possibly they will forget faster when the learning has been done. (2) they do not understand about the width formula of kite because they never made the formula by them selves. (3) several students are no used to with question “how to find the formula?” because they used to get the information from their teacher or book without their own finding process…

The report is supported by introduction learning result in Dadaha 1, Galunggung, and Citapen elementary school. The introduction learning result showed that essentially, the student did not understand yet about kite area dimensional concept completely. Here are some responses about introduction learning question:

Figure 1 the first student’s response about introduction learning question
Figure 2 the second student response about introduction learning question

Figure 1 and 2 showed that student faced difficulty to count kite area dimensional because of additional information on kite picture. It showed that there was an obstacle that is well known by epistemologist obstacle. Duroux (Suryadi, 2010: 14) expressed that ‘essentially, epistemologist obstacle is obstacle which appears because of limited knowledge of someone on certain context’. This epistemologist obstacle is one of student’s learning obstacle factors.

Suryadi (2010: 8) revealed that “…the relation between didactic and pedagogic could not be looked partially but it must be understood completely, because in fact, two those relations can be happened simultaneously”. To explain the relation, Suryadi illustrated them by modifying Kansanen didactical triangle, as follows:

The most principle of teacher’s role in this didactical triangle context is creating a didactical situation so that happened a learning process in student’s self (learning situation) (Suryadi 2010: 8). Therefore the teacher have to have an metapedadidactical ability, as Suryadi expressed (2010: 12) that:

The needed teacher’s ability will be called as metapedadidactic then, which can be meant as teacher ability for: (1) looking the modified didactical triangle components that is anticipation of didactic-pedagogic (ADP), didactical relation (DR), and pedagogical relation (PR) as a whole unity, (2) developing an action until it will be created a situation of didactic and pedagogic appropriate with student’s need, (3) identifying and analyzing student’s response as a result of didactical and pedagogical action done, (4) doing the next didactical and pedagogical action based on the result of student’s response to get learning target.

According to Suryadi (2012: 12), “metapedadidactic covers three integrated components those are unity, flexibility and coherence.” The unity related to ability looks sides of modified didactical triangle as a whole related unity, flexibility related to ability does improvisation on the true learning, coherence related to ability
restrains a dynamic and developing didactical and pedagogical situation.

2. RESEARCH METHOD

This research used the didactical design research (DDR), and the data was analyzed by qualitative method. The focus of this research is arranging and developing a didactical design for overcoming student’s learning obstacle. The research design used is didactical design research from Suryadi (2010: 15) expressed that:

“Didactical design research consists of 3 steps: (1) didactical situation analysis before learning process that forms hypothesis didactical design including ADP, (2) metapedadidactical analysis, and (3) retrospective analysis, that is analysis that relates the result of hypothesis didactical situation analysis with the result of metapedadidactical analysis. From the three steps, will be got the empirical didactical design that is possible to be enhanced by the three steps of didactical design research.”

The research steps are:

Step I: didactical situation analysis before learning process
a. Determining subject
b. Discussing with guiding lecturer and elementary teacher
c. Analyzing literature
d. Arranging the first instrument
e. Introduction learning
f. Doing interview
g. Analyzing the result of introduction learning
h. Concluding kinds of learning obstacle
i. Making HLT
j. Arranging the first didactical design

Step II: metapedadidactical analysis
a. Implementation of didactical design has been done
b. Interview
c. Analyzing implementation result

Step III: retrospective analysis
a. Relating the first response prediction with the true student’s response
b. Developing revision didactical design

The instrument of this research are researcher as the main instrument and introduction learning question and student work sheet as the additional instrument. The data collection done by test instrument and triangulation technique, that is combining participatory observation, semi-structured, and documentation learning. Implementation of didactical design done in two cycles, each cycles is three times meeting.

3. RESULT AND ANALYSIS

The result and discussing research are learning obstacle about kite area dimensional concept, didactical design and revision design.

The Learning obstacle about kite area dimensional concept
a. Learning obstacle of type 1 student, student’s difficulty about understanding of kite’s wide formula

Student’s response showed learning difficulty as follow:

![Figure 4 student’s response in doing question no. 3](image-url)
Figure 3 and 4 showed that student used the wrong way to determine kite area dimensional. It showed that student did not master yet about kite area dimensional concept related kite area dimensional formula and use.

b. Learning obstacle of type 2 student, student’s difficulty about understanding of diagonal concept
Here are student’s response showed about the learning difficulty

Figure 5 student’s response in doing question no. 4

Figure 6 Example LO 2
Figure 6 showed that the students knew the formula but they did not understand about diagonal concept.

c. Learning obstacle of type 3 student, student’s difficulty about relation between flat form

Student’s response showed learning difficulty as follow:

Figure 7 student’s response on LO 3

Figure 7 showed that the student did not understand yet about relation between kite and rectangle.

d. Learning obstacle of type 4 student, student’s difficulty about understanding concept on story question context

Here is student’s response showed the learning difficulty:

Figure 8 student’s response on LO 4

Based on the result of response analysis can be looked that student did not master yet about kite area dimensional concept completely. It caused students’ learning difficulty when they faced different question. The result of introduction learning proved that student experienced learning obstacle. The learning obstacle must be overcome by the right way in order to it does not disturb the student’s thinking developing process. Therefore, it is made a didactical design to overcome the learning obstacle then.

3.1 Didactical Design of kite area dimensional Concept
This didactical design is arranged by noticing the learning obstacle, SK, KD, and learning indicator. Then it is arranged a hypothetical learning trajectory (HLT) consists of learning goal, learning activity and anticipation of didactical pedagogic (ADP). The learning goals have been developed are:

a. By manipulative activity of concrete thing, the students can find the kite area dimensional formula concept through rectangle’s width approach correctly.

b. By manipulative activity of concrete thing, student can find the kite area dimensional concept through triangle’s width approach correctly.

c. By manipulative activity of concrete thing, student can find the kite area dimensional concept through parallelogram approach appropriately.

d. By picture observation, student can show diagonal in a kite’s picture.

e. By picture observation, student can count the kite area dimensional if the diagonal known.

f. By finding of pattern and question practice, student can determine the other diagonal length if the width and one of diagonals known.

g. By intensive reading and modeling, student can overcome the story question about daily life.

h. By finding pattern and modeling, student can solve the story question about finding the length of a diagonal appropriately.

The learning goal designed for 3 times meeting by each times is 2x35 minutes.

Didactical design (prospective analysis)

The first meeting started by a contextual problem, then researcher showed kite toys and did debriefing with the students. The main activity was finding formula of kite area dimensional through the width of rectangle, triangle, and parallelogram approach by using folding paper, scissors, glue, ruler and student work sheet.

The second meeting started by a problem about finding diagonal’s length, then student was assigned to name a kite’s picture and count the width of kite’s picture. The last question was challenge that was developing one of introduction learning question.

The third meeting started by a contextual problem, a child who was making a kite. Then the student given 5 story questions in a different context completed by guiding question.

3.2 Implementation of the first didactical design

Developing of student’s understanding about kite area dimensional concept (the first meeting)

In the first meeting, student studied to find the kite area dimensional formula by width approach of triangle, rectangle, and parallelogram. Generally, the student’s response was appropriate with prediction before. The student was difficult to cut paper, combine and find the formula. But it can be overcome well. Anticipation done was giving guiding comprehensively or giving guiding to certain student.

Developing of student’s understanding on width concept of kite and diagonal in picture context (second meeting)

In the second meeting, the learning in line with the expectation. The student’s response in line with prediction before because the student can master the material in the first meeting well. But there is student who is still difficult. It is fair because of student’s different perception. student’s response largely in line with prediction. It is out of prediction about the time, because the student’s speed is different in doing question.

Developing of kite area dimensional understanding on story questions context

In the third meeting, student’s response largely in line with prediction, student can understand question meaning given and do it. When student did question no 4, they faced difficulty, because researcher made too difficult question context, so the question given did not appropriate with thinking level of fifth grade elementary student about kite area dimensional concept.

3.3 Reflection (retrospective analysis)

Some revisions done related context:

<table>
<thead>
<tr>
<th>No</th>
<th>Aspect</th>
<th>Problem</th>
<th>Situation</th>
<th>Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Linguistic</td>
<td>Difficult to understand</td>
<td>Language adaption</td>
<td>Student work sheet 1,3</td>
</tr>
<tr>
<td>2.</td>
<td>Question</td>
<td>Too difficult, too many questions</td>
<td>Reducing question and activity</td>
<td>Student work sheet 3</td>
</tr>
<tr>
<td>3.</td>
<td>Time allocation</td>
<td>Time is not appropriate</td>
<td>Adapted with the fact in the field</td>
<td>Student work sheet 1,2,3</td>
</tr>
<tr>
<td>4.</td>
<td>Presentation form</td>
<td>Less appropriate with the goal</td>
<td>Changing first page format</td>
<td>Student work sheet 1,2,3</td>
</tr>
<tr>
<td>5.</td>
<td>Material adding</td>
<td>Less explanation</td>
<td>Adding diagonal concept</td>
<td>Student work sheet 1</td>
</tr>
</tbody>
</table>
4. CONCLUSIONS

Didactical design has been arranged and developed based on the learning obstacle by noticing at SK, KD, indicator and learning theories can overcome student’s learning obstacle. This didactical design can be a learning alternative at elementary school, but for the application, it must be adapted with the student’s characteristic.

REFERENCES

This research is based on a learning problem in junior high school material cubes cuboids that have not obtained the results of student learning to the fullest, because they studied up three-dimensional space but is presented in a two-dimensional static images, so the lack of student learning outcomes. Researchers developed the mathematical learning in extensive material cube and cuboids-based Anchored Instruction in order to solve the problem. This study uses the theory of the development of four-D by Thiagarajan which consists of four main phases, namely: (1) Define, (2) Design, (3) Develop, and (4) Disseminate. After researchers define execute phase (curriculum analysis, analysis of learners, content analysis, task analysis, and specification of learning objectives), and design (Syllabus, Lesson Plan, VCD learning, worksheets and test instrument), researchers conducted a phase development that begins with expert validation. Validation of experts involves six people who are experts in their fields, both in education and in the multimedia field. Expert validation results show that the learning of mathematics that are well developed and can be continued for a limited trial on condition that there are some minor revisions in accordance with the input validator.

Keywords : Expert Judgment Develop mathematic education

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1. INTRODUCTION

In the era of globalization, modern convenience, technology and full competition, requires every person to master different areas of life, so that Human Resources ( HR ) it can be increased. With qualified human resources and highly competitive, the country will be able to compete with other countries in various fields as well. One area of increasing support Indonesian human resources is education. Educational success is inseparable from the role of teachers and schools, both public and private schools. One of the lessons in the school who had a role in the progress of technology and human resources is math. Mathematics is a discipline that has distinctive properties compared with other disciplines, namely with regard to the ideas or abstract concepts are arranged hierarchically and deductive reasoning. Teaching and learning of mathematics should not be equated with other sciences. Teaching and learning activities should be regulated with respect to the ability of students and understand the essence of mathematics because students are learning different math abilities.

Math is one of the compulsory subjects that must be mastered by junior high school students, with a mathematical proof that the subjects included in accounted for junior high school students’ graduation requirements. Learning mathematics will succeed if good learning process, one that involves students actively. Zaini (2008: 14) reveals that when students learn by active, meaning those who dominate the learning activities. By this they actively use the brain, either to find the main idea of the subject matter, solving problems, or apply it in everyday life. Anni (2004: 11) states that learning outcomes are influenced by many factors, both factors of the ( internal) and external factors ( external ). Internal factors such as physiological and psychological factors
According to Nurhadi (2004: 3), learning will be more meaningful if the kids 'experience' itself what he learned, not just know it. Mastery goal oriented learning materials proved successful in the competition 'remember' the short-term, but failed to provide children solve problems in life long term. Furthermore, according to Bruner in Trianto (2009: 91), starts to look for solutions and the accompanying knowledge, generating knowledge that is really meaningful. A logical consequence, because the effort to find a solution to the problem independently will give a concrete experience, the experience will be used also solving a similar problem-solving, because the experience it gives a special meaning for students.

Learning model that includes strategies, approaches, techniques and instructional methods to optimize the learning activities of students when learning occurs with meaningful, fun and can encourage students to build and develop their knowledge. In this case the teacher must be able to be a designer to teach the theory and implementation theory to students in learning to achieve learning objectives (Joyce, 1994:34).

According to Dwyer (in Waluya, 2006: 2), packaging learning material in the form of audio-visual impressions to capture a 94% influx channel messages or information into the human psyche that is through the eyes and ears. Audio-visual media can make people generally remember 50% of what they see and hear, although only once aired. Or, in general, people will remember 85% of what they see from the show, after 3 hours later and 65% after 3 days later. This is consistent with the proverbial china: I hear and I forget, I see and I remember, I do and I understand (Zaini et al, 2008: 15).

Expository learning and instructional media in the form of cubes and cuboids up models that have been used by junior high school teachers have not been able to bridge between the broad concept of cubes and cuboids are still abstract to students who still think concrete. This is due to the media being used only media silent and not moving (static visual), and often times the cube and the cuboid is presented in the two-dimensional visually, but that are discussed are three-dimensional.

In this connection it is necessary-oriented learning model students, can engage students actively, and students can use the knowledge he already has to build new knowledge in order to solve a problem, so the learning process to be meaningful, contextual and not boring. Media is also necessary to involve more than one sense on students, the media can move/animated, and can guide students in constructing knowledge, so as to attract students and make the learning environment more enjoyable.

Anchored Instruction (AI) is a technology-based learning model developed by the Cognition and Technology Group at Vanderbilt University led by John Bransford. More students helped in solving math problems in class with the help of AI (Bottge, 2004: 1). Concepts in science becomes more evident when students can explore their capabilities in a variety of settings. AI has been able to help students understand the usefulness of the concept by creating a video scenarios involving contextual objects (Rabinowitz, 1993: 39). Lee (2002) also states that students who have been given a lesson by using AI learning media had significantly different effects in problem solving. In AI there are videos that include complex issues with contextual stories that help achieve attainment of the learning activity concepts (Bransford, 1990). According to Hasselbring in Heo (2007: 23), in constructing and observing the situation, instructional video is part of learning support students who are used to form a variety of origin, complex, and contextual learning experience.

So far, the results are often only provide advice to the public to use the models / strategies / specific approach (according to the study), without generating devices/products that can be used directly. Therefore, it is important that research produces products/devices that use multimedia learning, so that the research results not only provide advice to the public but also produces learning tools that have been developed, so that student learning outcomes can be maximized and in accordance with the desired learning objectives without having to developing a learning device again.

Starting from the above background, it is the learning of mathematics with AI models need to be developed device, so it can be used as learning tools in an effort to condition the particular geometry of broad learning cubes and cuboids the eighth grade students to be meaningful, contextual, not boring, increase student activity and raise students' motivation, so that student learning outcomes can be improved and in accordance with the desired learning objectives. This article discusses only at the stage of development that is only part of How math learning software development with extensive AI materials and cuboid cube eighth graders valid according to experts.

Expert Judgement, conducted by the respondent experts or experts in the field related to the product being developed. Expert validation is done to review the initial product, in order to obtain input for the initial repair. As a learning device is a good criterion is that if at least 75% of the validator it gives the general assessment is good or very good for such a learning device. If the validator assessment against each indicator there are values that are less or not good to be used as consideration to revise the developed learning tools.

Learning tools is a set of media devices or means used by teachers and learners in the learning process in the classroom, as supporting the learning process to run smoothly and effectively. Device developed in the form of Syllabus, Lesson Plan, VCD learning, students' worksheet, and Test Results Learning.
A. Four-D Model

Development of learning tools is an attempt to generate new learning tools that were developed from the existing learning to make learning more effective. Thiagarajan develop a model of learning known as the Four-D Model or Model 4-D. This model consists of four stages, namely: the definition, planning, development and disseminate.

![Diagram of Four-D Model]

B. Anchored Instruction

Anchored Instruction (AI) has been developed by the Cognition and Technology Group at Vanderbilt University led by John Bransford. Anchored Instruction arise from issues surrounding education in 1929, when it saw the students knowledge is often " inert " and not be able to respond to many changes in different situations or different problems. In 1929 noted that students are often asked to study the individual concepts and procedures that they remember when explicitly asked to do a multiple choice test. However, when asked to solve problems in which concepts and procedures used, most students often fail to do. knowledge they remain silent (inert). (Rabinowitz, 199 : 36). AI has been developed and involves a special design, based on a video - based format called “anchor ” or ” case “ that provide the basis for exploration and collaboration in solving problems. The story in the video depict real life that can be explored on many levels. The video is designed to enable teachers and students to connect mathematics with other subjects of knowledge by exploring the environment from a different perspective. (Rabinowitz, 1993 : 43). AI is a learning model in which teachers try to help students become active within the conditioned learning in an exciting instruction and solving real problems, where students later saw the video "anchor" and solve the problem that there are performance video story (Barab, 2001 : 2).
1) Seven Principles and Advantages of using AI learning by Woodbury as follows:
   a. Video - Based Format
      Video-based format provides several advantages in the study:
      (1) foster students' motivation
      (2) it is easier to understand
      (3) support the understanding of complex
      (4) is very useful for children who do not like to read
   b. Narrative with Realistic Problems
      Narrative with realistic problems in learning some advantages, namely:
      (1) easy to remember
      (2) more attractive
      (3) assist students in solving mathematical problems
   c. Generative format (ie, at the end of the story the students have to find the problems to be solved)
      Generative format gives several advantages in learning, namely:
      (1) leads students to discover and define the problems that must be solved.
      (2) improve student reasoning
   d. Embedded Data Design (ie, all the data needed to solve the problem is in the video)
      Embedded data is design to give some advantage in learning, namely:
      (1) make it easier to take decisions
      (2) motivate students to find problems and solutions
   e. Problem Complexity
      Problem complexity in learning some advantages, namely:
      (1) the tendency of students to overcome despair when faced with complex problems
      (2) to introduce students to the level of complexity of the characteristics of a real problem
      (3) assist students in solving the problem that complexity
      (4) increase the confidence of students
   f. Adventure parts Mutually Related
      The parts are interrelated adventures give some advantage in learning, namely:
      (1) help clarify what can and what cannot be
      (2) describe the reasoning by analogy
   g. Links Across the Curriculum
      Links across the curriculum gives some advantages in learning, namely:
      (1) helps to think mathematically for other subjects
      (2) encourage the integration of knowledge
      (Rabinowitz, 1993: 46)

2) AI steps according to Oliver (1999) is as follows:
   a. Using multimedia or other interactive technologies are used to tell a story (issue)
   b. Divide the class into small groups (3-4 students)
   c. Teachers encourage students to gather keywords, facts, and data issues presented in the instructional video
   d. Students are pushed back to "play-back" or "re-explore" to retrieve the data needed to solve the problem
   e. Students each float solution and present the results of the development of the solution in front of the class
   f. Pros and Cons of each student discussed ideas expressed (discussed) together
   g. analogize to the issue of new data to assist students in understanding the deeper issues related to the topic, usually used the word "what if".
   h. Expanding problem that requires skill and strategy are the same as those used in solving the problem in the story in order to improve students' ability to solve problems in a variety of issues that vary.

2. RESEARCH METHOD
   This study, including the type of research that emphasizes the development of the study of mathematics by developing AI. Device developed in the form of video-based anchors (VCD learning), Syllabus, Lesson Plan, Student Worksheet, and Test Results Learning. As for the validation of research instruments in the form of sheet syllabi, lesson plans sheet validation, validation sheet worksheets, teacher observation sheet management, achievement test validation sheets, activity sheets students in learning and student motivation questionnaire.
   A. Development Procedures
      Learning software development model with modifications from model 4 - D (Four D Model) proposed by Thiagarajan, Semmel and Semmel, ie starting from the definition (define), stage design (design), until the development stage (development). The learning phase of software development can be described as follows.
1) Definition Phase
This stage aims to determine and define the conditions needed in the learning by analyzing the purpose and limitations of the material. Activities undertaken at this stage is the front-end analysis (analysis of the curriculum), learner analysis, material analysis, task analysis, and specification of learning objectives.

2) Design Phase
This stage aims to design a learning device, in order to obtain the prototype learning device. Examples of the resulting design is a learning device (1) Syllabus, (2) Lesson Plan, and (3) learning VCD. Activity at this stage is the preparation of the test criteria, the selection of media, the selection and design format that begins after the initial set specific learning objectives.

3) Development Phase
The purpose of this stage is to produce a draft of the revised learning device based on the input of experts and the data obtained from the test results after validation expert.

B. Research Instruments
The research instrument is a tool to collect data on the learning of mathematics with AI on extensive material cubes and cuboids. Instrument in this study consisted of observations of student activity sheets, student motivation questionnaire. Before learning achievement test instruments given to students, first test instrument (achievement tests) to see the validity and realibitas the instrument and to analyze different power and level of difficulty of the questions.

C. Data Collection Method
Development of data collection methods were conducted in this study is the result of data validation specialists, documentation of data, learning outcomes data, activity data and student motivation.

3. RESULT AND ANALYSIS
From the research that has been carried out, both in the preparation, device fabrication, validation and implementation of the research instruments, the following results are obtained.

A. Define Phase
1) Front–end Analysis
With the method of literature obtained initial results of the analysis are the following tip. In UU No.20 Year 2003 on National Education System in Chapter II, Section 3. Schools are not only required to produce graduates who are capable of higher cognitive as well as to keep abreast of information and technology, but also creative, independent and responsible. Learning activities and high motivation is needed to make it happen, students can solve problems in AI are not only math but also other problems in social life (Bransford, 1997).

2) Student and Environmental Analysis
With the method of documentation and analysis of the results obtained literature students in junior high school Setiabudhi Semarang. Semarang SMP students Setiabudhi potential, ie the average NEM elementary school that is in each year ranged from 6.0 to 8.5. Of academic ability is quite good, but for math result is still not as expected. This study conducted during a teacher–centered to student behavior in the classroom. Often teachers only teach theory only, not application in daily life, so that motivation and learning activities of students in the low.

3) Concept Analysis
There are many materials in class VIII KTSP mathematical concept can be developed through previous concepts have been accepted by the students, one that is large cube and cuboid material. Extensive material and cuboid cube in it there are things or material that is a prerequisite that must be understood about the vast square and rectangular and cube nets and cuboids. Once you can determine the surface area of cubes and cuboids, then later asked to break the day-to-day problems of the solution using a broad concept behind the cube and cuboid.

4) Task Analysis
Based on the analysis of the curriculum for the vast task cubes and cuboids the kinds of tasks that will be acquired by the students, including: students are able to identify the properties of cubes and cuboids wide, define and draw a cube nets and cuboid, determine the extent of the web of cubes and cuboids, determine the extent of cubes and cuboids, completing the contextual problems (word problems) associated with the concept of surface area of cubes and cuboids.

B. Design Phase
Having done the analysis on the level of appropriation, then compiled form Syllabus learning tool, RPP, VCD learning, Student Worksheet, the question of Test Results Study, the result is called draft 1.

1) Arrangement Syllabus, Student Worksheet, and the question of Test Results Study
Basis of preparation of the syllabus in this study is Permendiknas No.41 Year 2007 for junior high school and developed in accordance with AI learning models, and for the preparation of the RP developed in accordance with the steps of learning AI and syllabus has been designed. For the preparation of the test is the basic material analysis, task analysis and formulation of objectives. The test in question is a test of learning outcomes are arranged in the form of multiple choice and make a description that was preceded by a reference grating and scoring items.

2) Media Arrangement

Based on the results of the analysis and the range of end - selected early learning VCD media. In learning VCD are learning and learning videos. In learning CD containing about SK, KD. Indicators and learning purposes, in learning CD are also available material on the cube nets and cuboids, the surface area of the material and cuboid cube, examples and practice questions. Learning CD can also be taken home by students for training to learn at home.

In learning videos containing movie in which there are facts and problems in everyday life that can be broken using the concept of surface area of cube and cuboid. Video learning is very helpful to train students meemecahkan kareana issue with not only read and listen then imagine the problem but students can view, explore and solve problems in front of a group of his way, at the time of this berkelompom also trained students to discuss, menyekidiki, fact-finding and solving problems.

3) Selection Formats

In the selection of the format of guided student learning refers to the process standards (BSNP, 2007); observation of student activity sheet refers to the kinds of activity coinciding with the pace of learning and AI models.

4) Preliminary Design Learning Tool

This activity is the writing of learning tools, which include the syllabus, lesson plans, instructional VCD, Student Worksheet (LKS), Learning about the test results (THB), the result is called a draft 1.

C. Tahap Pengembangan

Expert assessment includes validation of products, which includes all the learning that was developed at the design stage. Validation is done by six people who are competent to assess the feasibility of the study. Revisions were made based on suggestions / guidance from the validator. Results of revision based validators produce draft assessment II.

Based on expert validation results obtained are described in Table 1 below.

| Table 1 Validator Against a valuation recapitulation Learning Tool |
|---------------------------------|--------|--------|-------|-----|-----|
| **Validator** | **Silabus** | **RP** | **LKS** | **THB** | **Video** |
| I | 3.6 | 3.4 | 2.9 | 3.3 | 3.1 |
| II | 3.1 | 3.1 | 3.1 | 3.3 | 3 |
| III | 3.6 | 3.4 | 3.1 | 3.3 | 3.1 |
| IV | 3.1 | 3.4 | 2.9 | 3.3 | 3 |
| V | 3.1 | 3.1 | 3.1 | 3.3 | 3 |
| VI | 3.6 | 3.4 | 3.1 | 3.3 | 3.1 |
| **Average** | 3.35 | 3.34 | 3.03 | 3.3 | 3.06 |

From Table 1 shows that for validation validator syllabus of six mean 3.35, validation RP averaged 3.34, validation LKS averaged 3.03, validation THB averaged 3.30, and validation of learning acquired VCD average of 3.06 of the maximum value of 4.

Based on the results of expert validation study results obtained in the form of suggestions for improvements as follows:

1) Syllabus

In general, both validators stated syllabus and can be used with revisions. Revised syllabus of the validator correction results can be seen in Table 2 below.
Table 2 Revised Syllabus Based on input from the Validator

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) The numbering by using bullets</td>
<td>a) The numbering using the numbering</td>
</tr>
<tr>
<td>b) Time has not been detailed</td>
<td>b) The time specified / split on each activity in the syllabus</td>
</tr>
<tr>
<td>c) In the syllabus has been no exploration, elaboration and confirmation</td>
<td>c) It was given the stage of exploration, elaboration and confirmation</td>
</tr>
</tbody>
</table>

2) Lesson Plan
In general, a good lesson plan validator states and can be used with the revision. lesson plan revision of the validator correction results can be seen in Table 3 below.

Table 3 Revised RP Based on input from the Validator

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) the formulation of learning objectives are too broad,</td>
<td>a) the formulation of specific learning objectives are made,</td>
</tr>
<tr>
<td>b) the formulation of objectives should be an activity, (determining and drawing nets cubes and cuboids)</td>
<td>b) the formulation of the objectives of the activity, (i. determine nets cubes and blocks, ii. draw a cube nets and cuboids)</td>
</tr>
<tr>
<td>c) at the end of the RP activities must be able to know the purpose of which is planned to be achieved or not,</td>
<td>c) has been made an instrument to know that the learning objectives have been achieved,</td>
</tr>
<tr>
<td>d) need to be declared in the planning stages of exploration, elaboration, and confirmation,</td>
<td>d) has made plans set forth in the exploration phase, elaboration, and confirmation,</td>
</tr>
<tr>
<td>e) operational use of words.</td>
<td>e) has been using words operational.</td>
</tr>
</tbody>
</table>

3) VCD
Validation results for VCD Learning validator says is good and can be used with the revision. Based on the results of the expert validation, several revisions were made to the instructional VCD can be seen in Table 4 below.

Table 4 Revised VCD Student Learning Based on input from the Validator

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) examples in the CD does not reflect the learning objectives</td>
<td>a) the examples on the CD are converted into an example that reflects the learning objectives</td>
</tr>
<tr>
<td>b) layer beginning to be filled, Prodi, year, etc.,</td>
<td>b) the initial appearance was made, study program, year, etc.</td>
</tr>
<tr>
<td>c) numbering should not use the &quot;#&quot;</td>
<td>c) the numbering is to use numbers</td>
</tr>
<tr>
<td>d) color graphics made more interesting</td>
<td>d) graphical color is made more interesting</td>
</tr>
</tbody>
</table>

4) work sheet
In general, both validators declared worksheet and can be used with the revision. Based on the results of the expert validation, several revisions were made to the worksheet can be seen in Table 5 below.

Table 5 Revision Worksheet Based Input Validator

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) there is no SK, KD and objectives</td>
<td>a) has been added there SK, KD and learning objectives on LKS</td>
</tr>
<tr>
<td>b) material for nets has been no</td>
<td>b) has been made materials for nets</td>
</tr>
<tr>
<td>c) drawing cube letters must be neatly arranged</td>
<td>c) is laid out neatly and clearly</td>
</tr>
<tr>
<td>d) measures anchored in worksheet</td>
<td>d) has been clarified step anchored instruction on LKS</td>
</tr>
</tbody>
</table>
5. THB

In general, states about THB validator is good and can be used with the revision. Based on the results of the expert validation, several revisions were made to the THB Problem can be seen in Table 6 below.

Table 4.6 Revised THB Based Input Validator

<table>
<thead>
<tr>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) does not reflect the programmed measurements, for example, there is no question about the nets,</td>
<td>a) is made in accordance with the measurement problem that has been programmed and made a matter of nets,</td>
</tr>
<tr>
<td>b) needs to be grating,</td>
<td>b) has been made the grille,</td>
</tr>
<tr>
<td>c) mathematical symbols must be written with symbols</td>
<td>c) has been created using a mathematical symbol by symbol</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

a. Expansion device in this study using a model that consists of the development of a definition, the planning and development stage. At the level of development validation performed by a device that produces suggestions and improvements, followed by testing the testing device to choose a question to be used in THB, the last is to know the outcome of trial development of learning tools.

b. The tools developed in this study is valid, this can be seen from the opinion of the state validator well and can be used after revised.

REFERENCES

DESIGN RESEARCH: PLACE VALUE IN DECIMAL NUMBERS USING METRIC SYSTEM

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ABSTRACT
There were many studies which had been investigated how students learn decimal numbers. Some of them observed it in detail. The goal is to scaffold students’ thinking and to help them be motivated in practice. The idea was an understanding of place value and relationship between numbers and decimal numbers using metric system. Realistic Mathematics Education underlies the design of the activities and the contexts; design research was done in two cycles. It was conducted in three phases: preliminary design, teaching experiment, and retrospective analysis. Six students and 34 students of fifth grade were involved as data; each group of students was divided into two cycles. It was held at MIN 1 Palembang, one of PMRI participant school in Palembang. Students would enable to work with contextual situations within measurement activities; number line is used as a model for supporting their understanding and reasoning. In the analysis, their reasoning was used to develop the new one of learning trajectory. The result of this study could show that the activities could bring students’ understanding from the contextual situation to the formal situation. Students would not use integer number system when doing decimal number problem. Learning trajectory on this study can be used for elementary school.

Keywords:
Decimal numbers
Place value
Design research
Realistic mathematics education

INTRODUCTION
The extensive studies from around the world on decimals have documented students’ difficulties and weak conceptual understanding of decimals from elementary to college levels, Steinle dan Stacey [1]. Ubuz dan Yayan [2] explained about the knowledge in the domain of decimals and investigated students’ performance, and also observed their difficulties in reading scales, ordering numbers, and the operation of decimals. Because of those difficulties, the teaching and learning process on decimals needs more attention.

Much of the research literature on teaching learning process about decimals separated decimals from meaningful contexts, often in order to teach students about place value, Irwin and Britt [3]. One level of knowledge involved in processing decimal numbers consists of position knowledge (involving place number names), the base ten system, and order of the places (hundredths are larger than tenth, etc.), Widjaja [4]. Practically, some students use the meaning of whole numbers and apply this knowledge on decimal number concepts when they try to solve the problem of decimals.

There are some studies investigated how students might be thought the sense of decimals which requires the application of four basic arithmetic operations. The fact that there are current studies which are provide the misconceptions of students in doing decimals without further investigation, has established a need to conduct this topic deeply. Based on what present studies have found, researcher hopes that this study can extend previous knowledge in some way and give a contribution continuing the research about decimal number.

Although previous studies have addressed students’ difficulty on decimals addition, none of them have described explicitly the way to solve it. For example, in his research, Irwin [5] stated that when you do something to one side of the point or comma, you also do it to the other side (e.g., $2.5 + 1 = 3.6$). Another
example, Ubuz and Yayan [2] stated that the most common errors in the addition tasks were adding the last digit of number behind the comma adding up 0,1 to the number 4,256 and 6,98, students gave the answers of 4,257 and 6,99 instead of 4,356 and 6,98. In order to overcome the difficulty, this study presents a sequence of classroom activities aimed at constructing the understanding of place value in decimal numbers.

Furthermore, according to the approach of some Indonesian textbooks, the way of teaching and learning decimals is conducted directly using algorithm. It does not present any concrete models. It is in line with the previous research from Zulkardi [6]. He stated that the topic of decimals in Indonesian textbook mainly contains sets of rules and algorithms and it lacks applications that are experientially real to the students. The meaningful situation is important in order to avoid students’ misconception about decimals, Pramudiani [7]. Supporting students with a model (e.g. number line) can engage them to find some strategies which lead them to the meaningful learning situation. So, this study provides some classroom activities which are related to the experiential world of the students, it is purposed to bring students’ informal knowledge or out of school reasoning experiences into school mathematics or formal knowledge.

Considering the misconception of students and also the approach of the textbook in Indonesia, this research has an aim to investigate the development of students’ understanding about place value in decimals through the activities. The research question is: “How can a metric system support students to understand the meaning of place value in decimals?”

2. RESEARCH METHOD

The research method of this research that will be discussed are: (a) research approach, (b) data collection including preparation phase, pre-teaching experiment, teaching experiment, post-test, validity and reliability; and (c) data analysis including pre-test, pre-teaching experiment, teaching experiment, and post-test.

2.1. Research approach

The main object of this research is to investigate students’ learning of understanding the place-value of decimals. For this purpose, design research is chosen as an approach for achieving the research goals and answering the research questions. Gravemeijer & Cobb [8] stated that design research is a type of research methods aimed to develop theories about both the process of learning and the means that are designed to support that learning. Therefore, in this research, a sequence of activities is designed as means to improve educational practices in understanding of place value in decimals for grade 5 elementary school.

According to Gravemeijer & Cobb [8], there are three phases of conducting a design experiment, as follows:

a. Preliminary design

In the preliminary design, the ideas, which are implemented here, are inspired by studying literature. A sequence of instructional activities containing conjectures of students’ strategies and students’ thinking is developed. The conjectured hypothetical learning trajectory is developed based on literature; it is adjusted to students’ actual learning during the pilot and teaching experiment.

b. Teaching experiment

In teaching experiment, instructional activities are tried, revised, and designed on a daily basis during the teaching experiment Gravemeijer [9]. The teaching experiment is aimed at collecting data for answering the research questions. In this research, it is conducted through activity in one meeting which the duration is 70 minutes for the lesson. But before that, the teacher and the researcher discuss the upcoming activity. And after each lesson ends, teacher and researcher make a reflection in order to improve the designed activities.

c. Retrospective analysis

In the retrospective analysis, all the data collected during the teaching experiment are analyzed. The hypothetical learning trajectory is used as a guideline in answering the research questions; it is compared with students’ actual learning.

2.2. Data Collection

a. Preparation phases

In the preparation phase, the data collection is aimed to investigate pre-knowledge of students. It will be collected by doing observation class, interview, and pre-test for all students. The information about students’ pre-knowledge will be used to fit the initial HLT (hypothetical learning trajectory) considering the aspect of starting point of students. It can be adjusted before the first cycle is started. The classroom situation is also important to be concerned about how the learning process works in the class. It concerns about social norms and socio-mathematical norms. Data is collected during the lesson of the observation class; it can be received from audio or video recording, and field notes of researcher. The researcher writes field notes based on the lists of the observation points.

b. Preliminary teaching experiment (first cycle)

In the preliminary teaching experiment, the instructional activities are given to four students with respect to the differences in level of understanding (1 high level of students, 2 middle levels of students, and 1 low level of students) which are not different too far. Researcher expects that choosing 4 students will represent the ability of the other students. Four students who are selected are not from the observation class for the next following phase. In this phase, they will be taught by the researcher or one of mathematics teacher who will do teaching experiment later on; expecting that the teacher will know better the learning trajectory before the second cycle of teaching experiment is begun. During the learning process, it is recorded by one video recording which is focused...
on all students; also field notes of researcher. The aim of the preliminary teaching experiment is collecting data to support the adjustment of the initial HLT.

c. Teaching experiment (second cycle)

In the teaching experiment phase, the HLT which has been improved will be tried out. It will be given to all students in one class, but for the analysis of the experiment it will be focused on four students only (one group of four students). Data is gathered through two video recording, one camera, and field notes. One video recording is put in the corner of the classroom in order to record most of the situation of learning process. And another video is placed in front of the group which consists of four students. Also, one camera is used to take some pictures in which there are interesting moments related to the experiment, such as students’ strategy when they try to solve the problem, and so on. Here, the researcher has a role as an observer and makes some notes; researcher is only focus to the group of four. Moreover, the teaching experiment of the second cycle has an aim to answer the research question.

d. Post-test

In the post-test phase, the test is used to assess students’ understanding after the lesson is finished. It can measure students’ ability whether the lesson is succeeded or not. The test is in written form which consists of 10 problems; the problems are in the same form with the pre-test. The post-test will be given both in the first cycle and the second cycle at the end of the activities. Four students who are focused on this study are interviewed to know more about their answer on the post-test problem. It is used to find out what their thinking and reasoning toward the problem. In this phase, data is collected through one video recording (during the post-test and interview session) and field notes of researcher.

e. Validity and reliability

In this study, the different types of data are involved, such as video observations, students’ worksheet, field notes, and interview data. The method of triangulation data will be done by involving different types of data. Then, the triangulation data and also testing conjectures of the HLT during the teaching experiment contribute to the internal validity of the data. Data registration will convince ourselves that researcher works in a reliable way because the data was collected by different methods; collecting data by a video recording is more objective than making field notes.

2.3. Data Analysis

a. Pre-test

In the pre-test phase, the result of data pre-test (students’ answer and calculation) is analyzed to investigate starting points of students in learning about decimals. The test result is expected to reveal students’ prior knowledge about decimals; it can direct the HLT in such a way in order to make it appropriates for students.

b. Preliminary teaching experiment (first cycle)

In the preliminary teaching experiment phase, the video recording and the students’ worksheet are analyzed to find out the useful of the learning process. Officially there is a possibility that the conjectures of our HLT does not appropriate with real situation. Here, the HLT needs improvement because sometimes it fits students in learning process and sometimes it does not appropriate for them.

c. Teaching experiment (second cycle)

In the teaching experiment phase, the video and the students’ worksheet are analyzed; these four students will be focused more than the others. Their thinking and also their development from the beginning of the study until the end will be analyzed. However, it is still possible to the other students to be analyzed. If there is a situation or a statement which is supported the learning process, it can be that (s)he will be included to be analyzed in this study.

d. Post-test

In the post-test phase, the result of the test is analyzed to measure students’ understanding after the lesson has finished. It also can be compared to the result of the pre-test finding out whether there is any improvement or not. A post-test has an aim to investigate students’ development in understanding the concept of place value in learning decimals.

e. Reliability

In this study, the reliability of the data analysis covers two aspects, track-ability and inter-subjectivity. Giving a clear description on how the work on this study so that the reader will easily understand the way of track-ability. The description contains the explanation of the process of how the preparation phase is done, how the teaching experiment phase (first and second cycle) is happened, and how the research analyzes the data; also provide conclusion. In addition, discussing with colleagues can avoid the researcher’s own viewpoint toward data analysis; it is needed to attain inter-subjectivity.

3. RESULT AND ANALYSIS

The activity was inspired from the book written by C. Barnett, D. Goldenstein, dan B. Jackson [10]. They said that metric system could be used in learning decimal number; it succeeded enhancing students’ ability of reasoning. Now, in Indonesia, we have an advantage of Indonesia situation that know measuring system already. This fact supports researcher to use metric system in Indonesia. Measuring system as we know that something which can be divided by 10, such as an unit become tenth, tenth become hundredth, hundredth become thousandth, and so on.

Practically, students could be experienced with the real situation and the real media/tool also. In this activity, they were given a customized carton. It has been formed a square which has the length 1 meter. One cartoon was given to each group. They have to find the form of square which has the length 0,1 meter and
0.01 meter (if possible). Here, on the first cycle, researcher acted as a teacher. Researcher guides students learning a little bit, not much. Some students who have an idea made pieces of carton with an equal length already (Figure 1). They did not forget to measure it carefully. However, there are two students who still got a difficulty to grasp the idea of this activity.

Therefore, researcher needs to help the students to find them right way as soon as possible. Researcher gave an apperception of relation between unit length (meter, decimeter, and centimeter) while showing one meter length of carton. After that, researcher asks students to form a 0.1 meter from the one meter length of carton, which means that students have to divide the carton into 10 equal parts by folding and draw a vertical line on each fold or cutting the carton. Next, they divided the tenth into 10 equal parts to make a hundredth (some students use a ruler to find 1 cm which is equally to hundredth, it showed on figure 2). Then, if it is still possible (if students still want to know how the form of thousandth), teacher could ask them to make hundredths into 10 parts to form thousandth.

Researcher facilitates the students realizing the relationship between unit, tenth, and hundredth from the carton that they have created. Through this activity, we could see the ability of students understanding of place value in decimal numbers. When they did not able to determine the tenth or hundredth to measure carton, such as when they measured the book length of 0.21 m, they tend to use a single piece of ‘tenth’ and eleven piece ‘hundredth’, this means that they did not concern the relationship of place value in decimal number. Although they had been studying fractions and decimal already in grade 4, there was no guarantee that they had been understood the place value of decimal number. Therefore, through this activity the researchers want to explore how far the students' understanding of decimal numbers, particularly the relationship of place value in decimal number.

Through carton, students could find the relationship of decimals with the unit of measurement (figure 3). Starting from unit in meter, it would be continued by tenth (decimeter), hundredth (centimeter), thousandth (millimeters), ten (decameter), hundred (hectometer), and thousand (kilometer). They realized the condition if we define the unit in centimeter (length measurement) or unit in gram (weight measurement). Gradually, they would know the difference between each step is ten (tenth is equal to hundredth multiply by ten, tenth is equal to unit divide by ten). In addition, they would also know the difference interval of two unit length, for example tenth is equal to hundredth multiply by a hundred (two times ten), and so on. That is the result and also the analysis from the first cycle. We could say that overall it was success. So, there are no big obstacle which is affected the researcher to change the activity or the instrument. It is only the way of teaching that has to be changed, especially in the apperception phase. Researcher seemed too much in giving
the apperception, in the second cycle it has to be minimalized. And also, we need to transform the structure of
the instrument language easier to be understood than before.

![Figure 3. Student is trying to integrate decimals with the unit of measurement](image3)

On the second cycle, researcher acted as an observer. The real teacher acted as a model teacher in
this research, and she gives students only a little bit guidance, not much. Here, the treatment is also the same
with the first cycle; teacher divided the students in the class to some groups. Each group was consisted four
students. Then, teacher gave them one meter carton (figure 4). After that, the teacher asked them to find the
representative of decimals using the carton. It means that the students have to find tenth, hundredth, and
thousandth (if possible). Students with the high level of thinking did not find any difficulty finding the
relationship among unit, tenth, and hundredth. Some groups also did not find any obstacle finding tenth
(showed in figure 5), but the problem is hundredth. Somehow they stacked when they involved with more
numbers behind the comma (decimal point).

![Figure 4. Teacher is distributing one meter carton](image4)

![Figure 5. One group of students is trying to figure out tenth](image5)

Here, the guidance has to be given, before the students jump into a wrong perception of decimal
concept. Different case also happen, there was student in one group who think critically. He found something
that the other students usually thought wrong of it. He said that 0,01 meter is not the same with 0,10 meter,
because both of them have a different length of carton; 0,10 meter is longer than 0,01 meter. One student had
a different thinking with the others, he thought of decimals as a part-whole relationship. In the carton, he use one meter carton as a whole, tenth meter as a part-whole in decimeter, and so on.

Further question, teacher asked students to measure the length of two different objects using the carton. The activity before, made students to have a collection of piece cartoon with a different length, such as tenth, hundredth, and so on. In this moment, they used it to find the structure of decimals. To find the length structure of one object, for instance the length of ballpoint 0.12 meter, some students use one tenth and two hundredth (figure 6). But there were also students who use 12 of tenth or 12 cm. From this, we could find the level of students thinking of decimals. Students’ point of view with the metric system activity was nice; they were happy learning mathematics with different approach. Finally, they knew that decimals were not only numbers with comma.

Many hypotheticals of students made by researcher did actually match with the real situation. But, there are some things that beyond researcher doubt. This can be due to many things, such as the design of the activities, teacher’s explanation, the way of content delivery, miscommunication between teacher and researcher, and so on.

Meanwhile, seeing the implementation of instructional design of PMRI reflects how characteristics of RME be a grounded theory for the activity where the concept of place value in decimals. It is inspired by Bakker [11]. The first characteristic of RME is phenomenological exploration, better understanding of the basic conceptual mathematics in decimal numbers, rich context and meaningful problems and also necessary activities. Using context which is familiar with students, this activity motivated students in learning and made a meaningful teaching and learning process.

The second characteristic of RME is using models and symbols for progressive mathematization, bridging ‘the gap’ between the concrete and the abstract level, the models and symbols used. Diversity of models and symbols, and also the design of activity intended to bring students to develop their knowledge. Here, a context of place value in decimals as: metric system.

The third characteristic is using students’ own construction and productions. Students are free to use their own strategies; it became the foundation for them as a solution that could be used in the further studies. Through the activity and also class discussions, students could build their own understanding of decimals with minimal guidance. They made some groups depending on the place value in decimals, separating the tenth, hundredth, and thousandth.

The fourth characteristic is interactivity. Student learning is not only an individual learning process but also a process of social learning. From the models made by them (e.g. drawing a representation of carton) the interaction among students is occurred; it made the discussion became more meaningful, such as learning from the others. Role of the teacher is only as a facilitator that connects student with others so that the students could find by themselves the understanding of the place value concept in decimal numbers.
The fifth characteristic, intertwinement, integration of learning topics could help students learn math in an effective way, and learn place value concept of decimal number could be combined with other topics such as fractions, percent, ratio, and measurement.

4. CONCLUSIONS

The goal of this research is to scaffold students’ thinking and to help them be motivated in teaching learning process. The idea was an understanding of place value and relationship between numbers and decimal numbers using metric system. There is one research question needed to be answered. The research question is: “How can a metric system support students to understand the meaning of place value in decimals?” From the results and analysis, it has been clear that the application of metric system is success overall. In the beginning of the study, metric system is not a new thing for students so that it is realistic for them. They adapted with it easily, not needed much time to learn decimals through the context.

Through metric system, students could learn place value in decimals. They knew the representation of decimal numbers. Starting from unit, they were introduced tenth, hundredth, and so on. The learning process of their invention is by themselves; the role of the teacher here as a guidance only. Moreover, this research can be developed more in the other topics. Not only restricted in decimal numbers but also other topics especially the topic which needs an implantation of concept. As a final conclusion, this research result can be used as guidance for teacher who wants to apply the activity and also can be used for the other researcher as a reference for further studies.

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THE ENHANCEMENT OF MATHEMATICAL COMMUNICATION THROUGH METACOGNITIVE SCAFFOLDING APPROACH AMONG PRESERVICE ELEMENTARY SCHOOL TEACHERS IN BANDUNG

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ABSTRACT
This research aims to investigate the enhancement of students’ mathematical communication through teaching under metacognitive scaffolding approach. This research used a quasi-experimental design with pretest-posttest control. The subjects were preservice elementary school teachers in Bandung. In this study, there were two groups of subjects: experimental and control groups. The experimental group consists of 60 students under metacognitive scaffolding approach, while the control group consists of 58 students under direct approach. Based on results of prior mathematical ability test, the students were classified into three ability levels, namely high, middle, and low. Data collection instrument was mathematical communication test instrument. By using t test, t’ test, Mann-Whitney test, and Kruskal-Wallis, some conclusions of the research were: (1) there was a significant difference in enhancing mathematical communication ability between students who attended the course under metacognitive scaffolding approach and students who attended the course under direct approach, and (2) there was no significant interaction effect between teaching approaches and ability levels based on prior knowledge in enhancing students’ mathematical communication. Teaching approaches applied in this study provided influence in enhancing of students’ mathematical communication significantly, while the ability level based on mathematical prior knowledge did not give it significantly.

Keywords: Ability levels, Direct approach, mathematical communication, metacognitive scaffolding approach.

1. INTRODUCTION
Mathematical communication is an important part of learning mathematics. It is explicitly stated in Regulation of National Education Minister of Republic of Indonesia Number 22 (Departemen Pendidikan Nasional RI, 2006), Curriculum and Evaluation Standards for School Mathematics [CESSM] (Romberg, et al., 1995) and the Principles and Standards for School Mathematics [PSSM] (Carpenter & Gorg, 2000). However, some of the survey results (Mullis, et al., 2000, Mullis, et al., 2008, OECD, 2005: OECD, 2007, and OECD, 2010) showed that Indonesian students’ mathematical communication ability were quite low compared to some other countries. If it was seen from the content of mathematics in the surveys, a lot of the topics had been studied by the students while they were in elementary school. Thus the students’ lack of mathematical communication was related to lack of their mathematical communication in elementary school.

Elementary students' mathematical ability related to the teachers' ability. This linkage revealed in study reports. In their study report, Hill, Rowan, & Ball (2005) stated that the mathematical ability of elementary school teachers was significantly related to students' mathematical achievement. Passos (2009) reported that there was a relationship between elementary school teacher competence and student achievement in reading and math. Therefore, it could be said that the development of mathematical communication of preservice elementary school teachers is very important. One effort to develop their mathematical communication ability was looking for factors expected to enhance their mathematical communication ability. One of the factors was a teaching approach.

Carpenter & Gorg (2000) recommended an approach that includes strategy, planning, monitoring, and evaluation during the learning process. This approach was known as metacognitive (Schoenfeld, 1992). Although the approach had been recommended by experts, this approach still has drawbacks, such as when a student realized that he/she did not find a way to solved mathematical problems, he/she would pause in his own confusion. Goos & Galbraith (1996) and Yee (2002) reported that the metacognitive approach (without
scaffolding) was not able to raise students' success in learning mathematics. Instead, Van Der Stuyf (2002) and Peter (2011) revealed that if the scaffolding approach run itself (not involving metacognitive) the students were weak in developing their own ways to solve problems. To overcome this, metacognitive approach needs to be combined with scaffolding.

For learning in the classroom that involves a lot of students, usually more than 30 people, metacognitive scaffolding approach was almost impossible to be implemented. Therefore, this approach needed to be combined with cooperative learning. In cooperative learning, the lower mathematical ability students could learn of mathematical work habits of higher mathematical ability students, and in the process of explaining the material, the higher mathematical ability students could develop mastery stronger and deeper understanding for themselves about the mathematical tasks. Brown & Goren (1993) revealed that in cooperative learning, there was indication the incorporation of students' ability to work together in solving mathematical tasks. Thus, metacognitive scaffolding approach in cooperative learning pattern was expected to enhance the students' mathematical communication ability. However, cooperative learning was not entirely a positive impact on students' academic performance (Lucas, 1999; Iqbal, 2004 and Hecox, 2010). Lucas (1999) reported that there was no significant difference in academic performance in algebra among students who attended teaching under the cooperative learning and students under traditional approach at Midwestern University. Iqbal (2004) reported that low-ability students of middle school at Rawalpindi got a benefit from cooperative learning, while high ability students did not have it, although they remain in position at the top in math achievement. Hecox (2010) found no difference in scores Florida Comprehensive Assessment Test (FCAT) among fourth-grade students who obtained teaching under cooperative learning and students who obtained traditional teaching in Polk County, Florida.

Despite many studies that examine the influence of metacognitive approach, scaffolding or cooperative learning to academic performance in certain educational levels, but it did not found studies that aimed to review the enhancement of mathematical communication on low, middle and high ability students under metacognitive scaffolding approach in cooperative learning pattern, hereinafter referred to as metacognitive scaffolding approach. Thus, the purpose of this study was to find the students' mathematical communication after obtaining metacognitive scaffolding approach.

Research Questions and Hypotheses

The research questions were, “Is there a difference of enhancement of mathematical communication ability between students who acquire teaching under metacognitive scaffolding approach and students who acquire teaching under direct approach?” and “Is there an interaction effect between teaching approaches (metacognitive scaffolding approach and direct approach) and students’ prior mathematical ability (high, middle, and low) to the enhancement of mathematical communication ability?” To address that question, enhancement of mathematical communication of students taught by metacognitive scaffolding approach compared with that of a control group. Besides that, the enhancement both of students taught by metacognitive scaffolding approach and that of a control group based on the prior mathematical ability compared each others. In this study, the hypotheses were, “There is a difference of enhancement of mathematical communication ability between students who acquire teaching under metacognitive scaffolding approach and students who acquire teaching under direct approach” and “There is an interaction effect between teaching approaches and students’ prior mathematical abilities toward enhancement of mathematical communication ability”.

In this study, mathematical communication was considered as students’ ability to understand and communicate mathematical ideas both in writing, and drawing. The indicators used in this study were: (1) stating a picture or diagram into mathematical ideas, (2) stating a daily occurrence in the mathematical symbols, and (3) explaining the idea, situation, or a mathematical relation with graphs or algebraic. Metacognitive scaffolding approach was considered as a teaching approach that was characterized by activities: (1) teacher raised a mathematical problem, (2) students tried to solve the problem; and (3) teacher provided temporary metacognitive assistance, which is gradually reduced and eventually the student can independently take full responsibility for mathematical tasks that must be completed; whereas traditional teaching, direct approach, was consider as a teaching approach that was characterized by activities: (1) explanation or manipulation concept by teacher, (2) providing an opportunity for students to ask, (3) demonstrating completion of example problems, (4) giving exercises to be completed by the students, (5) asking some students to write again their answer on the board, (6) commenting on student answers, and (7) providing homework assignments if it deemed necessary.

2. METHOD

Research Design

The method used in this study was aquasi-experimental method. There were two groups of students. As the experimental group was students who acquire teaching mathematics under metacognitive scaffolding approach, while the control group were students who acquire teaching mathematics under direct approach. This
After providing a treatment, posttest on the control group pretest-posttest, and expressed as follows.

\[
\begin{array}{ccc}
O & X & O \\
\hline
\end{array}
\]

Description:
1. O: pretest-Posttest on mathematical communication.
2. X: Treatment in the form of learning with metacognitive scaffolding approach.

The study design involves two factors, namely learning approaches and student groups based on factors prior mathematical abilities. The first factor consisted of metacognitive scaffolding and direct approach. The second factor consisted of a group of students based on prior mathematical ability (high, middle, and low). This research design could be described as the relationship between the factors as presented in Table 2.1.

**Table 2.1**

<table>
<thead>
<tr>
<th>Prior Mathematical Ability</th>
<th>Teaching Approach</th>
<th>Metacognitive Scaffolding Approach (B₁)</th>
<th>Direct Approach (B₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High (A₁)</td>
<td>A₁B₁</td>
<td>A₁B₂</td>
<td></td>
</tr>
<tr>
<td>Middle (A₂)</td>
<td>A₂B₁</td>
<td>A₂B₂</td>
<td></td>
</tr>
<tr>
<td>Low (A₃)</td>
<td>A₃B₁</td>
<td>A₃B₂</td>
<td></td>
</tr>
</tbody>
</table>

**Sample**

Through this study, the authors try to uncover how the students’ communication abilities after obtaining mathematics teaching under metacognitive scaffolding approach. Taking into account the results of international studies (TIMSS and PISA), it appears that the most highlighted of the low ability of mathematical communication of Indonesian students were elementary and junior high school. When it seen from the mathematical content of the topics in TIMSS questions, generally, it were studied by elementary students.

In terms of mathematical communication abilities that should be found, it was closely related to aspects of the elaboration of the mathematical idea. Furthermore, to reduce misinterpretation of the students’ idea, especially those expressed in writing, investigators determined the study was not done in the Elementary School. Some studies (Hill, Rowan, & Ball, 2005 and Passos, 2009) showed that elementary students’ mathematics achievement was influenced by teachers’ mathematical knowledge. Finally, taking into account the constraints of the researcher, the research conducted at the Elementary School Teacher Education Program at a university in Bandung. Thus, population of the study was all students of Elementary School Teacher Education Program who received mathematics education course, at a university in Bandung. Whereas the sample was 118 students; 60 students as an experiment group and 58 students as a control group.

**Research Procedure**

Research activities initiated by determining the study sample. After the sample was set, each student was given a prior mathematical ability test. The test is intended to classify students based on prior mathematical abilities (high, middle, and low). After the experimental and the control groups were formed, the students were given the pretest about the mathematical communication ability After providing a treatment, posttest on mathematical communication was given for the students. For data analysis, researchers used the help of Statistical Package for Social Science (SPSS) for Windows version 20 software.

**Data Analysis**

There were two main hypotheses to be tested. The first one was related to test two independent samples with the interval ratio of measurement. The data was analyzed by t test, t’ test, and Mann-Whitney. In the second hypothesis, data could be tested using two-ways ANOVA if the conditions were available. If the conditions were not available, the interaction effect would be seen by the diagram and one way ANOVA or Kruskal-Wallis.

**3. RESULT AND DISCUSSION**

**Result**

Descriptive statistical analysis of the results of students’ mathematical communication ability was presented in Table 3.1 below.
Table 3.1 Description of Students’ Mathematical Communication Ability (MCA)

<table>
<thead>
<tr>
<th>Group</th>
<th>Level</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Gain of MCA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
</tbody>
</table>

From the Table 3.1, it was appeared that the enhancement of communication ability that students acquire teaching under metacognitive scaffolding approach was relatively higher than students who acquire teaching under direct approach, the well-viewed as a whole and viewed based on the level of prior mathematical ability. Inferential statistical analysis of the results of students’ mathematical communication ability to experimental and control groups were presented in Table 3.2 below.

Table 3.2 The Difference of Students’ Mathematical Communications Ability (MCA) Between Experiment and Control Groups (The level of significance α = 0.05)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Group</th>
<th>Difference Test</th>
<th>Test Statistic</th>
<th>Sig.</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCA-1 (Pretest)</td>
<td>Experiment</td>
<td>M-W test</td>
<td>0.000</td>
<td>Different</td>
<td></td>
</tr>
<tr>
<td>Mix</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gain of MCA</td>
<td>Experiment</td>
<td>M-W test</td>
<td>0.000</td>
<td>Different</td>
<td></td>
</tr>
<tr>
<td>Mix</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCA-1 (Pretest)</td>
<td>Experiment</td>
<td>t-test</td>
<td>0.000</td>
<td>Different</td>
<td></td>
</tr>
<tr>
<td>Low Ability</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gain of MCA</td>
<td>Experiment</td>
<td>t-test</td>
<td>0.002</td>
<td>Different</td>
<td></td>
</tr>
<tr>
<td>Low Ability</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCA-1 (Pretest)</td>
<td>Experiment</td>
<td>M-W test</td>
<td>0.017</td>
<td>Different</td>
<td></td>
</tr>
<tr>
<td>Middle Ability</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gain of MCA</td>
<td>Experiment</td>
<td>M-W test</td>
<td>0.000</td>
<td>Different</td>
<td></td>
</tr>
<tr>
<td>Middle Ability</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCA-1 (Pretest)</td>
<td>Experiment</td>
<td>t'-test</td>
<td>0.100</td>
<td>Not Different</td>
<td></td>
</tr>
<tr>
<td>High Ability</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCA-2 (Posttest)</td>
<td>Experiment</td>
<td>t-test</td>
<td>0.291</td>
<td>Not Different</td>
<td></td>
</tr>
<tr>
<td>High Ability</td>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the Table 3.2, it could be stated that there was difference in enhancing of mathematical communication ability significantly between students who attained teaching under metacognitive scaffolding approach (experimental group) and students who attained teaching under direct approach (control group), the well-viewed as a whole (mix) and viewed based on the prior mathematical ability levels (low and middle). If these results were associated with the results in Table 3.1 it can be concluded that the enhancement of students’ mathematical communication who attained teaching under metacognitive scaffolding approach were higher than students who attained teaching under direct instructional approach.

The interaction effect between teaching approaches and prior mathematical ability toward enhancement of students’ mathematical communication ability would be tested by using Two Ways ANOVA. Before using the Two Ways ANOVA, it was necessary to be viewed whether the data of each factor was distributed normally. The result of Distribution Normality was presented in Table 3.3.
Table 3.3
Data Distribution Normality on Mathematical Communication Ability
Based on Group and Prior Mathematical Ability
(The level of significance $\alpha=0.05$)

<table>
<thead>
<tr>
<th>Group</th>
<th>Ability</th>
<th>Distribution</th>
<th>Normality</th>
<th>Implications of the Use of Two Ways ANOVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>-</td>
<td>0.006</td>
<td>Not Normal</td>
<td>Two Ways ANOVA is not used</td>
</tr>
<tr>
<td>Control</td>
<td>-</td>
<td>0.006</td>
<td>Not Normal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0.163</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Middle</td>
<td>0.072</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.022</td>
<td>Not Normal</td>
<td></td>
</tr>
</tbody>
</table>

From Table 3.3 it appeared that the condition for using Two Ways ANOVA was not sufficient. Therefore, the interaction effect was analyzed using diagram 3.1 and Table 3.4.

Diagram 3.1
Interaction Effect between Learning Approach and Prior Mathematical Ability toward the Enhancement of Students’ Mathematical Communication Ability

The diagram 3.1 indicated that there was interaction effect between teaching approaches and prior mathematical ability toward enhancement of students’ mathematical communication ability. To confirm the presence of this interaction effect is significant, it was necessary to be tested the difference of the gain among prior mathematical ability levels (low, middle, and high), both in the experimental and the control groups, as presented in Table 3.4 below.

Table 3.4
Test of Difference of Mathematical Communication Ability Gain (Gain of MCA) among Ability Levels of Experimental and Control Groups
(The level of significance $\alpha=0.05$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Ability Level</th>
<th>Difference Test</th>
<th>Statistic Test</th>
<th>Sig.</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCA Gain of</td>
<td>Low</td>
<td>Kruskal-Wallis</td>
<td>0.332</td>
<td>Not Different</td>
<td></td>
</tr>
<tr>
<td>Experimental Group</td>
<td>Middle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MCA Gain of</td>
<td>Low</td>
<td>Kruskal-Wallis</td>
<td>0.288</td>
<td>Not Different</td>
<td></td>
</tr>
<tr>
<td>Control Group</td>
<td>Middle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 3.4, it could be seen that there were no differences of enhancement of students’ mathematical communication ability in experiment group among the low, middle, and high level students. The same result occurred in the control group. It can be concluded that there is no significant interaction effect between learning
approaches and prior mathematical ability toward enhancement of students’ mathematical communication ability.

Discussion

The results showed that there was a significant enhancement of mathematical communication in which students gain hands-on approach to learning significantly. In addition, the results also showed that the enhancement of communication ability of students who acquired teaching under metacognitive scaffolding approach is significantly higher than students who acquired teaching under direct approach. Thus it can be said that learning mathematics with metacognitive scaffolding approach significantly positive impact on enhancement of students’ mathematical communication ability. The effectiveness of this approach to improving communication ability supported King & Roshenshine (1993) who found that guidance through the submission of questions had increased the ability of representation and understanding of the concept.

Clark et al. (2005) stated that giving problems that triggered discussionis one of the strategies to develop mathematical communication. By giving mathematical problems, students in small groups tried to understand and showed the model of the problem solution. The ability to understand mathematical problems and show a model of the solution was identified by Har (2008) as part of mathematical communication aspects. Mathematical problems posed by a teacher and the solving models developed students would be object of discussion, reflection or revision of the understanding of mathematical problem faced. Carpenter & Gorg (2000) stated that when students think, respond, discuss, elaborate, write, read, listen, and discover mathematical concepts, they have done two activities deals with communications, namely (1) communicate to learn mathematics and (2) learn mathematical communication. At the time of discussion to solve mathematical problems encountered, students used communication as part of the problem solving process. Teacher encouraged students to actively engaged in discussions, and provided assistance if there was a group of students came to a halt in understanding or solving the problem. The presence of these metacognitive questions encouraged students to identify the problem, identify relevant information, display ideas, and explain mathematical ideas to the friends group. The findings of this study are consistent with the recommendation Clark, et al. (2005) which stated that teachers should encourage students to actively explain the mathematical ideas and Carpenter & Gorg (2000) stated that teachers should encourage students to explain and justify mathematical ideas to his friends.

In the face of a mathematical problem, it appeared students in some groups (cooperative learning) argued for their opinion, while in other group it appeared that a student explained his idea to friends in his group. Thus, through cooperative learning, students not only got an understanding of mathematical as stated by Hatano & Inagaki (1991), but they also had opportunity to represented and evaluated mathematical ideas to test and compare it with the mathematical idea of their friend.

Although it was not as high as in teaching under metacognitive scaffolding approach, there was asignificant enhancement of mathematical communication ability of students who obtained teaching under direct approach. Thus, teaching under direct approach not only able to enhance academic achievement in general as reported by Engelmann (1999) or to enhance academic achievement, particularly for low ability students, as presented by the American Federations of Teachers (1998); Walberg & Paik (2000); and DeJager (2002), but it also can enhance students’ mathematical communication ability both as a whole and it based on their prior mathematical ability level.

Teaching under direct approach is characterized by teacher activities, such as explain a concept. When the teacher explains a concept, students gain an understanding of the concept needed to build mathematical communication ability. New knowledge that was acquired by students encourage them to match the existing cognitive structure and frequently it was preceded by cognitive conflict (disequilibrium); furthermore through student’s question and teacher’s answer, this conflict can be resolved, so that the cognitive structures remain in equilibrium. It encouraged the development of thinking and understanding of concepts, which is required to solve mathematical problems, especially problems related to mathematical communication ability.

The presence of solving mathematical problem samples by teacher, especially those related to mathematical communication could encourage students learn meaningfully, because those samples based on the concepts that already explained by their teacher and understood by the students. In addition, samples can be used as a model completion by students in solving mathematical problems, especially those related to mathematical communication ability. Thus the teacher activity could be seen as a stimulus for students, so that it encouraged the enhancement of students’ mathematical communication ability. In addition, the Time required on teaching under direct approach was efficient, so that teachers can provide additional practice materials. The presence of the materials made students have chance to practice more and represent mathematical ideas in a variety of forms, so that adds to the experience in the face of problems related to mathematical communication. According to Pressley (1995), with experiences, gradually students were able to implement relevant strategies flexibly and precisely. Although the samples of solving mathematical problems could encourage mathematical communication ability, the samples could also lead students tend to imitate those procedures. As a result,
students solved the problems easily if the problems were similar to the samples, but he had trouble when facing new problems. This was consistent with Heibert & Wearne (1986) stated that teaching under a direct approach could lead students being unable to resolve new problems. Furthermore, in learning the direct approach, there is a very limited social interaction, almost nothing, especially the interaction among students. Meanwhile, Vygotsky considers that an individual’s cognitive development depends on social interaction (Bonk & Reynolds, 1997).

From the above, it could be predicted that the presence of mathematical problems, metacognitive questions and cooperative learning on teaching under metacognitive scaffolding approach on the one hand, and the explanation of the concepts and samples of solving mathematical problems on teaching under direct approach on the other hand, are factors that could explain one of the results of this study, enhancement of students’ mathematical communication ability who acquired teaching under metacognitive scaffolding approach was higher than students who acquired teaching under direct approach.

Data analysis showed that there was no interaction effect between learning approaches (scaffolding metacognitive and the direct) and students’ prior mathematical ability (high, middle, and low) toward the enhancement of students’ mathematical communication. Statistically, the absence of the interaction effect was as result of: (1) there was no difference enhancement significantly of students’ mathematical communication ability who obtained teaching under metacognitive scaffolding approach among low, middle, and high mathematical ability students, and (2) there was no difference enhancement significantly of students’ mathematical communication ability who obtained teaching under direct approach among low, middle, and high mathematical ability students. However, the result also showed that there was an enhancement of mathematical communication ability at high ability students who obtained teaching under metacognitive scaffolding approach.

By metacognitive scaffolding approach, a teacher posed metacognitive questions when students had difficulty on understanding or solving the problem. The questions posed by the teacher would be used by high ability students to link the mathematical problems encountered with mathematical ideas that will be displayed. This supported the study of King & Roshenshine (1993) who found that guidance through questioning had improved the ability of the problem representation. Meanwhile, Clark et al. (2005) argued that the posing of a problem was a strategy that could develop students’ mathematical communication.

On the teaching under metacognitive scaffolding approach, there were elements of cooperative learning. In cooperative learning, high ability students could explain their mathematical ideas to friends in the group. The activities to explain mathematical ideas were part of the mathematical communication (Carpenter & Gorg, 2000). When dealing with mathematical problems, some high ability students argued their ideas, while in another group, one student explained his idea to friends in his group. Thus, through the cooperative learning, high ability students had an opportunity to evaluate their mathematical ideas and compare it with their friends in their group.

Besides enhancing mathematical communication ability for high ability students, the result of this study also showed that there was a significant enhancement of student mathematical communication ability for low and middle ability students who obtained teaching under metacognitive scaffolding approach. On teaching under this approach, low and middle ability students had support from high ability students. The support was obtained through discussions, explanations, and examples of mathematical representation. By paying attention, reflect, and ask about mathematical ideas shown high ability students, low and middle ability students faced a cognitive conflict between what they would develop and what was submitted by the high ability student. Yackel, Cobb, & Wood’s (1993) reported that the difference of idea between group members was an opportunity to learn.

Interesting finding in this study was that the low and middle ability students who obtained teaching under metacognitive approach got benefit as much as high ability students who obtained teaching under the same approach, particularly with regard to the enhancement of the mathematical communication. This finding could also be interpreted in the context of the theory of Piaget and Vygotsky. In terms of the theory of cognitive development (Piaget, 1970), metacognitive scaffolding approach played an important role in improving the cognitive development of low ability students. Cognitive conflicts on high ability students might be initiated by teacher’s metacognitive questions that impacted on tension on the students and that gave a mismatch between what had been understood and the fact that be faced. It result an imbalance (disequilibrium) condition in the cognitive system then they tried to overcome it through mental activities (thinking). In this case, the approach encourages high ability students to overcome the disequilibrium condition by relying on their knowledge and experiences, especially on their success in the tasks of mathematical representation.

For low and middle ability students, cognitive conflicts might be accrued because of the cooperative learning with high student. In cooperative learning, low and middle ability students were faced with a mathematical representation from high ability student. The mathematical representation as often not in accordance with the ideas developed by the low and middle ability students. To resolve this conflict, low and middle ability students could ask for an explanation from the high ability students, so that a balance in their cognitive system was recovered (re-equilibrium). In teaching under metacognitive scaffolding approach, low and middle ability students did not feel awkward discussed and applied their mathematical ideas to his friends,
including to the high ability students. Effort of low and middle ability students gave their argumentations were encouraged activate their prior knowledge with new mathematical problems. Thus, the discussion could activate of their schemata, so that allow the students elaborate and provide representation of the problem and the solution, either in the form of drawing, diagrams, as well as mathematical sentences. It can be concluded that low-ability students, who are usually weak in the use of learning strategies activation (Golinkoff, 1976; Meichenbaum, 1976; Ryan, 1981) through metacognitive scaffolding approach, their mathematical communication ability was enhanced, such as on the high ability students.

The idea of a cooperative learning approach in teaching under metacognitive scaffolding approach related to zone of proximal development [ZPD] of Vygotsky (1978). Through cooperative learning with students obtained model of representation and solving problems from the high ability students, so that the low and middle ability students were being able to achieve the level of mathematical communication that cannot be achieved without the metacognitive scaffolding approach. Thus, the approach gave low and middle ability students could fully explore their potential capabilities, thus changing the position of the level of potential development into level of actual development, and the level of potential development moving into his new position. It could be predicted that the enhancement of mathematical communication ability of high ability students who obtained teaching under metacognitive scaffolding approach triggered by readiness of their mathematical knowledge and metacognitive assistance from their teacher, while the low and middle ability students were triggered by their interaction with high ability students.

In teaching under direct approach, high ability students showed an enhancement of their mathematical communication ability. This enhancement seemed to be related to their readiness of mathematical knowledge and learning experiences, particularly with regard to the mathematical communication tasks. In addition, this enhancement was apparently due to an explanation of mathematical concept and representation of solving problems. Explanation of mathematical concepts through illustrations that were easy to be understood and providing representation of solving problems step by step, would give a positive effect on mathematical communication of high ability students, as Rosenshine & Stevens (1986), stated that if teachers prepare the new material carefully then the students would be able to process it in the proper order and no parts were missing.

From the above, there were different aspects of teaching under metacognitive scaffolding approach with teaching under direct approach applied to high ability students. If in the scaffolding metacognitive approach, there was an aspect of metacognitive assistance, then in the direct approach, there were aspects of a concept explanation and an example of representation of solving a problem. Thus it was predicted that the metacognitive assistance on teaching under metacognitive scaffolding approach on the one hand, and the readiness of a concept explanation and the example on solving a problem on teaching under direct approach on the other hand were factors that could explain one of the results of this study, which was no difference of enhancement of mathematical communication ability between high ability students who obtained teaching under metacognitive scaffolding approach and those who obtained teaching under direct approach. This evidence suggested that teaching by applying principles of constructivism was not always better compared to teaching by applying principles of behaviorism. Conversely, teaching by applying principles of behaviorism is not always worse compared to teaching by applying principles of constructivism.

For low and middle ability students who obtained teaching under direct approach, the research result showed that there was significant enhancement of students’ mathematical communication ability. Explanation of mathematical concepts through illustrations that were easy to be understood and providing representation of solving problems step by step, would give a positive effect on mathematical communication of low ability students or in terms of Ausubel (in Ivie1998) referred to as meaningful cognitive structure. Explanation of the concept and representation of solving problem were suspected as factors in determining the enhancement of mathematical communication of low and middle students that obtained teaching under direct approach. However, although apparently students might understand only on the specific problems. Heibert & Wearne (1986) stated that learning with a direct approach can bring students learn procedures by rote, so that they were notable to solve new problems. Therefore, it could be indicated that the presence of cooperative learning in teaching under metacognitive scaffolding approach on the one hand, and the explanation of concepts and the examples of representation of solving problems on the other hand are factors that could explain one of the results of this study, that was the enhancement of mathematical communication ability of low and middle ability students who obtained teaching under metacognitive scaffolding approach higher than those who obtained teaching under direct approach.

These study results were only based on specific aspects of mathematical ability, the subject is limited, and the narrow subject matter. Even so, it was clear that the approach was effective in supporting students’ mathematical communication ability. In addition, the implementation of learning with metacognitive scaffolding approach did not require expensive. Therefore, this learning approach should be tested in other aspects of mathematical ability, other topics or other subject matter.
REFERENCES


USING DYNAMIC GEOMETRY SOFTWARE TO SOLVE CHALLENGING PROBLEM IN GEOMETRY

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Article Info

ABSTRACT

This paper discusses issues concerning the using of dynamic geometry software (DGS) to solve problem solving with extensions or variation called challenging problem in geometry. The using of DGS in the regular mathematics classroom has the potential of enriching and furthering the learning process. DGS can make learning geometry meaningful compare to the static objects drawn on paper. Some misconceptions can also be corrected in this kind of learning environment and precise drawings can be produced in a very short time with high accuracy. The graphic and numerical capabilities of DGS provide a rich new mathematical environment in which geometry user can experiment with shapes and relations. This exploratory experience offers promise of building strong geometric intuition and conjecturing spirit that is essential for problem solving and theory building in any branch of mathematics. DGS may support heuristic approaches to problem solving and to the study of problem solving aimed at making problem solving meaningful, interesting, and instructive to mathematics students at all levels and it is hoped be able to provide valuable underpinnings for work in more advance mathematics.

Keywords:
Dynamic geometry software
Challenging problem
Geometry

1. INTRODUCTION

Various studies indicate that computer technology has shown its role in improving students’ learning achievement and adding new strategies and approaches of teaching and learning. However, certainly computer is not a cure for all the problems of education, particularly mathematics [1]. We can’t expect a lot of enhancing students’ achievement if computer facilities are inadequate in number. Suitability of the material with a computer program, teacher skills in using computer software, and proper training and knowledge by teachers who will use the computers in the classroom are important factors in learning with computer software.

There are some software used in teaching and learning. One of the software which is widely used in the teaching of mathematics is a dynamic geometry software (DGS). Measuring with DGS tools allowed students to go deeper in their investigation. Students talked about geometric relationships that they had not discovered when using paper and pencil diagrams [2]. Exploration of using the software gave a good effect in solving of problem [3].

Problem solving has been the focus of learning in elementary education for several decades. In addition, NCTM also see that problem solving can be used as a tools to build students’ understanding of mathematical concepts [4]. Therefore, activities of problem solving using DGS presented in this article be able to be used as a reference of problem solving process.

2. RESEARCH METHOD

This paper is a literature review. The first section discuses about dynamic geometry software and its role in teaching and learning geometry. Next section discuses problem solving and how to solve problem in problem solving classroom. A problem and its solution is discussed as illustration how to use challenging geometry problem.
3. RESULT AND ANALYSIS

3.1. Dynamical Geometry Software

Dynamical Geometry Software (DGS) is a term used to describe a certain type of software which is used for the construction and analysis of tasks and problems in elementary geometry [5, 6]. DGS is one of the three major software pillars of technology-based mathematics education [7] and it is one of the most widely used software for mathematics in the education [8]. DGS lets the user to create complex geometric constructions or objects interactively. Precise constructions can be produced in a very short time with high accuracy.

According to [2] DGS developers intend to provide a new learning media in learning geometry, DGS has a feature which has the same role with compass and straightedge. In addition, user can measure and drag graphic in a screen of computer with DGS. It can be used as a tools of doing exploration in solving problem-solving in geometry.

Exploration activities can also be used to prove various mathematical postulate or theorem. Students' ability to acquire, analyze, measure, and compare several examples of mathematical arguments in DGS give them the opportunity to make conjectures and test a proposition [9]. Moreover, according to [10] DGS give opportunity to students to learn with directions given by the teacher. Students can acquire a new problem and the solution.

Positive results in the application of the use of DGS influenced of geometry teaching and learning in several countries. There is an increasing emphasis on implementation and use of computer software in mathematics teaching [11]. However, there are several factors that must be considered in order to regular DGS use will probably take place in the classroom. These factors are selection and processing of appropriate material, training of teachers on how to make use of it, and alignment of the developed teaching sequences with regular teaching conditions [12]. It is important that there will be no change in the geometry teaching and learning without investment of time and concepts to understand the software [13].

3.2. Problem Solving

Defining the terms problem is an important issue before further discussion of problem solving. Problem is the gap between expectations with reality. However, defining the problem in mathematical is difficult. The difficulty with defining the term problem is that problem solving is relatively [14]. The same tasks that call for significant efforts from some students may well be routine exercises for others, and answering them may just be a matter of recall for a given mathematician. So that a problem is not an inherent nature of a mathematical task or activity.

Sometimes a simple problem can be used in the learning activities and be able to be presented as a challenging problem for some students. However, some educators used a problem with a high degree of difficulty in teaching. According to [15] it is something that is fair and very natural because educators think that they should challenge their students in learning problem solving.

In this study the authors use the term challenging problem. In essence these challenging problem is same as with the ill-structured problem in problem-based learning or open problem in open-ended approach. So the problem presented in this article is an open problem that is a problem that has several possible ways to get it done and have some possible true answers [16].

According to [17] the best way to understand the solution of a problem is to write a new problem based on the concepts learned in the previous problem solving. Some problems can be solved in this way. In general G. Polya describes four basic stages in problem solving. These steps are understand the problem, devising a plan, carrying out the plan, and looking back [18].

The steps of problem solving presented by Polya is a general problem-solving activities. In particular, a mathematical problem can be solved by translating words to mathematical expression, executing, and test results in the initial equation [19]. However, [20] also expressed some activities that can be done in solving the problem, namely by looking for patterns, draw a figure, formulate an equivalent problem, modify the problem, choosing an effective notation, exploit symmetry, divide into cases, works backward, argue by contradiction, pursue parity, consider extreme cases, and generalize.

There are eleven important problem solving strategy or tools in solving a problem, namely the find and use a pattern, act it out, build a model, draw a picture or diagram, make a table and/or a graph, write a mathematical sentence, guess and check, or trial and error, account for all possibilities, solve a simpler problem, or break the problem into parts, work backward, break set, or change point of view [1]. Sometimes one strategy may led to solve a problem. However, the combination of several techniques required to solve more complex problems. Successful solutions of problems are dependent on the learner not only having the knowledge and skills required but also being able to tap into the relevant networks and structures in the mind [21].

3.3. DGS Use in Problem Solving

This section will discussion a geometry problem and an illustration how to solve it.
Given an angle $\angle BAC$. Line $l_1$ is bisector $\angle BAC$ with $0 < m(\angle BAC) < \pi$ and point $D$ in side it as shown in Figure 1. Construct a circle inscribed in the angle $BAC$ and through the point $D$!

**Procedures of Problem Solving I**

Based on the problems presented is known that the center of the circle must lie on the line $l_1$. First of all, of course we can draw a lot of circles centered on the line $l_1$ and inscribed in the angle $BAC$. We can draw a circle centered at $F$ and intersect line $AC$ at $E$ as shown in Figure 2, then we can draw line $l_3$ from point $A$ through point $D$ and so intersect a circle centered at $F$ on point $G$. The next step is drawing the line $l_4$ through the points $F$ and $G$. We can draw a parallel line $l_4$, namely $l_5$ and the line through point $D$. Point $H$ is intersection point between the lines $l_1$, $l_3$, and $l_5$ and point $H$ is one of the central point of the circle as the solution of the problem solving.

**Procedures of Problem Solving II**

The solution of problem solving in this procedure is obtained as a result of analyzing the pattern of points. Pattern of points is a pattern point of intersection between a circle centered at $l_1$ with a line through the center of the circle and perpendicular with line $AC$. Based on Figure 3 we can see that the pattern of points form a curve.

In CaRMeta there is *Conic Section tool* can be used to create a curve through the five points. Therefore, the next step is to draw any circle centered at line $l_1$, as shown in Figure 4.
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Figure 4

The next step is to create line $l_2$ to $l_6$. Line $l_2$ to $l_6$ through the points E, F, G, H, and I and perpendicular to the line AC. The line $l_2$ to $l_6$ intersect the circle at point J, K, L, M, and N. The points J, K, L, M, and N are linked by using Conic Section tool. These five points are turned to form curves as shown in Figure 5.

Figure 5

The next activity is to draw intersection between the curve formed by the line AC. We call the point formed by point O. Point O is used as the point through perpendicular line to the line AC and we call the line with line $l_7$. Line $l_7$ intersect $l_1$ at the point P as shown in Figure 6.

Figure 6

Point P is estimated as a point formed a circle as the solution of the problems presented. Therefore, we can form a circle centered at the point P and through O. Figure 7 shows that the circular was also formed through the point D. This can be checked by Lies on a line tools contained in the CaRMetal.
The next step is to check whether there is a point of tangency between the circle centered at P and line AB. *Intersection tool* can be used as a tool in the investigation of the existence of the tangent point in question. Figure 8 shows that there is a point Q which is the point of intersection between circle and line AB. Circle centered at the point P is a solution of the problem. It is indicated that the circle centered at $l_1$ through point D, and inscribed in the angle BAC.

**4. CONCLUSIONS**

Computer technology is not a cure for all for every problem in education so education problem, especially mathematics cannot be solved by it. Nevertheless, the developers of DGS packages intend to provide new instructional tools for the study of geometry so it be able to help learning activities and facilitate students in understanding a concept. We have to place enough computer in a classroom if we expect improvement. Improvement will not come even if every child has her or his own computer. Along with the technology must come appropriate use of the computer, effective programs that take advantage of the computers, and proper training and knowledge by teachers who will use the computers in the classroom.

*Dynamic geometry software* widely used in mathematics learning have some of the advantages. One of the advantages of configuration software menus is that it can solve certain problem while others are made impossible. The ability of a student to obtain, analyze, measure and compare the many instances of a mathematical proposition in a dynamic geometry environment gives him/her opportunities to make conjectures and test a proposition. These roles for dynamic geometry software are widely acknowledged as having the potential to enrich the teaching of geometry.

The use of software both in learning geometry and algebra should be maintained on a regular teaching conditions. An appropriate number of hours or equal portion of computer using in learning activities is expected to motivate teachers to be able to use the software in teaching their students. Adequate of computer facilities is a factor supporting learning activities with computer be able to be done. In addition, training of the use of software for teachers is also a factor so role of DGS can be maximized in improving the quality of learning activities and facilitate students in learning and understanding the concepts in geometry.
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ENHANCING STUDENTS’ MATHEMATICAL REASONING IN JUNIOR HIGH SCHOOL THROUGH GENERATIVE LEARNING

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ABSTRACT

This paper will discuss the result of a research on the enhancement of students’ mathematical reasoning under generative learning model, viewed from student ability. This research is quasi experiment with pre test-post test control group design. The population consists of all students in a Junior High School in Bandung. The sample comprises 7-grade students from 2 groups, which forms experimental group and control group. Both groups are given pretest and post test on mathematical reasoning, and the experiment group was also given questionnaire. To find the result of the research, statistical analyses have been utilized. The finding indicates that the students’ enhancement on mathematical reasoning under generative learning model is much better than those who have been given conventional teaching. Viewed from the quality of their enhancement, it can be observed that their enhancement falls into middle category. The students who are categorized as high-level students have high enhancement in terms of mathematical reasoning ability, whereas the students who have middle-level ability show middle enhancement. The students who work under generative learning model have positive attitude towards mathematics, generative learning model and mathematical reasoning test.

Keywords:
Generative teaching models, mathematical reasoning skills

1. INTRODUCTION

Based on National Council of Teachers of Mathematics (NCTM, 2000), the vision of mathematics teaching has the following formulation: mathematical communication, mathematical reasoning, mathematical problem solving, mathematical connections. If we refer to this description, it is obvious that mathematical reasoning is one of important standard in mathematics teaching, which should be developed for students understanding.

The question is “How to enhance students’ mathematical reasoning?” As a matter of fact, reasoning is related to formal objectives and connected to students’ daily life. Mathematical reasoning is very important in order to understand mathematics in almost all levels. This also means that all mathematical problems which are based on problem solving will depend on the students’ understanding and ability in reasoning.

There are some efforts that can be carried out for improving the quality of teaching and learning by creating the situation of learning more conducively and constructively. One of the learning process that can be applied for this purposes is generative learning, a model of teaching which was introduced by Osborne and Wittrock in 1985. This model of teaching is based on constructivism, which gives emphasis on the integration actively new knowledge and the previous students’ knowledge. The use of this model of teaching can foster students to work actively, particularly in constructing their own knowledge. The main contributive aspects of generative teaching model is that teacher has a central role in designing teaching situation and managing the content of teaching material to create interesting learning (Grabowski, 2001).

Basically generative learning comprises 4 phases: orientation, challenge and restructuring, application, and evaluation. All this phases encourage students to construct their own knowledge. In generative learning students are free to raise questions and to present ideas and problems, so learning mathematics would...
be even effective and meaningful (Wittrock, 1992). Marrison (2011) emphasizes that “generative learning models are those that require learners consciously and deliberately to relate new information to existing knowledge”.

Theoretically, the phases of generative learning model can develop student skills related to mathematical reasoning ability. This can be obtained in application and challenge phase, as students will be encouraged to predict answer and process, give explanation by using pictures, facts, or even the relationship in solving problems, where in these phases students have to demonstrate their logical arguments and draw a conclusion.

Based on the above discussion, a research on the application of generative learning model in enhancing students’ mathematical reasoning has been conducted, taking the title: “The application of Generative Learning Model for enhancing junior high school students’ mathematical reasoning”.

2. RESEARCH METHOD

In this study, a quasi experimental design with pre-test post test control group design was used to determine the effect of generative learning model toward students’ mathematical reasoning.

2.1 Subjects

The population in this research is students of Junior High School 47 Bandung, West Java. The sample, selected randomly, consists of 2 groups, in which the first group was given generative learning (experimental group), and the second group was given conventional one. The experimental one is grouped into 3 categories (high, middle, low) based on their daily achievement in mathematics.

2.2 Instruments

To measure students’ ability in mathematical reasoning under the implementation of generative learning, some instruments of test were used in the research. In construction phase of mathematical reasoning test, an essay form is designed to observe students’ way of thinking. In constructing the test, a blue print covering basic competence, indicators, item test, accompanied by its solution and the answer key of the test.

Score are distributed to each level of competency of reasoning, based on Thomson (2006): 4 (when the solution is completely correct; 3 (when the solution is partly incorrect with 1 mistake); 2 (when the solution is partly incorrect with more than one mistake); 1 (when the solution is not complete with at least 1 correct argument; 0 (when the solution is totally incorrect or no answer is given).

To measure the level of students’ enhancement in reasoning ability, the following formulation is used:

\[ \text{Normalized Gain (g)} = \frac{\text{Post test score} - \text{Pre test score}}{\text{Maximal ideal score} - \text{Pre test Score}} \]

The resulted gain is then interpreted by using the following criteria Hake (Meltzer, 2002):

<table>
<thead>
<tr>
<th>Gain (g)</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 - 1.0</td>
<td>High</td>
</tr>
<tr>
<td>0.3 - 0.7</td>
<td>Middle</td>
</tr>
<tr>
<td>0.0 - 0.3</td>
<td>Low</td>
</tr>
</tbody>
</table>

3. RESULT AND ANALYSIS

The data obtained in this research are based on the result of pre test and post test score. These tests are used to find out the quality level of students’ enhancement in mathematical reasoning ability. The program of SPSS version 16.00 for Windows for analyzing the data has been utilized.
Table 1

Mean and Standard Deviation of Mathematical Reasoning Ability

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Statistic</th>
<th>Mathematical Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pretest</td>
</tr>
<tr>
<td>Conventional</td>
<td>33</td>
<td>$\bar{x}$</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>1.42</td>
</tr>
<tr>
<td>MPG</td>
<td>34</td>
<td>$\bar{x}$</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>1.62</td>
</tr>
</tbody>
</table>

Note: Maximal Ideal Score is 12.

The students’ score in post tests are 6.94 (conventional group) and 8.26 (MPG group), which means that these two results are not so high, compared to the maximal ideal score 12, although the achievement of MPG group looks higher than that of conventional group. The students’ enhancement in mathematical reasoning ability under generative learning model reached 0.61 (middle level); whereas the students’ enhancement in this aspect under conventional model was 0.44 (middle level), which is statistically (after t-test is used) lower than the above result.

Based on the independent sample t-test, it can be shown that the mean values of both students’ mathematical knowledge under generative model and conventional model are not significantly different. In this research, the t-test was conducted after normality and homogeneity test had been successfully commenced.

With $\alpha = 0.05$, it has been obtained that 1-tailed test gives significant value of 0.001 which is less than 0.05. This means that the students’ enhancement of mathematical reasoning who learn mathematics under generative model is higher than that of students who learned mathematics under conventional model.

Table 4.12

Students’ Mathematical Reasoning under Generative Model based on the Category of Students’ Skills

<table>
<thead>
<tr>
<th>Mathematical Prior Knowledge</th>
<th>Statistic</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(n) Pre Test</td>
<td>Post Test</td>
</tr>
<tr>
<td>High</td>
<td>9</td>
<td>4.90</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>1.05</td>
</tr>
<tr>
<td>Middle</td>
<td>16</td>
<td>3.30</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.79</td>
</tr>
<tr>
<td>Low</td>
<td>9</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Note: Ideal maximal Score is 12.

From the above table, after further analysis using t-test, it can be shown that the students in high-level group earned excellent reasoning enhancement (0.9), the student’s from middle-level group earned middle level reasoning enhancement (0.6), and the student’s from low level group earned low level reasoning enhancement (0.3). By using 1-way ANOVA, with significant level of $\alpha = 0.05$, it was found that the students’ mathematical reasoning skills under generative learning is higher than those of students who learned mathematics under conventional model. This result indicates that reasoning skills can be strengthened by generative model of teaching as this model allows students to construct their own knowledge, interpretation on the phenomena they encounter, and they are encouraged to explain the procedure and the deductive steps in each part of solutions. The students who are in high-level group obtained excellent reasoning skills. It can be understood easily, as students who have good capacity, will be much easier to understand and interpret the concepts. In addition, they
are rich with concepts and procedures they needs in dealing with the problems. Students’ brain which is not a blank slate actively record all important information, facts and procedures, and retrieved them when students needs these tools in handling the problems. Based on the Scheffe test, it is obvious that the students’ gain of mathematical reasoning skill, viewed from student level of skills (high, middle, and low level) is higher than that of students from middle and low level group; and students from middle level have better reasoning mathematical skills compared to the mathematical reasoning skills of low-level group of students. All these results gives information that the higher the level of students’ skill, the better their mathematical reasoning skills would be.

4. CONCLUSION

Based on the data analysis and the previous discussion, the following conclusions are resulted:

1. The students’ enhancement of mathematical reasoning skills under generative model of learning are higher than those of students under conventional model of learning. These two groups have the same level: middle level.

2. Viewed from student level of skills (high, middle, low), the students’ enhancement of mathematical reasoning skill is higher than that of students from middle and lower level group; and student from middle level have better reasoning mathematical skills compared to students from the lower-level group.

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